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# Multi Node Tandem Queuing Model with Binomial Bulk Size Distribution Having Load Dependent Service

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***Abstract:*** ***In this article we study a multi node tandem queuing model consisting of K-nodes in which the customers arriving in batches to the first queue and after receiving service they will be directed with some node specific probability to join any one of the (K-1) parallel queues which are connected to first queue in series and exit from the system after getting service. It is assumed that the arrival and service completions follow Poisson processes and service rates depend on number of customers in the queue connected to it. Here the bulk arrivals are assumed to be Binomially distributed. Using difference differential equations the joint probability function is derived and performance measures such as average number of customers, waiting time of customer, throughput of each service station, utilization of each server, variance of number of customers in each queue are derived explicitly. A numerical illustration is provided to understand the theoretical results. Sensitivity analysis of the system behavior with regards to the arrival rates and load dependent service distribution parameters is carried out. A comparison between transient and study state behavior is also done .***

***Key words: Poisson Process, Bulk arrivals, Binomial Distribution, Forked queuing model, Load dependent service rates, Performance measures***.

## I. INTRODUCTION

In recent works the authors analyzed multi node tandem queuing models with bulk arrivals which are distributed Uniformly ([18]) and Geometrically ([19]). In the present paper we are going to study a multi node tandem queuing model with bulk arrivals. The model describes a system in which the bulk arrivals are Binomially distributed. The model consists of K-node series in which the customers arrive at the first node, receive their service there and then directed to (K-1) parallel nodes for various services offered in the system. Such models are called tandem queuing models since the output of first node is the input for the others ([2], [5], [8], [9], [11], [13], [17]).

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Further the arrivals could be in batch to represent natural phenomenon observed in animal population due to their societal nature. Such systems should be able to receive, monitor and control the populations properly by providing the requisite services ([1], [6], [7]).

Beginning with the basic application to telephonic conversations ([10]), applications of queuing theory has gained prominence over the decades ([6], [7]). Further applications in real time situations are data handling in communication networks ([9], [10], [12], [21], [26], [27]), management information systems/ hierarchical systems where instructions are passed on to the subordinates ([1], [13]), production units and transport systems for cargo handling ([16]). When the service time at each node has to be adjusted depending on the number of customers the models are called load dependent systems ([3], [4], [14], [15]), while the systems which spare time irrespective of number of customers are termed as load independent.The present study analyses a multi node tandem queuing model with bulk arrivals that follow a Binomial distribution. The motivation for considering a Binomial distribution is from the following observations. It is well known that the Binomial distribution is applicable for situations that require a specified result such as yes or no ,failure or success, accepted or rejected etc., Some of the situations that attract a Binomial distribution are acceptable or defective in quality test control of inventory, data received correctly or incorrectly in data communications, a customer is satisfied or not at a particular server, service is provided in a given time slot or not etc., A Binomial distribution clearly indicates whether the task assigned /specified is completed or not within the given frame work or setup. This tells upon the performance of the system whether the system is working satisfactorily or not. For some interesting works on queuing models the readers are referred to [25].Recalling the model studied in [17] with Binomial arriving bulk population here, we present the joint probability generating function of the number of customers for each queue and system characteristics such as average number of customers in the queue, probability of idleness of each server, throughput of nodes, average waiting time of a customer in each queue, utilization of each server etc., numerical illustrations are provided and a sensitivity analysis is done. Steady state behavior of the model is also discussed. Our steady here provides new insights in to the theory and enriches the literature.

## II. QUEUEING MODEL WITH GENERAL BULK SIZE DISTRIBUTION

In this section a queuing model with K buffers $B_1, B_2, \dots B_k$ of infinite capacity and K servers $S_1, S_2, \dots S_k$ connected as forked network is considered. It is assumed that the customers arrive in batches to the first q

ueue and after getting service at first server they may join any of the (K-1) queues connected to the servers $S_2, S_3, \dots S_k$ which are parallel and connected to first server in tandem, with some probability i.e., the customers after getting service at $S_1$ in batches may join second buffer with probability $\theta_1$or third buffer with probability $\theta_2$or K$^{th}$ buffer with probability $\theta_{k-1}$. Let us assume that the actual number of customers in any arriving module is a random variable X with probability C(X). Let $\lambda_x$be the arrival rate of batches of size $x$ and $\lambda$ is the composite arrival rate. Then $\lambda = \sum \lambda_x$ . Therefore the arrival process follows a compound Poisson process with arrival rate $\lambda$. Further it is assumed that the service completion in each service station is random and follows a Poisson process with service rates $\mu_1, \mu_2, \mu_3, \dots, \mu_k$ respectively. Here we assume that service rate in each server is linearly dependent on the content of buffer connected to it and queue discipline is first come first serve(FCFS).

Let $P(n_1, n_2, \dots, n_k; t)$ be the probability that there are $n_1$costumers in first queue, $n_2$ customers in second queue and $n_k$ customers in $k^{th}$ th queue at time t. The customers arrive in batches of size X .The probability generating function of X is $C(Z) = \sum_{m=1}^{\infty} C_m z^m$.

Then difference differential equations governing the system are

$$\frac{\partial P}{\partial t}(n_1, n_2, \dots n_k; t) = -\left[\lambda + \sum_{i=1}^{k} n_i \mu_i\right] P(n_1, n_2, \dots, n_k; t) + (n_1+1)\mu_1[\theta_1 P(n_1+1, n_2-1, n_3, \dots, n_k; t) + \theta_2 P(n_1+1, n_2, n_3-1, \dots, n_k; t) + \dots + \theta_{k-1} P(n_1+1, n_2, \dots, n_k-1; t)] + (n_2+1)\mu_2 P(n_1, n_2+1, n_3, \dots, n_k; t) + (n_3+1)\mu_3 P(n_1, n_2, n_3+1, \dots, n_k; t) + \dots + (n_k+1)\mu_k P(n_1, n_2, \dots, n_k+1; t) + \lambda \sum_{m=1}^{n_1} C_m P(n_1-m, n_2, \dots, n_k, t) \quad (1)$$

$$\frac{\partial P}{\partial t}(0, n_2, \dots n_k; t) = -\left[\lambda + \sum_{i=2}^{k} n_i \mu_i\right] P(0, n_2, \dots, n_k, t) + \mu_1[\theta_1 P(1, n_2-1, n_3, \dots, n_k; t) + \theta_2 P(1, n_2, n_3-1, \dots, n_k; t) + \dots + \theta_{k-1} P(1, n_2, n_3, \dots, n_k-1; t)] + (n_2+1)\mu_2 P(0, n_2+1, n_3, \dots, n_k; t) + (n_3+1)\mu_3 P(0, n_2, n_3+1, \dots, n_k; t) \dots + (n_k+1)\mu_k P(0, n_2, n_3, \dots, n_k+1; t) \quad (2)$$

$$\frac{\partial P}{\partial t}(n_1, 0, \dots n_k; t) = -\left[\lambda + \sum_{i=1, i\neq 2}^{k} n_i \mu_i\right] P(n_1, 0, \dots, n_k, t) + (n_1+1)\mu_1[\theta_2 P(n_1+1, 0, n_3-1, \dots, n_k; t) + \dots + \theta_{k-1} P(n_1+1, n_2, \dots, n_k-1; t)] + \mu_3 P(n_1, 0, n_3+1, \dots, n_k; t) + \dots + (n_k+1)\mu_k P(n_1, 0, \dots, n_k+1; t) + \lambda \sum_{m=1}^{n_1} C_m P(n_1-m, 0, \dots, n_k, t) \quad (3)$$

$$\frac{\partial P}{\partial t}(n_1, n_2, \dots, 0; t) = -\left[\lambda + \sum_{i=1}^{k-1} n_i \mu_i\right] P(n_1, n_2, \dots, 0, t) + (n_1+1)\mu_1[\theta_1 P(n_1+1, n_2-1, n_3, \dots, 0; t) + \theta_2 P(n_1+1, n_2, n_3-1, \dots, 0; t) + \dots + \theta_{k-2} P(n_1+1, n_2, \dots, n_{k-1}-1, 0; t)] + (n_2+1)\mu_2 P(n_1, n_2+1, n_3, \dots, 0; t) + (n_3+1)\mu_3 P(n_1, n_2, n_3+1, \dots, 0; t) + (n_k+1)\mu_k P(n_1, n_2, \dots, 1; t) + \lambda \sum_{m=1}^{n_1} C_m P(n_1-m, n_2, \dots, n_{k-1}, 0; t) \quad (4)$$

$$\frac{\partial P}{\partial t}(0, 0, \dots n_k; t) = -\left[\lambda + \sum_{i=3}^{k} n_i \mu_i\right] P(0, 0, \dots, n_k, t) + \mu_1[\theta_2 P(1, 0, n_3-1, \dots, n_k; t) + \dots + \theta_{k-1} P(1, 0, \dots, n_k-1; t)] + \mu_2 P(0, 1, n_3, \dots, n_k; t) + (n_3+1)\mu_3 P(0, 0, n_3+1, \dots, n_k; t) \dots + (n_k+1)\mu_k P(0, 0, n_3, \dots, n_k+1; t) \quad (5)$$

$$\frac{\partial P}{\partial t}(0, n_2, \dots n_{k-1}, 0; t) = -\left[\lambda + \sum_{i=2}^{k-1} n_i \mu_i\right] P(0, n_2, \dots, n_{k-1}, 0; t) + \mu_1[\theta_1 P(1, n_2-1, n_3, \dots, n_{k-1}, 0; t) + P(1, n_2+1, n_3, \dots, n_{k-1}, 0; t) + \dots + \theta_{k-2} P(1, n_2, \dots, n_{k-1}-1, 0; t)] + (n_2+1)\mu_2 P(n_1, n_2+1, n_3, \dots, 0; t) + (n_3+1)\mu_3 P(0, n_2, n_3+1, \dots, 0; t) + \dots + (n_k+1)\mu_k P(0, n_2, \dots, 1; t) \quad (6)$$

$$\frac{\partial P}{\partial t}(0, 0, 0, n_4, \dots n_k; t) = -\left[\lambda + \sum_{i=4}^{k} n_i \mu_i\right] P(0, 0, 0, n_4, \dots n_k; t) + \mu_1 \theta_3 P(1, 0, 0, n_4-1, \dots, n_k; t) + \dots + \mu_k \theta_{k-1} P(1, 0, 0, n_4, \dots, n_k-1; t)] + \mu_2 P(0, 1, 0, n_4, \dots, n_k; t) + \mu_3 P(0, 0, 1, n_4, \dots, n_k; t) + \dots + (n_k+1)\mu_k P(0, 0, 0, n_4, \dots, n_k+1; t) \quad (7)$$

$$\frac{\partial P}{\partial t}(0, 0, n_3, \dots, n_{k-1}, 0; t) = -\left[\lambda + \sum_{i=3}^{k-1} n_i \mu_i\right] P(0, 0, n_3, \dots, n_{k-1}, 0; t) + \mu_1[\theta_2 P(1, 0, n_3-1, \dots, 0; t) + \dots + \theta_{k-2} P(1, 0, n_3, \dots, n_{k-1}-1, 0; t)] + \mu_2 P(0, 1, n_3, \dots, n_{k-1}, 0; t) + (n_3+1)\mu_3 P(0, 0, n_3+1, \dots, n_{k-1}, 0; t) + \dots + \mu_k P(0, 1, 0, n_3, \dots, 1; t) \quad (8)$$

$$\frac{\partial P}{\partial t}(n_1, 0, 0, n_4, \dots, 0; t) = -\left[\lambda + \sum_{i=1\neq 2,3}^{k-1} n_i \mu_i\right] P(n_1, 0, 0, n_4, \dots, 0; t) + (n_1+)\mu_1 [\theta_3 P(n_1+1, 0, 0, n_4-1, \dots, 0; t) + \dots + \theta_{k-2} P(n_1+1, 0, 0, \dots, n_{k-1}-1, 0; t)] + \mu_2 P(n_1, 1, 0, n_4, \dots, 0; t) + \mu_3 P(n_1, 0, 1, n_4, \dots, 0; t) + \dots + \mu_k P(n_1, 0, 0, n_4, \dots, 1; t) + \lambda \sum_{m=0}^{n_1} C_m P(n_1-m, 0, 0, n_4, \dots, t) \quad (9)$$

$$\frac{\partial P}{\partial t}(0, 0, \dots, 0; t) = -\lambda P(0, 0, \dots, 0; t) + \mu_2 P(0, 1, 0, \dots, 0; t) + \mu_3 P(0, 0, 1, \dots, 0; t) + \dots + \mu_k P(0, 0, \dots, 0, 1; t) \quad (10)$$

Let $P(Z_1, Z_2, \dots, Z_k; t) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \dots \sum_{n_k=0}^{\infty} p(n_1, n_2, \dots, n_k; t) z_1^{n_1} z_2^{n_2} \dots z_k^{n_k}$ be the probability generating function of $p(n_1, n_2, \dots, n_k; t)$.

Multiplying equations (1) to (10)with probability generating function and summing over $n_1, n_2, \dots, n_k$ from 0 to $\infty$ we get the Joint Probability generating function of number of customers in first, second,…, k$^{th}$ queues respectively at any time t as

$$P(Z_1, Z_2, \dots, Z_k; t) == \exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} \sum_{r_3=0}^{r_2} \dots \sum_{r_k=0}^{r_{k-1}} (-1)^{r_2} \binom{m}{r_1} \binom{r_1}{r_2} \binom{r_2}{r_3} \dots \binom{r_{k-1}}{r_k} C_m \left\{(z_1 - 1) + \frac{\theta_1 \mu_1 (z_2 - 1)}{\mu_2 - \mu_1} + \frac{\theta_2 \mu_1 (z_3 - 1)}{\mu_3 - \mu_1} + \dots + \frac{\theta_{k-1} \mu_1 (z_k - 1)}{\mu_k - \mu_1}\right\}^{r_1 - r_2}\right.$$



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$$\left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_2-r_3}..\left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_3-r_4} \quad (11)$$

## III . CHARACTERISTICS OF THE MODEL

Putting $z_1=0, z_2=0,\dots,z_k=0$ in (11) and expanding we get the probability that the
k-server system is empty at any time t as

$$P(0,0,\dots,0;t)=$$

$$\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\dots\sum_{r_k=0}^{r_{k-1}}(-1)^{r_2+r_1}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}..\binom{r_{k-1}}{r_k}C_m\right.$$

$$\left(1+\frac{\theta_1\mu_1}{\mu_2-\mu_1}+\frac{\theta_2\mu_1}{\mu_3-\mu_1}+..+\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_1-r_2}\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_3-r_4}\dots\left(\frac{\theta_{k-2}\mu_1}{\mu_{k-1}-\mu_1}\right)^{r_{k-1}-r_k}\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^{r_2-r_3}\left(\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_k}$$

$$\left.\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+..+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+..+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k}\right\}\right] \quad (12)$$

## III. PERFORMANCE ANALYSIS

Using the joint probability generating function of number of customers in each queue the performance of the model is analysed by deriving explicit expressions for system characteristics such as mean number of customers in each queue, probability of emptiness of each queue, utilization of each server, average waiting time of a customer in each queue, variance and coefficient of variation of number of customers in each queue.

## IV. PERFORMANCE ANALYSIS OF FIRST QUEUE

Putting $z_2=1, z_3=1,\dots,z_k=1$ in (11) we get probability generating function of first queue size as

$$P(Z_1;t)=\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}C_m\binom{m}{r_1}(z_1-1)^{r_1}\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (13)$$

Mean number of customers in first queue is

$$E(N_1)=L_1(t)=\frac{\lambda}{\mu_1}(1-e^{-\mu_1 t})E(X) \quad (14)$$

Where E(X) is the mean of batch size arrivals to first queue given by $E(X)=\sum_{m=1}^{\infty}m.C_m$

Putting $Z_1=0$ in (13) we get the probability that the first queue is empty as

$$P(0,\dots,\dots;t)=\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}(-1)^{r_1}C_m\binom{m}{r_1}\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (15)$$

Utilization of first server is $U_1(t)=1-P(0,\dots,\dots;t)$

$$=1-\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}(-1)^{r_1}C_m\binom{m}{r_1}\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (16)$$

Throughput of first server is $\text{Thp1}(t)=\mu_1.U_1(t)$

$$=\mu_1\left\{1-\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}(-1)^{r_1}C_m\binom{m}{r_1}\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\} \quad (17)$$

Average waiting time of a customer in first queue is

$$W_1(t)=\frac{L_1(t)}{Thp_1(t)}=\frac{\left(\frac{\lambda}{\mu_1^2}\right)(1-e^{-\mu_1 t})E(X)}{\left\{1-\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}(-1)^{r_1}C_m\binom{m}{r_1}\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\}} \quad (18)$$

Variance of number of customers in first queue is

$$V(Z_1)=V_1(t)=\lambda\sum_{m=2}^{\infty}\left[\binom{m}{2}.C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right] \quad (19)$$

Coefficient of variation of the number of customers in first queue is

$$CV_1(t)=\frac{\sqrt{V_1(t)}}{L_1(t)}x100=\frac{\sqrt{\lambda\sum_{m=2}^{\infty}\left[\binom{m}{2}.C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right]}}{\frac{\lambda}{\mu_1}(1-e^{-\mu_1 t})E(X)}x100 \quad (20)$$

## V. PERFORMANCE ANALYSIS OF $i^{th}$ QUEUE FOR i =2,3,..,k.

Putting $z_1=1, z_2=1, z_3=1,\dots,z_{k-1}=1$ in (11) we get the probability generating function of $i^{th}$ queue size distribution

$$P(Z_i;t)=$$

$$\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_1}\binom{m}{r_1}\binom{r_1}{r_i}C_m\left\{\frac{\theta_{i-1}\mu_1(z_i-1)}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (21)$$

Mean number of customers in $i^{th}$ queue is $E(N_i)=L_i(t)$

$$=\left(\frac{\lambda\theta_{i-1}}{\mu_i}\right)\left[1-\left\{\frac{\mu_i e^{-\mu_1 t}-\mu_1 e^{-\mu_i t}}{\mu_i-\mu_1}\right\}\right]E(X) \quad (22)$$

Where E(X) is the mean of batch size arrivals to first queue and is given by $E(X)=\sum_{m=1}^{\infty}m.C_m$

Putting $z_i=0$ in (21)we get the probability that the $i^{th}$ queue is empty as

$$P(.,.,0,..;t)=$$

$$\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_1+r_i}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (23)$$

Utilization of $i^{th}$ server is $U_i(t)=1-P(.,..,0,.;t)=$

$$\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_1+r_i}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (24)$$

Throughput of $i^{th}$ server is $Thp_i(t)=\mu_i.U_i(t)=$

$$\mu_i\left|1-\right.$$

$$\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_1+r_i}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right]\Bigg| \quad (25)$$

Average waiting time of a customer in $i^{th}$ queue is

$$W_i(t)=\frac{L_i(t)}{Thp_i(t)}$$

$$=\frac{\left(\frac{\lambda\theta_{i-1}}{\mu_i^2}\right)\left[1-\left\{\frac{\mu_i e^{-\mu_1 t}-\mu_1 e^{-\mu_i t}}{\mu_i-\mu_1}\right\}\right]E(X)}{\left[1-\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_1+r_i}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right]\right]} \quad (26)$$

Variance of the number of customers in $i^{th}$ queue is

$$V(z_i)=V_i(t)=$$

$$\lambda\left[\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)^2\sum_{m=1}^{\infty}\binom{m}{2}.C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_i+\mu_1)t}}{\mu_i+\mu_1}\right)+\left(\frac{1-e^{-2\mu_i t}}{\mu_i}\right)\right\}+\right.$$

$$\left.\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_i t}}{\mu_i}\right)\right\}E(X)\right] \quad (27)$$

Coefficient of variation of the number of customers in $i^{th}$ queue is

$$CV_i(t)=\frac{\sqrt{V_i(t)}}{L_i(t)}x100=$$

$$\frac{\sqrt{\lambda\left[\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)^2\sum_{m=1}^{\infty}\binom{m}{2}C_m\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_i+\mu_1)t}}{\mu_i+\mu_1}\right)+\left(\frac{1-e^{-2\mu_i t}}{\mu_i}\right)\right\}+\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_i t}}{\mu_i}\right)\right\}E(X)\right]}}{\left(\frac{\lambda\theta_{i-1}}{\mu_i}\right)\left[1-\left\{\frac{\mu_i e^{-\mu_1 t}-\mu_1 e^{-\mu_i t}}{\mu_i-\mu_1}\right\}\right]E(X)}x100 \quad (28)$$

## IV. PERFORMANCE MEASURES OF THE MODEL WHEN BATCH SIZE DISTRIBUTION IS BINOMIAL

Let $P(n_1,n_2,\dots,n_k;t)$ be the probability that there are $n_1$costumers in first queue, $n_2$ customers in second queue and $n_k$ customers in $k^{th}$ th queue at any time t. It is assumed that the customers arrive in batches of size 'm' and the probability generating function is $C(Z)=\sum_{m=1}^{\infty}C_m z^m$.The performance of the model is influenced by the batch size arrival distribution. It is assumed that the customers in any arriving module is random and follows Binomial distribution with parameters A and p.

Which means the number of customers in a batch has Binomial distribution with parameter p. Then probability mass function of Zero Truncated Binomial batch size distribution is $C_m = \frac{\binom{A}{m} p^m q^{A-m}}{1-q^A}$ , $m = 1,2,\ldots,A$ and $0 < p < 1$, where $p+q=1$. The mean of Binomial distribution is $E(X) = \frac{Ap}{1-q^A}$.

Then the Joint Probability generating function of number of customers in first, second,…, k^th^ queues respectively at any time t is

$$P(Z_1,Z_2,\ldots Z_k;t) = \exp\Bigg[\lambda \sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\ldots\sum_{r_k=0}^{r_{k-1}}(-1)^{r_2}\binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\ldots\binom{r_{k-1}}{r_k}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{(z_1-1)+\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}+\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}+\ldots+\frac{\theta_{k-1}\mu_1(z_k-1)}{\mu_k-\mu_1}\right\}^{r_1-r_2}\left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_2-r_3}\left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_3-r_4}\cdots\left\{\frac{\theta_{k-2}\mu_1(z_{k-1}-1)}{\mu_{k-1}-\mu_1}\right\}^{r_{k-1}-r_k}\left\{\frac{\theta_{k-1}\mu_1(z_k-1)}{\mu_k-\mu_1}\right\}^{r_k}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\ldots+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\ldots+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k}\right\}\Bigg] \quad (29)$$

## IV. CHARACTERISTICS OF THE MODEL

Putting $z_1=0, z_2=0, \ldots, z_k=0$ in (29) and expanding we get the probability that the k-server system is empty at any time t as $P(0,0,\ldots 0;t)=$

$$\exp\Bigg[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\ldots\sum_{r_k=0}^{r_{k-1}}(-1)^{r_2+r_1}\binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\ldots\binom{r_{k-1}}{r_k}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left(1+\frac{\theta_1\mu_1}{\mu_2-\mu_1}+\frac{\theta_2\mu_1}{\mu_3-\mu_1}+\ldots+\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_1-r_2}\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^{r_2-r_3}\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_3-r_4}\cdots\left(\frac{\theta_{k-2}\mu_1}{\mu_{k-1}-\mu_1}\right)^{r_{k-1}-r_k}\left(\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_k}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\ldots+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\ldots+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k}\right\}\Bigg] \quad (30)$$

## IV. PERFORMANCE ANALYSIS OF FIRST QUEUE

Putting $z_2=1, z_3=1, \ldots, z_k=1$ in (29) we get the probability generating function of first queue size as

$$P(Z_1;t) = \exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}\binom{A}{m}\binom{m}{r_1}\left(\frac{p^m q^{A-m}}{1-q^A}\right)(z_1-1)^{r_1}\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (31)$$

Mean number of customers in first queue is

$$L_1(t) = \left[\frac{\lambda}{\mu_1}\right]\left(\frac{Ap}{1-q^A}\right)(1-e^{-\mu_1 t}) \quad (32)$$

Putting $Z_1 = 0$ in (31) we get the probability that the first queue is empty as

$$P(0,\ldots,\ldots;t) = \exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}(-1)^{r_1}\binom{A}{m}\binom{m}{r_1}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (33)$$

Utilization of first server is $U_1(t) = 1 - P(0,\ldots,\ldots;t) =$

$$1-\exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}(-1)^{r_1}\binom{A}{m}\binom{m}{r_1}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (34)$$

Throughput of first server is $Thp1(t) = \mu_1 . U_1(t) =$

$$\mu_1\left\{1-\exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}(-1)^{r_1}\binom{A}{m}\binom{m}{r_1}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\} \quad (35)$$

Average waiting time of a customer in first queue is

$$W_1(t) = \frac{L_1(t)}{Thp_1(t)} = \frac{\left[\frac{\lambda}{\mu_1^2}\right]\left(\frac{Ap}{1-q^A}\right)(1-e^{-\mu_1 t})}{\left[1-\exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}(-1)^{r_1}\binom{A}{m}\binom{m}{r_1}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right]} \quad (36)$$

Variance of number of customers in first queue is

$$V_1(t) = \lambda\sum_{m=2}^{A}\left[\binom{A}{m}\binom{m}{2}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right] \quad (37)$$

Coefficient of variation in number of customers in first queue is

$$CV_1(t) = \frac{\sqrt{V_1(t)}}{L_1(t)}x100 = \frac{\sqrt{\lambda\sum_{m=2}^{A}\left[\binom{A}{m}\binom{m}{2}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right]}}{\left[\frac{\lambda}{\mu_1}\right]\left(\frac{Ap}{1-q^A}\right)(1-e^{-\mu_1 t})}x100 \quad (38)$$

## IV. PERFORMANCE ANALYSIS OF i^th^ QUEUE FOR i =2,3,..k.

Putting $z_1=1, z_2=1,\ldots, z_{i-1}=1, z_{i+1}=1,\ldots, z_k=1$ in ( 29) we get probability generating function of i^th^ queue size distribution as $P(Z_i;t) =$

$$\exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_1}\binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_i}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_{i-2}\mu_1(z_i-1)}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (39)$$

Mean number of customers in i^th^ queue is

$$E(N_i) = L_i(t) = \left(\frac{\lambda\theta_{i-2}}{\mu_i}\right)\left(\frac{Ap}{1-q^A}\right)\left[1-\left\{\frac{\mu_i e^{-\mu_1 t}-\mu_1 e^{-\mu_i t}}{\mu_i-\mu_1}\right\}\right] \quad (40)$$

Putting $Z_i = 0$ in (39) we get probability that the i^th^ queue is empty as $P(\ldots,0,\ldots;t) =$

$$\exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}\binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_i}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_{i-2}\mu_1}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (41)$$

Utilization of i^th^ server is

$$U_i(t) = 1-P(\ldots,\ldots,0;t) = 1-\exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}\binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_i}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_{i-2}\mu_1}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (42)$$

Throughput of i^th^ server is $Thp_i(t) = \mu_i . U_i(t) =$

$$\mu_i\left\{1-\exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}\binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_i}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_{i-2}\mu_1}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right]\right\} \quad (43)$$

Average waiting time of a customer in i^th^ queue is

$$W_i(t) = \frac{L_i(t)}{Thp_i(t)} = \frac{\left[\left(\frac{\lambda\theta_{i-2}}{\mu_i^2}\right)\left(\frac{Ap}{1-q^A}\right)\left[1-\left\{\frac{\mu_i e^{-\mu_1 t}-\mu_1 e^{-\mu_i t}}{\mu_i-\mu_1}\right\}\right]\right]}{\left[1-\exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}\binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_i}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_{i-2}\mu_1}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1-e^{-(\mu_1(r_1-r_i)+\mu_i r_i)t}}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right]\right]} \quad (44)$$

Variance of number of customers in i^th^ queue is

$$V_i(t) = \lambda\left[\left(\frac{\theta_{i-2}\mu_1}{\mu_i-\mu_1}\right)^2\sum_{m=1}^{A}\binom{A}{m}\binom{m}{2}\left(\frac{p^m q^{A-m}}{1-q^A}\right).\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_i+\mu_1)t}}{\mu_i+\mu_1}\right)+\left(\frac{1-e^{-2\mu_i t}}{\mu_i}\right)\right\}+\left\{\frac{\theta_{i-2}\mu_1}{\mu_i-\mu_1}\right\}\left(\frac{Ap}{1-q^A}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)-\left(\frac{1-e^{-\mu_i t}}{\mu_i}\right)\right\}\right] \quad (45)$$

Coefficient of variation of number of customers in i^th^ queue is

$$CV_i(t) = \frac{\sqrt{V_i(t)}}{L_i(t)}x100 = \frac{\sqrt{\lambda\left[\left(\frac{\theta_{i-2}\mu_1}{\mu_i-\mu_1}\right)^2\sum_{m=1}^{A}\binom{A}{m}\binom{m}{2}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_i+\mu_1)t}}{\mu_i+\mu_1}\right)+\left(\frac{1-e^{-2\mu_i t}}{\mu_i}\right)\right\}+\left\{\frac{\theta_{i-2}\mu_1}{\mu_i-\mu_1}\right\}\left(\frac{Ap}{1-q^A}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)-\left(\frac{1-e^{-\mu_i t}}{\mu_i}\right)\right\}\right]}}{\left[\left(\frac{\lambda\theta_{i-2}}{\mu_i}\right)\left(\frac{Ap}{1-q^A}\right)\left[1-\left\{\frac{\mu_i e^{-\mu_1 t}-\mu_1 e^{-\mu_i t}}{\mu_i-\mu_1}\right\}\right]\right]}x100 \quad (46)$$

## V. NUMERICAL ILLUSTRATION

For numerical illustration we take k=4 and calculate equations and performance measures.

The Joint Probability generating function of number of customers in first, second, third and fourth queues respectively at any time t is $P(Z_1,Z_2,Z_3,Z_4;t) =$

$$\exp\Bigg[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\sum_{r_4=0}^{r_3}(-1)^{r_2}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\binom{r_3}{r_4}C_m\left\{(z_1-1)+\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}+\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}+\frac{\theta_3\mu_1(z_4-1)}{\mu_4-\mu_1}\right\}^{r_1-r_2}\left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_2-r_3}\left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_3-r_4}\left\{\frac{\theta_3\mu_1(z_4-1)}{\mu_4-\mu_1}\right\}^{r_4}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4}\right\}\Bigg] \quad (47)$$

### A.CHARACTERISTICS OF THE MODEL

Putting $z_1 = 0, z_2 = 0, z_3 = 0, z_4 = 0$ in (47) and expanding we get the probability that the 4-server

system is empty at any time t as $P(0,0,0,0;t) =$

$$exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} \sum_{r_3=0}^{r_2} \sum_{r_4=0}^{r_3} (-1)^{r_2+r_1} \binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\binom{r_3}{r_4} C_m \left(1+\frac{\theta_1\mu_1}{\mu_2-\mu_1}+\frac{\theta_2\mu_1}{\mu_3-\mu_1}+..+\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_1-r_2} \left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^{r_2-r_3} \left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_3-r_4} \left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^{r_4} \left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4}\right\}\right] \quad (48)$$

### B. PERFORMANCE ANALYSIS OF FIRST QUEUE

Putting $z_2 = 1, z_3 = 1, z_4 = 1$ in (47) we get probability generating function of first queue size as

$$P(Z_1;t) = exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} C_m \binom{m}{r_1} (z_1-1)^{r_1} \left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (49)$$

Mean number of customers in first queue is

$$E(N_1) = L_1(t) = \left(\frac{\lambda}{\mu_1}\right)(1-e^{-\mu_1 t})E(X) \quad (50)$$

Where $E(X)$ is the mean of batch size arrivals to first queue and is given by $E(X) = \sum_{m=1}^{\infty} m.C_m$

Putting $Z_1 = 0$ in (49) we get the probability that the first queue is empty as

$$P(0,\dots,;t) = exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} (-1)^{r_1} C_m \binom{m}{r_1} \left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (51)$$

Utilization of first server is $U_1(t) = 1 - P(0,\dots,;t) =$

$$1 - exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} (-1)^{r_1} C_m \binom{m}{r_1} \left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \quad (52)$$

Throughput of first server is $Thp1(t) = \mu_1 . U_1(t) =$

$$\mu_1\left\{1 - exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} (-1)^{r_1} C_m \binom{m}{r_1} \left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\} \quad (53)$$

Average waiting time of a customer in first queue is

$$W_1(t) = \frac{L_1(t)}{Thp_1(t)} = \frac{\left(\frac{\lambda}{\mu_1}\right)(1-e^{-\mu_1 t})E(X)}{\left[1-exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} (-1)^{r_1} C_m \binom{m}{r_1} \left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right]} \quad (54)$$

Variance of the number of customers in first queue is

$$V(Z_1) = V_1(t) = \lambda \sum_{m=2}^{\infty} \left[\binom{m}{2}.C_m \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) + \left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right] \quad (55)$$

Coefficient of variation in number of customers in first queue is

$$CV_1(t) = \frac{\sqrt{V_1(t)}}{L_1(t)} x100 = \frac{\sqrt{\lambda \sum_{m=2}^{\infty}\left[\binom{m}{2} C_m \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right]}}{\left(\frac{\lambda}{\mu_1}\right)(1-e^{-\mu_1 t})E(X)} x100 \quad (56)$$

### C.PERFORMANCE ANALYSIS OF SECOND QUEUE

Putting $z_1 = 1, z_3 = 1, z_4 = 1$ in (47) we get probability generating function of second queue size distribution as

$$P(Z_2;t) = exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2} C_m \binom{m}{r_1}\binom{r_1}{r_2} \left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right] \quad (57)$$

Mean number of customers in second queue is

$$E(N_2) = L_2(t) = \left[\left(\frac{\lambda\theta_1}{\mu_2}\right)\left\{1-\frac{(\mu_2 e^{-\mu_1 t}-\mu_1 e^{-\mu_2 t})}{(\mu_2-\mu_1)}\right\}\right] E(X) \quad (58)$$

Where $E(X)$ is the mean of batch size arrivals at second queue and $E(X) = \sum_m m.C_m$

Putting $z_2 = 0$ in (57) we get the probability that the second queue is empty as

$$P(.,0,\dots;t) = exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_2} \left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right] \quad (59)$$

Utilization of second server is $U_2(t) = 1 - P(.,0,\dots;t) =$

$$1 - exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_2} \left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right] \quad (60)$$

Throughput of second server is $Thp_2(t) = \mu_2 . U_2(t) =$

$$\mu_2\left[1 - exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_2} \left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]\right] \quad (61)$$

Average waiting time of a customers in the second queue is

$$W_2(t) = \frac{L_2(t)}{Thp_2(t)} = \frac{\left[\left(\frac{\lambda\theta_1}{\mu_2}\right)\left\{1-\left(\frac{\mu_2 e^{-\mu_1 t}-\mu_1 e^{-\mu_2 t}}{(\mu_2-\mu_1)}\right)\right\}\right]E(X)}{\left[1-exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_2} \left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]\right]} \quad (62)$$

Variance of the number of customers in second queue is $V(z_2) = V_2(t) =$

$$\lambda\left[\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^2 \sum_{m=1}^{\infty}\binom{m}{2}.C_m \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) - 4\left(\frac{1-e^{-(\mu_2+\mu_1)t}}{\mu_2+\mu_1}\right) + \left(\frac{1-e^{-2\mu_2 t}}{\mu_2}\right)\right\} + \left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_2 t}}{\mu_2}\right)\right\}E(X)\right] \quad (63)$$

Coefficient of variation of the number of customers in second queue $CV_2(t) = \frac{\sqrt{V_2(t)}}{L_2(t)} x100 =$

$$\frac{\sqrt{\lambda\left[\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^2 \sum_{m=1}^{\infty}\binom{m}{2} C_m \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_2+\mu_1)t}}{\mu_2+\mu_1}\right)+\left(\frac{1-e^{-2\mu_2 t}}{\mu_2}\right)\right\}+\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_2 t}}{\mu_2}\right)\right\}E(X)\right]}}{\left[\left(\frac{\lambda\theta_1}{\mu_2}\right)\left\{1-\frac{(\mu_2 e^{-\mu_1 t}-\mu_1 e^{-\mu_2 t})}{(\mu_2-\mu_1)}\right\}\right]E(X)} x100 \quad (64)$$

### D. PERFORMANCE ANALYSIS OF THIRD QUEUE

Putting $z_1 = 1, z_2 = 1, z_4 = 1$ in (47) we get probability generating function of third queue size distribution as

$$P(Z_3;t) = exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3} C_m \binom{m}{r_1}\binom{r_1}{r_3} \left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right] \quad (65)$$

Mean number of customers in third queue is

$$E(N_3) = L_3(t) = \left[\left(\frac{\lambda\theta_2}{\mu_3}\right)\left\{1-\left(\frac{\mu_3 e^{-\mu_1 t}-\mu_1 e^{-\mu_3 t}}{\mu_3-\mu_1}\right)\right\}\right] E(X) \quad (66)$$

Where $E(X)$ is the mean of batch size arrivals at third queue and $E(X) = \sum_m m.C_m$

Putting $z_3 = 0$ in (65) we get the probability that the third queue is empty as

$$P(.,.,0,.;t) = exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_3} \left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right] \quad (67)$$

Utilization of third server is $U_3(t) = 1 - P(.,.,0,.;t) =$

$$1 - exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_3} \left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right] \quad (68)$$

Throughput of third server is $Thp_3(t) = \mu_3 . U_3(t) =$

$$\mu_3\left[1 - exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_3} \left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]\right] \quad (69)$$

Average waiting time of a customer in third queue is

$$W_3(t) = \frac{L_3(t)}{Thp_3(t)} = \frac{\left[\left(\frac{\lambda\theta_2}{\mu_3}\right)\left\{1-\frac{(\mu_3 e^{-\mu_1 t}-\mu_1 e^{-\mu_3 t})}{(\mu_3-\mu_1)}\right\}\right]E(X)}{\left[1-exp\left[\lambda \sum_{m=1}^{\infty} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} C_m \binom{m}{r_1}\binom{r_1}{r_3} \left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1} \left\{\frac{1-e^{-(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]\right]} \quad (70)$$

Variance of the number of customers in third queue is $V(z_3) = V_3(t) =$

$$\lambda\left[\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^2 \sum_{m=1}^{\infty}\binom{m}{2}.C_m \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_3+\mu_1)t}}{\mu_3+\mu_1}\right)+\left(\frac{1-e^{-2\mu_3 t}}{\mu_3}\right)\right\}+\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_3 t}}{\mu_3}\right)\right\}E(X)\right] \qquad (71)$$

Coefficient of variation of the number of customers in third queue is $CV_3(t)=\frac{\sqrt{V_3(t)}}{L_3(t)}x100=$

$$\frac{\sqrt{\lambda\left[\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^2 \sum_{m=1}^{\infty}\binom{m}{2}C_m \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_3+\mu_1)t}}{\mu_3+\mu_1}\right)+\left(\frac{1-e^{-2\mu_3 t}}{\mu_3}\right)\right\}+\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_3 t}}{\mu_3}\right)\right\}E(X)\right]}}{\left[\left(\frac{\lambda\theta_2}{\mu_3}\right)\left\{1-\left(\frac{\mu_3 e^{-\mu_1 t}-\mu_1 e^{-\mu_3 t}}{\mu_3-\mu_1}\right)\right\}\right]E(X)}x100 \qquad (72)$$

## E. PERFORMANCE ANALYSIS OF FOURTH QUEUE

Putting $z_1=1, z_2=1, z_3=1$ in (47) we get probability generating function of fourth queue size distribution as

$P(Z_4;t)=$

$$exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4}C_m\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1(z_4-1)}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right] \qquad (73)$$

Mean number of customers in fourth queue is

$$E(N_4)=L_4(t)=\left[\left(\frac{\lambda\theta_3}{\mu_4}\right)\left\{1-\left(\frac{\mu_4 e^{-\mu_1 t}-\mu_1 e^{-\mu_4 t}}{\mu_4-\mu_1}\right)\right\}\right]E(X) \qquad (74)$$

Where $E(X)$ is the mean of batch size arrivals at fourth queue and $E(X)=\sum_m m.C_m$

Putting $z_4=0$ in (73) we get the probability that the fourth queue is empty as $P(.,.,.,.,0;t)$

$=$

$$exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right] \qquad (75)$$

Utilization of fourth server is $U_4(t)=1-P(.,.,.,.,0;t)=$

$1-$

$$exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right] \qquad (76)$$

Throughput of fourth server is $Thp_4(t)=\mu_4.U_4(t)=$

$$\mu_4\Bigg|1-exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right]\Bigg] \qquad (77)$$

Average waiting time of a customer in fourth queue is

$W_4(t)=\frac{L_4(t)}{Thp_4(t)}$

$$=\frac{\left[\left(\frac{\lambda\theta_3}{\mu_4{}^2}\right)\left\{1-\frac{(\mu_4 e^{-\mu_1 t}-\mu_1 e^{-\mu_4 t})}{(\mu_4-\mu_1)}\right\}\right]E(X)}{\left[1-exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_4}\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^{r_1}\left\{\frac{1-e^{(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right]\right]} \qquad (78)$$

Variance of the number of customers in fourth queue is $V(z_4)=V_4(t)=$

$$\lambda\left[\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^2 \sum_{m=1}^{\infty}\binom{m}{2}.C_m \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_4+\mu_1)t}}{\mu_4+\mu_1}\right)+\left(\frac{1-e^{-2\mu_4 t}}{\mu_4}\right)\right\}+\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_4 t}}{\mu_4}\right)\right\}E(X)\right] \qquad (79)$$

Coefficient of variation of the number of customers in fourth queue $CV_4(t)=\frac{\sqrt{V_4(t)}}{L_4(t)}x100=$

$$\frac{\sqrt{\lambda\left[\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^2 \sum_{m=1}^{\infty}\binom{m}{2}C_m \left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_4+\mu_1)t}}{\mu_4+\mu_1}\right)+\left(\frac{1-e^{-2\mu_4 t}}{\mu_4}\right)\right\}+\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_4 t}}{\mu_4}\right)\right\}E(X)\right]}}{\left[\left(\frac{\lambda\theta_3}{\mu_4}\right)\left\{1-\left(\frac{\mu_4 e^{-\mu_1 t}-\mu_1 e^{-\mu_4 t}}{\mu_4-\mu_1}\right)\right\}\right]E(X)}x100 \qquad (80)$$

## F . NUMERICAL ILLUSTARTION WITH BINOMIAL DISTRIBUTION

It is assumed that the customers arrive in batches of size 'm' and the probability generating function is $C(Z)=\sum_{m=1}^{\infty}C_m z^m$. The performance of the model is influenced by the batch size arrival distribution. It is assumed that the customers in any arriving model is random and follows Binomial distribution with parameters A and p. Which means the number of customers in a batch has Binomial distribution with parameter p. The probability mass function of Zero Truncated Binomial batch size distribution is $C_m=\frac{\binom{A}{m}p^m q^{A-m}}{1-q^A}, m=1,2,..A$ $and\ 0<p<1, where\ p+q=1.$ The mean of Binomial distribution is $E(X)=\frac{Ap}{1-q^A}$.

The Joint Probability generating function of number of customers in first, second, third and fourth queues respectively at any time t is

$P(Z_1,Z_2,Z_3,Z_4;t)=$

$$exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\sum_{r_4=0}^{r_3}(-1)^{r_2}\binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\binom{r_3}{r_4}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{(z_1-1)+\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}+\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}+\frac{\theta_3\mu_1(z_4-1)}{\mu_4-\mu_1}\right\}^{r_1-r_2}\left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_2-r_3}\left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_3-r_4}\left\{\frac{1-e^{(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4}\right\}\right] \qquad (81)$$

## G. CHARACTERISTICS OF THE MODEL

Putting $z_1=0, z_2=0,....z_k=0$ in (81) and expanding we get the probability that the 4-server system is empty at any time t as

$$P(0,0,0,0;t)=exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\sum_{r_4=0}^{r_3}(-1)^{r_2+r_1}\binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\binom{r_3}{r_4}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left(1+\frac{\theta_1\mu_1}{\mu_2-\mu_1}+\frac{\theta_2\mu_1}{\mu_3-\mu_1}+\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^{r_1-r_2}\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^{r_2-r_3}\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_3-r_4}\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^{4}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4}\right\}\right] \qquad (82)$$

## H. PERFORMANCE ANALYSIS OF FIRST QUEUE

Putting $z_2=1, z_3=1, z_4=1$ in (81) we get probability generating function of first queue size as

$$P(Z_1;t)=exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}\binom{A}{m}\binom{m}{r_1}\left(\frac{p^m q^{A-m}}{1-q^A}\right)(z_1-1)^{r_1}\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \qquad (83)$$

Mean number of customers in first queue is

$$L_1(t)=\left[\frac{\lambda}{\mu_1}\right]\left(\frac{Ap}{1-q^A}\right)(1-e^{-\mu_1 t}) \qquad (84)$$

Putting $Z_1=0$ in (83) we get the probability that the first queue is empty as

$$P(0,.,.,.,.;t)=exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}(-1)^{r_1}\binom{A}{m}\binom{m}{r_1}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \qquad (85)$$

Utilization of first server is $U_1(t)=1-P(0,.,.,.,.;t)=$

$$1-exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}(-1)^{r_1}\binom{A}{m}\binom{m}{r_1}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right] \qquad (86)$$

Throughput of first server is $Thp1(t)=\mu_1.U_1(t)=$

$$\mu_1.\left\{1-exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}(-1)^{r_1}\binom{A}{m}\binom{m}{r_1}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\} \qquad (87)$$

Average waiting time of a customer in first queue is

$$W_1(t)=\frac{L_1(t)}{Thp_1(t)}=\frac{\left[\frac{\lambda}{\mu_1{}^2}\right]\left(\frac{Ap}{1-q^A}\right)(1-e^{-\mu_1 t})}{\left\{1-exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}(-1)^{r_1}\binom{A}{m}\binom{m}{r_1}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{1-e^{-\mu_1 r_1 t}}{\mu_1 r_1}\right\}\right]\right\}} \qquad (88)$$

Variance of number of customers in first queue is

$$V(Z_1)=V_1(t)=\lambda\sum_{m=1}^{A}\left[\binom{A}{m}\binom{m}{2}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right] \qquad (89)$$

Coefficient of variation of number of customers in first queue is

$$CV_1(t)=\frac{\sqrt{V_1(t)}}{L_1(t)}x100=\frac{\sqrt{\lambda\sum_{m=1}^{A}\left[\binom{A}{m}\binom{m}{2}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)+\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)\right\}\right]}}{\left[\frac{\lambda}{\mu_1}\right]\left(\frac{Ap}{1-q^A}\right)(1-e^{-\mu_1 t})}x100 \qquad (90)$$

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## I. PERFORMANCE ANALYSIS OF SECOND QUEUE

Putting $z_1 = 1, z_3 = 1, z_4 = 1$ in (81) we get probability generating function of second queue size distribution as $P(Z_2;t) =$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_2}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right] \quad (91)$$

Mean number of customers in second queue is

$$L_2(t) = \left[\left(\frac{\lambda\theta_1}{\mu_2}\right)\left(\frac{Ap}{1-q^A}\right)\left\{1-\left(\frac{\mu_2 e^{-\mu_1 t}-\mu_1 e^{-\mu_2 t}}{\mu_2-\mu_1}\right)\right\}\right] \quad (92)$$

Putting $Z_2 = 0$ in (91) we get probability that the second queue is empty as $P(.,0,.,.,;t) =$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_2}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right] \quad (93)$$

Utilization of second server is
$U_2(t) = 1 - P(.,0,.,.,;t) = 1-$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_2}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right] \quad (94)$$

Throughput of second server is $Thp_2(t) = \mu_2 . U_2(t) = \mu_2 . [1-$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_2}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right] \quad (95)$$

Average waiting time of a customer in second queue is $W_2(t) = \frac{L_2(t)}{Thp_2(t)}$

$$= \frac{\left[\left(\frac{\lambda\theta_1}{\mu_2}\right)\left(\frac{Ap}{1-q^A}\right)\left\{1-\left(\frac{\mu_2 e^{-\mu_1 t}-\mu_1 e^{-\mu_2 t}}{\mu_2-\mu_1}\right)\right\}\right]}{\left[1-exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_2}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_2)+\mu_2 r_2)t}}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]\right]} \quad (96)$$

Variance of number of customers in second queue is $V_2(t)$

$$= \lambda\left[\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^2 \sum_{m=1}^{A}\binom{A}{m}\binom{m}{2}\left(\frac{p^m q^{A-m}}{1-q^A}\right).\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_2+\mu_1)t}}{\mu_2+\mu_1}\right)+\left(\frac{1-e^{-2\mu_2 t}}{\mu_2}\right)\right\}+\left\{\frac{\theta_1\mu_1}{(\mu_2-\mu_1)}\right\}\left(\frac{Ap}{1-q^A}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)-\left(\frac{1-e^{-\mu_2 t}}{\mu_2}\right)\right\}\right] \quad (97)$$

Coefficient of variation of number of customers in second queue is

$$CV_2(t) = \frac{\sqrt{V_2(t)}}{L_2(t)} x100 =$$

$$\frac{\sqrt{\lambda\left[\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^2 \sum_{m=1}^{A}\binom{A}{m}\binom{m}{2}\left(\frac{p^m q^{A-m}}{1-q^A}\right).\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_2+\mu_1)t}}{\mu_2+\mu_1}\right)+\left(\frac{1-e^{-2\mu_2 t}}{\mu_2}\right)\right\}+\left(\frac{\theta_1\mu_1}{(\mu_2-\mu_1)}\right)\left(\frac{Ap}{1-q^A}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)-\left(\frac{1-e^{-\mu_2 t}}{\mu_2}\right)\right\}\right]}}{\left[\left(\frac{\lambda\theta_1}{\mu_2}\right)\left(\frac{Ap}{1-q^A}\right)\left\{1-\left(\frac{\mu_2 e^{-\mu_1 t}-\mu_1 e^{-\mu_2 t}}{\mu_2-\mu_1}\right)\right\}\right]} x100 \quad (98)$$

## J . PERFORMANCE ANALYSIS OF THIRD QUEUE

Putting $z_1 = 1, z_2 = 1, z_4 = 1$ in (81)we get probability generating function of third queue size distribution as $P(Z_3;t) =$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_3}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right] \quad (99)$$

Mean number of customers in third queue is

$$L_3(t) = \left[\left(\frac{\lambda\theta_2}{\mu_3}\right)\left(\frac{Ap}{1-q^A}\right)\left\{1-\left(\frac{\mu_3 e^{-\mu_1 t}-\mu_1 e^{-\mu_3 t}}{\mu_3-\mu_1}\right)\right\}\right] \quad (100)$$

Putting $Z_3 = 0$ in (99) we get the probability that the third queue is empty as

$P(.,.,0,.;t) =$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_3}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right] \quad (101)$$

Utilization of third server is $U_3(t) = 1 - P(.,.,0,.;t) = 1-$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_3}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right] \quad (102)$$

Throughput of third server is
$Thp_3(t) = \mu_3 . U_3(t) = \mu_3[1-exp$

$$\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_3}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right] \quad (103)$$

Average waiting time of a customer in third queue is $W_3(t) = \frac{L_3(t)}{Thp_3(t)}$

$$= \frac{\left[\left(\frac{\lambda\theta_2}{\mu_3}\right)\left(\frac{Ap}{1-q^A}\right)\left\{1-\left(\frac{\mu_3 e^{-\mu_1 t}-\mu_1 e^{-\mu_3 t}}{\mu_3-\mu_1}\right)\right\}\right]}{\left[1-exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_3}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_3)+\mu_3 r_3)t}}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]\right]} \quad (104)$$

Variance of number of customers in third queue is $V_3(t)$

$$= \lambda\left[\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^2 \sum_{m=1}^{A}\binom{A}{m}\binom{m}{2}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_3+\mu_1)t}}{\mu_3+\mu_1}\right)+\left(\frac{1-e^{-2\mu_3 t}}{\mu_3}\right)\right\}+\left\{\frac{\theta_2\mu_1}{(\mu_3-\mu_1)}\right\}\left(\frac{Ap}{1-q^A}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)-\left(\frac{1-e^{-\mu_3 t}}{\mu_3}\right)\right\}\right] \quad (105)$$

Coefficient of variation of number of customers in third queue is $CV_3(t) = \frac{\sqrt{V_3(t)}}{L_3(t)} x100 =$

$$\frac{\sqrt{\lambda\left[\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^2 \sum_{m=1}^{A}\binom{A}{m}\binom{m}{2}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_3+\mu_1)t}}{\mu_3+\mu_1}\right)+\left(\frac{1-e^{-2\mu_3 t}}{\mu_3}\right)\right\}+\left\{\frac{\theta_2\mu_1}{(\mu_3-\mu_1)}\right\}\left(\frac{Ap}{1-q^A}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)-\left(\frac{1-e^{-\mu_3 t}}{\mu_3}\right)\right\}\right]}}{\left[\left(\frac{\lambda\theta_2}{\mu_3}\right)\left(\frac{Ap}{1-q^A}\right)\left\{1-\left(\frac{\mu_3 e^{-\mu_1 t}-\mu_1 e^{-\mu_3 t}}{\mu_3-\mu_1}\right)\right\}\right]} x100 \quad (106)$$

## K. PERFORMANCE ANALYSIS OF FOURTH QUEUE

Putting $z_1 = 1, z_2 = 1, z_3 = 1$ in (81) we get probability generating function of fourth queue size distribution as $P(Z_4;t) =$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_4=0}^{r_1} (-1)^{r_4} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_4}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_3\mu_1(z_4-1)}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right] \quad (107)$$

Mean number of customers in fourth queue is

$$L_4(t) = \left[\left(\frac{\lambda\theta_3}{\mu_4}\right)\left(\frac{Ap}{1-q^A}\right)\left\{1-\left(\frac{\mu_4 e^{-\mu_1 t}-\mu_1 e^{-\mu_4 t}}{\mu_4-\mu_1}\right)\right\}\right] \quad (108)$$

Putting $Z_4 = 0$ in (107) we get probability that the fourth queue is empty as $P(.,.,.,.,0;t) =$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_4=0}^{r_1} (-1)^{r_4+r_1} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_4}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right] \quad (109)$$

Utilization of fourth server is $U_4(t) = 1 - P(.,.,.,.,0;t) = 1-$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_4=0}^{r_1} (-1)^{r_4+r_1} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_4}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right] \quad (110)$$

Throughput of fourth server is $Thp_4(t) = \mu_4 . U_4(t) = \mu_4 [1-$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_4=0}^{r_1} (-1)^{r_4+r_1} \binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_4}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1-e^{-(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right]\Bigg] \quad (111)$$

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Average waiting time of a customer in fourth queue is $W_4(t) = \frac{L_4(t)}{Thp_4(t)} =$

$$\frac{\left[\left(\frac{\lambda\theta_3}{\mu_4}\right)\left(\frac{Ap}{1-q^A}\right)\left[1-\left(\frac{\mu_4 e^{-\mu_1 t}-\mu_1 e^{-\mu_4 t}}{\mu_4-\mu_1}\right)\right]\right]}{\left[1-exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}\binom{A}{m}\binom{m}{r_1}\binom{r_1}{r_4}\left(\frac{p^m q^{A-m}}{1-q^A}\right)\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^{r_1}\left[\frac{1-e^{-(\mu_1(r_1-r_4)+\mu_4 r_4)t}}{\mu_1(r_1-r_4)+\mu_4 r_4}\right]\right]\right]} \quad (112)$$

Variance of number of customers in fourth queue is $V_4(t)$

$$= \lambda\left[\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^2\sum_{m=1}^{A}\binom{A}{m}\binom{m}{2}\left(\frac{p^m q^{A-m}}{1-q^A}\right).\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_4+\mu_1)t}}{\mu_4+\mu_1}\right)+\left(\frac{1-e^{-2\mu_4 t}}{\mu_4}\right)\right\}+\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}\left(\frac{Ap}{1-q^A}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)-\left(\frac{1-e^{-\mu_4 t}}{\mu_4}\right)\right\}\right] \quad (113)$$

Coefficient of variation of number of customers in fourth queue is

$CV_4(t) = \frac{\sqrt{V_4(t)}}{L_4(t)} x100 =$

$$\frac{\sqrt{\lambda\left[\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^2\sum_{m=1}^{A}\binom{A}{m}\binom{m}{2}\left(\frac{p^m q^{A-m}}{1-q^A}\right).\left\{\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right)-4\left(\frac{1-e^{-(\mu_4+\mu_1)t}}{\mu_4+\mu_1}\right)+\left(\frac{1-e^{-2\mu_4 t}}{\mu_4}\right)\right\}+\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)\left(\frac{Ap}{1-q^A}\right)\left\{\left(\frac{1-e^{-\mu_1 t}}{\mu_1}\right)-\left(\frac{1-e^{-\mu_4 t}}{\mu_4}\right)\right\}\right]}}{\left[\left(\frac{\lambda\theta_3}{\mu_4}\right)\left(\frac{Ap}{1-q^A}\right)\left[1-\left(\frac{\mu_4 e^{-\mu_1 t}-\mu_1 e^{-\mu_4 t}}{\mu_4-\mu_1}\right)\right]\right]} x100 \quad (114)$$

## VI. ANALYSIS

The transient behavior of the model is studied by considering Binomial batch size arrival distribution and the performance measures are calculated by varying system parameters as $t = 0.1,0.2,0.3,0.4,0.5;\ \lambda = 10,11,12,13,14;\ \mu_i = 10,11,12,13,14;\ i = 1,2,3,4;\ \theta_j = 0.1,0.2,0.3,0.4,0.5;\ j = 1,2,\ p = 0.1,0.2,0.3,0.4,0.5\ and\ A = 10,15,20,25,30.$

Here the mean number of customers in each queue $L_1(t), L_2(t), L_3(t), L_4(t)$ along with mean number of customers $L(t)$ in the entire system are calculated by varying the parameters $t, \lambda, \mu_1, \mu_2, \mu_3, \mu_4, \theta_1, \theta_2, \theta_3, p, A$ one at a time keeping all other constant and the values are tabulated in Table 1.The probability of emptiness of each server and also the utilization of servers are calculated correspondingly for each value of parameters as above and the calculations are recorded in Table.2.The throughputs of four servers $Thp_1(t), Thp_2(t), Thp_3(t), Thp_4(t)$ along with average waiting times of customers in four queues $W_1(t), W_2(t), W_3(t), W_4(t)$ are also calculate and tabulated in Table .3.The Variance of the number of customers $V_1(t), V_2(t), V_3(t), V_4(t)$ along with coefficient of variation of the number of customers in each queue are computed and the values are tabulated in Table 4.

From Table.1 it is observed that as time t is increasing from 0.1 to 0.5 the mean number of customers is also increasing in each queue. The same phenomenon is reflected in mean number of customers in the entire system. Also as the service rate of first server $\mu_1$ is increasing from 10 to 14 keeping $\mu_2, \mu_3, \mu_4$ unchanged the mean number of customers in first queue $L_1(t)$ is decreasing from 2.1245 to 1.8086 where as the mean number of customers in the remaining queues are increasing and mean number of customers in the entire system $L(t)$ is decreasing. Similarly when $\mu_2$ is increasing $L_2$(t) is decreasing , $\mu_3$ is increasing $L_3$ (t) is decreasing and no change in the other queue measures .The same phenomenon is observed with the fourth queue. This shows that the improvement in performance of first server improves the performance of entire system.

In the same pattern when the probability $\theta_1(or\,\theta_2)$ that the customers from first server join second server (or third server) increases the queue size at second server $L_2(t)$ (or at third server $L_3(t)$ ) is increasing correspondingly where as at the fourth server it is decreasing. As the batch size distribution parameters 'p' is increasing then $L_1(t), L_2(t), L_3(t)\ and\ L_4(t)$ are decreasing where as for the parameter A the phenomenon goes inversely.

Table.2. indicates that the probability of emptiness has shown decrease with respect to increase in time. In particular it has sudden decrease from 0.2281 to 0.0580 when t moves from 0.1 to 0.2 and decreasing normally thereafter for t=0.2 to 0.5. Similarly with increase in mean arrival rate $\lambda$ the probability of emptiness at each server is decrease while the utilizations of servers $U_1(t), U_2(t), U_3(t), U_4(t)$ increase. This clearly indicates that the system performs in accordance with time. As the service rate $\mu_1$ increases from $10\ to\ 14$ the probability of emptiness of first service station increases from 0.2726 to 0.2926 while utilization of first server decreases from 0.7274 to 0.7004 where as the probability of emptiness at other service stations decrease and utilization increase. The probability of emptiness of the system increases as the service rates $\mu_1, \mu_2, \mu_3, \mu_4$ increase. Similarly when the probability of customer joining a particular server increase the probability of emptiness decrease while it's utilization gets increased. Thus as $\theta_1$ increases from 0.1 to 0.5 the system emptiness decreases marginally from 0.2301 to 0.2295 and probability of emptiness of second server decreases from 0.9373 to 0.7425.This has an impact on the fourth server also since the joining probability of fourth queue is directly dependent on $\theta_1 and\ \theta_2$ ( $\theta_3 = 1-\theta_1-\theta_2$). Therefore the probability of emptiness at fourth server increases from 0.6838 to 0.8386 and it's utilization decreases from 0.3162 to 0.1614.Similarly as $\theta_2$ increases from 0.1 to 0.5 the probability of emptiness of third server decreases from 0.9391 to 0.7484 and it's utilization increases from 0.0609 to 0.2516. Also the probability of emptiness of fourth server increases from 0.6530 to 0.7944 and it's utilization decreases from 0.3470 to 0.2056.It is observed that as the batch size distribution parameter 'p' is increasing the probability of emptiness of the system as well as servers decrease and utilization of each server increase as well. It is also observed that as parameter 'A' increases the probability of emptiness of the system decrease and utilization increase.

From Table.3 it is observed that with increase in time the throughputs $Thp_1(t), Thp_2(t), Thp_3(t), Thp_4(t)$ and mean waiting time of customer at each of the queues $W_1(t), W_2(t), W_3(t), W_4(t)$ have shown increase. Similarly an increase in $\lambda$ led to an increase in throughputs as well as mean waiting times. Further we can observe that the increase in service rates at second, third and fourth servers lead to increase in throughputs and waiting times except at first server where it remain constant. It is also observed that the increase in $\mu_1$ leads to increase in $Thp_1(t), Thp_2(t), Thp_3(t), Thp_4(t)$ and decrease in $W_1(t), W_2(t), W_3(t), W_4(t)$. As the probability of joining second queue increases from $\theta_1$=0.1 to 0.5 the throughput $Thp_2(t)$ increases correspondingly from 0.4389 to 1.8025 . This in turn increase the waiting time $W_2(t)$ from 0.1417 to 0.1836.

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As $\theta_3$ depends on $\theta_1$ ($\theta_2$ is constant) which decreases from 0.7 to 0.3 the throughput $Thp_4$ $(t)$ decreases from 2.8458 to 1.4526 while mean waiting time $W_4(t)$ decreases from 0.1531 to 0.1286. A similar phenomenon is observed with variation in $\theta_2$, the probability of joining third queue after being served at first queue .

From Table .4.it is observed that with increase in time the variance of the number of customers in first, second , third and fourth queues increase and coefficient of variation of the number of customers decrease significantly. Similarly with increase in the variance of number of customers $V_1(t), V_2(t), V_3(t), V_4(t)$ at four queues increase and the coefficient of variation $CV_1(t), CV_2(t), CV_3(t), CV_4(t)$ decrease. Further it is observed that with increase in $\mu_1$ there is decrease in $V_1(t)$ and increase in $V_2(t), V_3(t), V_4(t)$ where as the increase in $\mu_2, \mu_3, \mu_4$ led to decrease in $V_2(t), V_3(t), V_4(t)$ respectively. The probability of joining the second (or third ) queue $\theta_1$(or $\theta_2$) increases the variance in second (or third ) queue $V_2(t)$ (or $V_3(t)$) increases where as the variance in fourth queue $V_4(t)$ decreases. It is also observed that the increase in parameter 'p' increases the variances $V_1(t), V_2(t), V_3(t), V_4(t)$ and the same phenomenon is observed with other batch size parameter ' A '.

**Table 1.Values of Expected Number (Mean) of Customers in the Queue in Transient State**

| t | $\lambda$ | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | p | A | $L_1(t)$ | $L_2(t)$ | $L_3(t)$ | $L_4(t)$ | $L(t)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0662 | 0.1283 | 0.4358 | 3.1576 |
| 0.2 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 3.9143 | 0.1782 | 0.3369 | 1.1162 | 5.5456 |
| 0.3 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.6755 | 0.2773 | 0.5133 | 1.6691 | 7.1352 |
| 0.4 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.0933 | 0.3504 | 0.6381 | 2.0454 | 8.1272 |
| 0.5 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.3226 | 0.3998 | 0.7191 | 2.2817 | 8.7232 |
| 0.1 | 10 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 1.6849 | 0.0441 | 0.0856 | 0.2905 | 2.1051 |
| 0.1 | 11 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 1.8534 | 0.0485 | 0.0941 | 0.3196 | 2.3156 |
| 0.1 | 12 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.0218 | 0.0529 | 0.1027 | 0.3486 | 2.5260 |
| 0.1 | 13 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.1903 | 0.0574 | 0.1120 | 0.3777 | 2.7366 |
| 0.1 | 14 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.3588 | 0.0618 | 0.1198 | 0.4067 | 2.9471 |
| 0.1 | 15 | 10 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.1245 | 0.0975 | 0.1889 | 0.6410 | 3.0519 |
| 0.1 | 15 | 11 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.0383 | 0.1041 | 0.2018 | 0.6843 | 3.0285 |
| 0.1 | 15 | 12 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 1.9572 | 0.1104 | 0.2138 | 0.7249 | 3.0063 |
| 0.1 | 15 | 13 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 1.8807 | 0.1162 | 0.2250 | 0.7629 | 2.9848 |
| 0.1 | 15 | 14 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 1.8086 | 0.1217 | 0.2356 | 0.7985 | 2.9644 |
| 0.1 | 15 | 6 | 10 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0604 | 0.1283 | 0.4358 | 3.1518 |
| 0.1 | 15 | 6 | 11 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0587 | 0.1283 | 0.4358 | 3.1501 |
| 0.1 | 15 | 6 | 12 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0570 | 0.1283 | 0.4358 | 3.1484 |
| 0.1 | 15 | 6 | 13 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0554 | 0.1283 | 0.4358 | 3.1468 |
| 0.1 | 15 | 6 | 14 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0539 | 0.1283 | 0.4358 | 3.1453 |
| 0.1 | 15 | 6 | 7 | 10 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1209 | 0.4358 | 3.1462 |
| 0.1 | 15 | 6 | 7 | 11 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1174 | 0.4358 | 3.1427 |
| 0.1 | 15 | 6 | 7 | 12 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1140 | 0.4358 | 3.1393 |
| 0.1 | 15 | 6 | 7 | 13 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1108 | 0.4358 | 3.1361 |
| 0.1 | 15 | 6 | 7 | 14 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1078 | 0.4358 | 3.1331 |
| 0.1 | 15 | 6 | 7 | 8 | 10 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1283 | 0.4230 | 3.1408 |
| 0.1 | 15 | 6 | 7 | 8 | 11 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1283 | 0.4108 | 3.1286 |
| 0.1 | 15 | 6 | 7 | 8 | 12 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1283 | 0.3991 | 3.1169 |
| 0.1 | 15 | 6 | 7 | 8 | 13 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1283 | 0.3880 | 3.1058 |
| 0.1 | 15 | 6 | 7 | 8 | 14 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1283 | 0.3773 | 3.0951 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1283 | 0.4358 | 3.1536 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.2 | 0.2 | 0.6 | 0.2 | 10 | 2.5273 | 0.1324 | 0.1283 | 0.3735 | 3.1615 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.3 | 0.2 | 0.5 | 0.2 | 10 | 2.5273 | 0.1985 | 0.1283 | 0.3113 | 3.1654 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.4 | 0.2 | 0.4 | 0.2 | 10 | 2.5273 | 0.2647 | 0.1283 | 0.2490 | 3.1693 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.5 | 0.2 | 0.3 | 0.2 | 10 | 2.5273 | 0.3309 | 0.1283 | 0.1868 | 3.1733 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.1 | 0.8 | 0.2 | 10 | 2.5273 | 0.0622 | 0.0642 | 0.4980 | 3.1517 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1283 | 0.4358 | 3.1536 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.3 | 0.6 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1925 | 0.3735 | 3.1555 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.4 | 0.5 | 0.2 | 10 | 2.5273 | 0.0622 | 0.2567 | 0.3113 | 3.1575 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.5 | 0.4 | 0.2 | 10 | 2.5273 | 0.0622 | 0.3208 | 0.2490 | 3.1593 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1283 | 0.4358 | 3.1536 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1283 | 0.4358 | 3.1536 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1283 | 0.4358 | 3.1536 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1283 | 0.4358 | 3.1536 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0622 | 0.1283 | 0.4358 | 3.1536 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.1 | 10 | 1.7318 | 0.0453 | 0.0879 | 0.2986 | 2.1636 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0662 | 0.1283 | 0.4358 | 3.1576 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.3 | 10 | 3.4823 | 0.0912 | 0.1768 | 0.6004 | 4.3507 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.4 | 10 | 4.5393 | 0.1189 | 0.2305 | 0.7827 | 5.6714 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.5 | 10 | 5.6454 | 0.1478 | 0.2867 | 0.9734 | 7.0533 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 2.5273 | 0.0662 | 0.1283 | 0.4358 | 3.1576 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 15 | 3.5073 | 0.0918 | 0.1781 | 0.6047 | 4.3819 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 20 | 4.5645 | 0.1195 | 0.2318 | 0.7870 | 5.7028 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 25 | 5.6612 | 0.1482 | 0.2875 | 0.9761 | 7.0730 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 30 | 6.7762 | 0.1774 | 0.3441 | 1.1684 | 8.4661 |

**Table 2.Probability of Emptiness and Utilization of Servers and System in Transient State**

| t | $\lambda$ | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | p | A | $P_{0000}t$ | $P_{0...}t$ | $P_{.0..}t$ | $P_{..0.}t$ | $P_{...0}t$ | $U_1(t)$ | $U_2(t)$ | $U_3(t)$ | $U_4(t)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2301 | 0.2625 | 0.9373 | 0.8841 | 0.6868 | 0.7375 | 0.0627 | 0.1159 | 0.3162 |
| | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.0619 | 0.1003 | 0.8407 | 0.7258 | 0.3871 | 0.8997 | 0.1593 | 0.2742 | 0.6129 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.0216 | 0.0539 | 0.7634 | 0.6134 | 0.2402 | 0.9461 | 0.2366 | 0.3866 | 0.7598 |
| | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.0101 | 0.0371 | 0.7106 | 0.5437 | 0.1710 | 0.9629 | 0.2894 | 0.4563 | 0.8290 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.0061 | 0.2999 | 0.6768 | 0.5023 | 0.1370 | 0.9701 | 0.3232 | 0.4977 | 0.8630 |
| 0. | 10 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.3755 | 0.4100 | 0.9577 | 0.9212 | 0.7761 | 0.5900 | 0.0423 | 0.0788 | 0.2239 |
| | 11 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.3404 | 0.3750 | 0.9536 | 0.9136 | 0.7567 | 0.6250 | 0.0464 | 0.0864 | 0.2433 |
| 0. | 12 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.3087 | 0.3430 | 0.9495 | 0.9062 | 0.7378 | 0.6570 | 0.0505 | 0.0938 | 0.2622 |
| | 13 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2799 | 0.3138 | 0.9454 | 0.8988 | 0.7193 | 0.6862 | 0.0546 | 0.1012 | 0.2807 |
| 0. | 14 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2538 | 0.2870 | 0.9413 | 0.8914 | 0.7013 | 0.7130 | 0.0587 | 0.1086 | 0.2987 |
| 0. | 15 | 10 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2344 | 0.2974 | 0.9098 | 0.8368 | 0.5900 | 0.7026 | 0.0902 | 0.1632 | 0.4100 |
| | 15 | 11 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2355 | 0.3069 | 0.9041 | 0.8273 | 0.5731 | 0.6931 | 0.0959 | 0.1727 | 0.4269 |
| 0. | 15 | 12 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2365 | 0.3166 | 0.8988 | 0.8186 | 0.5580 | 0.6834 | 0.1012 | 0.1814 | 0.4420 |
| | 15 | 13 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2375 | 0.3265 | 0.8940 | 0.8106 | 0.5445 | 0.6735 | 0.1060 | 0.1894 | 0.4555 |
| 0. | 15 | 14 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2385 | 0.3336 | 0.8894 | 0.8031 | 0.5323 | 0.6664 | 0.1106 | 0.1969 | 0.4677 |
| 0. | 15 | 6 | 10 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2303 | 0.2625 | 0.9424 | 0.8841 | 0.6868 | 0.7375 | 0.0576 | 0.1159 | 0.3162 |
| | 15 | 6 | 11 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2304 | 0.2625 | 0.9440 | 0.8841 | 0.6838 | 0.7375 | 0.0560 | 0.1159 | 0.3162 |
| 0. | 15 | 6 | 12 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2304 | 0.2625 | 0.9455 | 0.8841 | 0.6838 | 0.7375 | 0.0545 | 0.1159 | 0.3162 |
| | 15 | 6 | 13 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2305 | 0.2625 | 0.9470 | 0.8841 | 0.6838 | 0.7375 | 0.0530 | 0.1159 | 0.3162 |
| 0. | 15 | 6 | 14 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2306 | 0.2625 | 0.9484 | 0.8841 | 0.6838 | 0.7375 | 0.0516 | 0.1159 | 0.3162 |
| 0. | 15 | 6 | 7 | 1 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2304 | 0.2625 | 0.9373 | 0.8902 | 0.6868 | 0.7375 | 0.0627 | 0.1098 | 0.3162 |
| | 15 | 6 | 7 | | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2305 | 0.2625 | 0.9373 | 0.8931 | 0.6838 | 0.7375 | 0.0627 | 0.1069 | 0.3162 |
| 0. | 15 | 6 | 7 | 1 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2306 | 0.2625 | 0.9373 | 0.8958 | 0.6838 | 0.7375 | 0.0627 | 0.1042 | 0.3162 |
| | 15 | 6 | 7 | | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2308 | 0.2625 | 0.9373 | 0.8985 | 0.6838 | 0.7375 | 0.0627 | 0.1015 | 0.3162 |
| 0. | 15 | 6 | 7 | 1 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2309 | 0.2625 | 0.9373 | 0.9010 | 0.6838 | 0.7375 | 0.0627 | 0.0990 | 0.3162 |
| 0. | 15 | 6 | 7 | 8 | 1 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2306 | 0.2625 | 0.9373 | 0.8841 | 0.6902 | 0.7375 | 0.0627 | 0.1159 | 0.3098 |
| | 15 | 6 | 7 | 8 | | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2311 | 0.2625 | 0.9373 | 0.8841 | 0.6966 | 0.7375 | 0.0627 | 0.1159 | 0.3034 |
| 0. | 15 | 6 | 7 | 8 | 1 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2315 | 0.2625 | 0.9373 | 0.8841 | 0.7027 | 0.7375 | 0.0627 | 0.1159 | 0.2973 |
| | 15 | 6 | 7 | 8 | | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2320 | 0.2625 | 0.9373 | 0.8841 | 0.7087 | 0.7375 | 0.0627 | 0.1159 | 0.2913 |
| 0. | 15 | 6 | 7 | 8 | 1 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2324 | 0.2625 | 0.9373 | 0.8841 | 0.7145 | 0.7375 | 0.0627 | 0.1159 | 0.2855 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2301 | 0.2625 | 0.9373 | 0.8841 | 0.6838 | 0.7375 | 0.0627 | 0.1159 | 0.3162 |
| | 15 | 6 | 7 | 8 | 9 | 0.2 | 0.2 | 0.7 | 0.2 | 10 | 0.2299 | 0.2625 | 0.8809 | 0.8841 | 0.7174 | 0.7375 | 0.1191 | 0.1159 | 0.2826 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.3 | 0.2 | 0.7 | 0.2 | 10 | 0.2298 | 0.2625 | 0.8300 | 0.8841 | 0.7541 | 0.7375 | 0.1700 | 0.1159 | 0.2459 |
| | 15 | 6 | 7 | 8 | 9 | 0.4 | 0.2 | 0.7 | 0.2 | 10 | 0.2296 | 0.2625 | 0.7841 | 0.8841 | 0.7944 | 0.7375 | 0.2159 | 0.1159 | 0.2056 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.5 | 0.2 | 0.7 | 0.2 | 10 | 0.2295 | 0.2625 | 0.7425 | 0.8841 | 0.8386 | 0.7375 | 0.2575 | 0.1159 | 0.1614 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.1 | 0.8 | 0.2 | 10 | 0.2302 | 0.2625 | 0.9373 | 0.9391 | 0.6530 | 0.7375 | 0.0627 | 0.0609 | 0.3470 |
| | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2301 | 0.2625 | 0.9373 | 0.8841 | 0.6838 | 0.7375 | 0.0627 | 0.1159 | 0.3162 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.3 | 0.6 | 0.2 | 10 | 0.2300 | 0.2625 | 0.9373 | 0.8344 | 0.7174 | 0.7375 | 0.0627 | 0.1656 | 0.2826 |
| | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.4 | 0.5 | 0.2 | 10 | 0.2299 | 0.2625 | 0.9373 | 0.7893 | 0.7541 | 0.7375 | 0.0627 | 0.2107 | 0.2459 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.5 | 0.4 | 0.2 | 10 | 0.2299 | 0.2625 | 0.9373 | 0.7484 | 0.7944 | 0.7375 | 0.0627 | 0.2516 | 0.2056 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2301 | 0.2625 | 0.9373 | 0.8841 | 0.6868 | 0.7375 | 0.0627 | 0.1159 | 0.3162 |
| | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2301 | 0.2625 | 0.9373 | 0.8841 | 0.6838 | 0.7375 | 0.0627 | 0.1159 | 0.3162 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2301 | 0.2625 | 0.9373 | 0.8841 | 0.6838 | 0.7375 | 0.0627 | 0.1159 | 0.3162 |
| | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2301 | 0.2625 | 0.9373 | 0.8841 | 0.6838 | 0.7375 | 0.0627 | 0.1159 | 0.3162 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2301 | 0.2625 | 0.9373 | 0.8841 | 0.6838 | 0.7375 | 0.0627 | 0.1159 | 0.3162 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.1 | 10 | 0.2362 | 0.2894 | 0.8692 | 0.9175 | 0.4969 | 0.7106 | 0.1308 | 0.0825 | 0.5031 |
| | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2301 | 0.2625 | 0.8468 | 0.8841 | 0.4546 | 0.7375 | 0.1532 | 0.1159 | 0.5454 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.3 | 10 | 0.2224 | 0.2441 | 0.8253 | 0.8468 | 0.4207 | 0.7559 | 0.1747 | 0.1532 | 0.5793 |
| | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.4 | 10 | 0.2243 | 0.2331 | 0.8048 | 0.8086 | 0.3933 | 0.7669 | 0.1952 | 0.1914 | 0.6067 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.5 | 10 | 0.2235 | 0.2274 | 0.7853 | 0.7719 | 0.3710 | 0.7726 | 0.2147 | 0.2281 | 0.6290 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 0.2301 | 0.2625 | 0.9373 | 0.8841 | 0.6838 | 0.7375 | 0.0627 | 0.1159 | 0.3162 |
| | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 15 | 0.2265 | 0.2450 | 0.9150 | 0.8461 | 0.6122 | 0.7550 | 0.0850 | 0.1539 | 0.3878 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 20 | 0.2247 | 0.2349 | 0.8920 | 0.8084 | 0.5512 | 0.7651 | 0.1080 | 0.1916 | 0.4488 |
| | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 25 | 0.2238 | 0.2293 | 0.8693 | 0.7725 | 0.5012 | 0.7707 | 0.1307 | 0.2275 | 0.4988 |
| 0. | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 30 | 0.2234 | 0.2263 | 0.8473 | 0.7391 | 0.4608 | 0.7737 | 0.1527 | 0.2609 | 0.5392 |

**Table 3.Values of Throughput and Waiting Time of Customers in Queues in Transient State**

| t | λ | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | p | A | $Thp_1(t)$ | $Thp_2(t)$ | $Thp_3(t)$ | $Thp_4(t)$ | $W_1(t)$ | $W_2(t)$ | $W_3(t)$ | $W_4(t)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 2.8458 | 0.5711 | 0.1508 | 0.1384 | 0.1531 |
| 0.2 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.3982 | 1.1151 | 2.1936 | 5.5161 | 0.7251 | 0.1598 | 0.1536 | 0.2024 |
| 0.3 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.6766 | 1.6262 | 3.0928 | 6.8382 | 0.8236 | 0.1674 | 0.1660 | 0.2441 |
| 0.4 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7774 | 2.0258 | 3.6504 | 7.4610 | 0.8816 | 0.1730 | 0.1748 | 0.2741 |
| 0.5 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.8206 | 2.2624 | 3.9816 | 7.7670 | 0.9144 | 0.1767 | 0.1806 | 0.2938 |
| 0.1 | 10 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 3.5400 | 0.2961 | 0.6304 | 2.0151 | 0.4760 | 0.1489 | 0.1358 | 0.1442 |
| 0.1 | 11 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 3.7500 | 0.3248 | 0.6912 | 2.1897 | 0.4942 | 0.1493 | 0.1361 | 0.1460 |
| 0.1 | 12 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 3.9420 | 0.3535 | 0.7504 | 2.3598 | 0.5129 | 0.1496 | 0.1369 | 0.1477 |
| 0.1 | 13 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.1172 | 0.3822 | 0.8096 | 2.5263 | 0.5320 | 0.1502 | 0.1374 | 0.1495 |
| 0.1 | 14 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.2780 | 0.4109 | 0.8688 | 2.6883 | 0.5514 | 0.1504 | 0.1379 | 0.1513 |
| 0.1 | 15 | 10 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 7.026 | 0.6314 | 1.3056 | 3.6900 | 0.3024 | 0.1544 | 0.1447 | 0.1737 |
| 0.1 | 15 | 11 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 7.6241 | 0.6713 | 1.3816 | 3.8421 | 0.2673 | 0.1551 | 0.1461 | 0.1781 |
| 0.1 | 15 | 12 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 8.2008 | 0.7084 | 1.4512 | 3.9780 | 0.2387 | 0.1558 | 0.1473 | 0.1822 |
| 0.1 | 15 | 13 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 8.7555 | 0.7420 | 1.5152 | 4.0995 | 0.2148 | 0.1566 | 0.1485 | 0.1861 |
| 0.1 | 15 | 14 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 9.3296 | 0.7742 | 1.5752 | 4.2093 | 0.1939 | 0.1572 | 0.1496 | 0.1897 |
| 0.1 | 15 | 6 | 10 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.5760 | 0.9272 | 2.8458 | 0.5711 | 0.1049 | 0.1384 | 0.1531 |
| 0.1 | 15 | 6 | 11 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.6160 | 0.9272 | 2.8458 | 0.5711 | 0.0953 | 0.1384 | 0.1531 |
| 0.1 | 15 | 6 | 12 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.6540 | 0.9272 | 2.8458 | 0.5711 | 0.0872 | 0.1384 | 0.1531 |
| 0.1 | 15 | 6 | 13 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.6890 | 0.9272 | 2.8458 | 0.5711 | 0.0804 | 0.1384 | 0.1531 |
| 0.1 | 15 | 6 | 14 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.7224 | 0.9272 | 2.8458 | 0.5711 | 0.0746 | 0.1384 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 10 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 1.0980 | 2.8458 | 0.5711 | 0.1417 | 0.1101 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 11 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 1.1759 | 2.8458 | 0.5711 | 0.1417 | 0.0998 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 12 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 1.2504 | 2.8458 | 0.5711 | 0.1417 | 0.0912 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 13 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 1.3195 | 2.8458 | 0.5711 | 0.1417 | 0.0840 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 14 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 1.3860 | 2.8458 | 0.5711 | 0.1417 | 0.0778 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 8 | 10 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 3.0980 | 0.5711 | 0.1417 | 0.1384 | 0.1365 |
| 0.1 | 15 | 6 | 7 | 8 | 11 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 3.3374 | 0.5711 | 0.1417 | 0.1384 | 0.1231 |
| 0.1 | 15 | 6 | 7 | 8 | 12 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 3.5676 | 0.5711 | 0.1417 | 0.1384 | 0.1190 |
| 0.1 | 15 | 6 | 7 | 8 | 13 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 3.7869 | 0.5711 | 0.1417 | 0.1384 | 0.1025 |
| 0.1 | 15 | 6 | 7 | 8 | 14 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 3.9970 | 0.5711 | 0.1417 | 0.1384 | 0.0944 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 2.8458 | 0.5711 | 0.1417 | 0.1384 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.2 | 0.2 | 0.6 | 0.2 | 10 | 4.4250 | 0.8337 | 0.9272 | 2.5434 | 0.5711 | 0.1588 | 0.1384 | 0.1469 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.3 | 0.2 | 0.5 | 0.2 | 10 | 4.4250 | 1.1900 | 0.9272 | 2.2131 | 0.5711 | 0.1668 | 0.1384 | 0.1407 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.4 | 0.2 | 0.4 | 0.2 | 10 | 4.4250 | 1.5113 | 0.9272 | 1.8504 | 0.5711 | 0.1751 | 0.1384 | 0.1346 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.5 | 0.2 | 0.3 | 0.2 | 10 | 4.4250 | 1.8025 | 0.9272 | 1.4526 | 0.5711 | 0.1836 | 0.1384 | 0.1286 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.1 | 0.8 | 0.2 | 10 | 4.4250 | 0.4389 | 0.4872 | 3.1230 | 0.5711 | 0.1417 | 0.1318 | 0.1595 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 2.8458 | 0.5711 | 0.1417 | 0.1384 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.3 | 0.6 | 0.2 | 10 | 4.4250 | 0.4389 | 1.3248 | 2.5434 | 0.5711 | 0.1417 | 0.1453 | 0.1469 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.4 | 0.5 | 0.2 | 10 | 4.4250 | 0.4389 | 1.6856 | 2.2131 | 0.5711 | 0.1417 | 0.1523 | 0.1407 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.5 | 0.4 | 0.2 | 10 | 4.4250 | 0.4389 | 3.0128 | 1.5804 | 0.5711 | 0.1417 | 0.1594 | 0.1436 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 2.8458 | 0.5711 | 0.1417 | 0.1384 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 2.8458 | 0.5711 | 0.1417 | 0.1384 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 2.8458 | 0.5711 | 0.1417 | 0.1384 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 2.8458 | 0.5711 | 0.1417 | 0.1384 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 2.8458 | 0.5711 | 0.1417 | 0.1384 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.1 | 10 | 4.2636 | 0.9156 | 0.6600 | 4.5279 | 0.4062 | 0.0495 | 0.1332 | 0.0659 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 1.0724 | 0.9272 | 4.9086 | 0.5719 | 0.0617 | 0.1384 | 0.0888 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.3 | 10 | 4.5354 | 1.2229 | 1.2256 | 5.2137 | 0.7678 | 0.0746 | 0.1443 | 0.1152 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.4 | 10 | 4.6014 | 1.3664 | 1.5312 | 5.4603 | 0.9865 | 0.0870 | 0.1505 | 0.1433 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.5 | 10 | 4.6356 | 1.5029 | 1.8248 | 5.6610 | 1.2178 | 0.0893 | 0.1571 | 0.1719 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.4250 | 0.4389 | 0.9272 | 2.8458 | 0.5711 | 0.1508 | 0.1384 | 0.1531 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 15 | 4.5300 | 0.5950 | 1.2312 | 3.4902 | 0.7742 | 0.1543 | 0.1447 | 0.1733 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 20 | 4.5906 | 0.7560 | 1.5328 | 4.0392 | 0.9943 | 0.1581 | 0.1512 | 0.1948 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 25 | 4.6242 | 0.9149 | 1.8200 | 4.4892 | 1.2243 | 0.1620 | 0.1580 | 0.2174 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 30 | 4.6422 | 1.0689 | 2.0872 | 4.8528 | 1.4597 | 0.1660 | 0.1649 | 0.2408. |

**Table 4.Values of Variances and coefficients of Variation of Customers in Queues in Transient State**

| t | $\lambda$ | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | p | A | $V_1(t)$ | $V_2(t)$ | $V_3(t)$ | $V_4(t)$ | $CV_1(t)$ | $CV_2(t)$ | $CV_3(t)$ | $CV_4(t)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1490 | 1.7456 | 95.272 | 1563.7 | 835.48 | 303.17 |
| 0.2 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 8.1069 | 1.2066 | 1.4488 | 4.4180 | 72.740 | 616.42 | 357.28 | 188.31 |
| 0.3 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 9.1115 | 1.3394 | 1.7577 | 9.0585 | 64.561 | 417.36 | 258.29 | 180.32 |
| 0.4 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 9.5838 | 1.4451 | 2.0081 | 14.255 | 60.781 | 343.07 | 222.08 | 184.59 |
| 0.5 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 9.8191 | 1.5202 | 2.1853 | 18.612 | 58.872 | 308.40 | 205.57 | 189.08 |
| 0.1 | 10 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 3.8650 | 1.0471 | 1.0970 | 1.4498 | 116.68 | 2320.4 | 1223.6 | 414.48 |
| 0.1 | 11 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.2515 | 1.0519 | 1.1073 | 1.5016 | 111.25 | 2114.7 | 1118.3 | 383.80 |
| 0.1 | 12 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.6380 | 1.0568 | 1.1176 | 1.5616 | 106.51 | 1943.3 | 1029.4 | 358.47 |
| 0.1 | 13 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.0245 | 1.0616 | 1.1280 | 1.6207 | 102.34 | 1795.0 | 955.10 | 337.06 |
| 0.1 | 14 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.4110 | 1.0665 | 1.1385 | 1.6820 | 98.617 | 1671.1 | 890.66 | 318.89 |
| 0.1 | 15 | 10 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.5275 | 1.1091 | 1.2353 | 2.4527 | 100.16 | 1080.1 | 588.38 | 244.32 |
| 0.1 | 15 | 11 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.2796 | 1.1173 | 1.2550 | 2.6504 | 101.49 | 1015.4 | 555.14 | 237.91 |
| 0.1 | 15 | 12 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 4.0534 | 1.1252 | 1.2739 | 2.8549 | 102.87 | 960.83 | 527.91 | 233.09 |
| 0.1 | 15 | 13 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 3.8466 | 1.1327 | 1.2921 | 3.0653 | 104.28 | 915.91 | 505.20 | 229.49 |
| 0.1 | 15 | 14 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 3.6570 | 1.1398 | 1.3095 | 3.2809 | 105.74 | 879.25 | 485.71 | 226.84 |
| 0.1 | 15 | 6 | 10 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0648 | 1.1493 | 1.7456 | 95.272 | 1708.4 | 835.58 | 303.17 |
| 0.1 | 15 | 6 | 11 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0628 | 1.1493 | 1.7456 | 95.272 | 1756.2 | 835.58 | 303.17 |
| 0.1 | 15 | 6 | 12 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.1608 | 1.1493 | 1.7456 | 95.272 | 1806.9 | 835.58 | 303.17 |
| 0.1 | 15 | 6 | 13 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0590 | 1.1493 | 1.7456 | 95.272 | 1857.5 | 835.58 | 303.17 |
| 0.1 | 15 | 6 | 14 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0573 | 1.1493 | 1.7456 | 95.272 | 1907.7 | 835.58 | 303.17 |
| 0.1 | 15 | 6 | 7 | 10 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1390 | 1.7456 | 95.272 | 1664.2 | 882.75 | 303.17 |
| 0.1 | 15 | 6 | 7 | 11 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1344 | 1.7456 | 95.272 | 1664.2 | 907.23 | 303.17 |
| 0.1 | 15 | 6 | 7 | 12 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1300 | 1.7456 | 95.272 | 1664.2 | 932.49 | 303.17 |
| 0.1 | 15 | 6 | 7 | 13 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1258 | 1.7456 | 95.272 | 1664.2 | 957.62 | 303.17 |
| 0.1 | 15 | 6 | 7 | 14 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1219 | 1.7456 | 95.272 | 1664.2 | 982.56 | 303.17 |
| 0.1 | 15 | 6 | 7 | 8 | 10 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1493 | 1.7102 | 95.272 | 1664.2 | 835.58 | 309.16 |
| 0.1 | 15 | 6 | 7 | 8 | 11 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1493 | 1.6776 | 95.272 | 1664.2 | 835.58 | 315.29 |
| 0.1 | 15 | 6 | 7 | 8 | 12 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1493 | 1.6474 | 95.272 | 1664.2 | 835.58 | 321.60 |
| 0.1 | 15 | 6 | 7 | 8 | 13 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1493 | 1.6194 | 95.272 | 1664.2 | 835.58 | 327.98 |
| 0.1 | 15 | 6 | 7 | 8 | 14 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1493 | 1.5934 | 95.272 | 1664.2 | 835.58 | 334.56 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1493 | 1.7456 | 95.272 | 1664.2 | 835.58 | 303.17 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.2 | 0.2 | 0.6 | 0.2 | 10 | 5.7975 | 1.1545 | 1.1493 | 1.5883 | 95.272 | 811.54 | 835.58 | 337.4 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.3 | 0.2 | 0.5 | 0.2 | 10 | 5.7975 | 1.2511 | 1.1493 | 1.4523 | 95.272 | 563.49 | 835.58 | 387.1 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.4 | 0.2 | 0.4 | 0.2 | 10 | 5.7975 | 1.3635 | 1.1493 | 1.3346 | 95.272 | 441.14 | 835.58 | 463.96 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.5 | 0.2 | 0.3 | 0.2 | 10 | 5.7975 | 1.4944 | 1.1493 | 1.2325 | 95.272 | 369.43 | 835.58 | 594.32 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.1 | 0.8 | 0.2 | 10 | 5.7975 | 1.0715 | 1.0691 | 1.9281 | 95.272 | 1664.2 | 1610.6 | 278.83 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1490 | 1.7456 | 95.272 | 1664.2 | 835.48 | 303.17 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.3 | 0.6 | 0.2 | 10 | 5.7975 | 1.0715 | 1.2415 | 1.5883 | 95.272 | 1664.2 | 578.82 | 337.42 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.4 | 0.5 | 0.2 | 10 | 5.7975 | 1.0715 | 1.3486 | 1.4523 | 95.272 | 1664.2 | 452.39 | 387.12 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.5 | 0.4 | 0.2 | 10 | 5.7975 | 1.0715 | 1.4726 | 1.3346 | 95.272 | 1664.2 | 378.28 | 463.96 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1493 | 1.7456 | 95.272 | 1664.2 | 835.58 | 303.17 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1493 | 1.7456 | 95.272 | 1664.2 | 835.58 | 303.17 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1493 | 1.7456 | 95.272 | 1664.2 | 835.58 | 303.17 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1493 | 1.7456 | 95.272 | 1664.2 | 835.58 | 303.17 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1493 | 1.7456 | 95.272 | 1664.2 | 835.58 | 303.17 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.1 | 10 | 1.9864 | 1.0474 | 1.0959 | 1.4052 | 81.383 | 2259.2 | 1191.0 | 396.99 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1490 | 1.7456 | 95.272 | 1563.7 | 835.48 | 303.17 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.3 | 10 | 11.982 | 1.1019 | 1.2198 | 2.3425 | 99.404 | 1151.0 | 624.69 | 254.92 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.4 | 10 | 20.826 | 1.1377 | 1.3081 | 3.3825 | 100.53 | 897.08 | 496.19 | 234.98 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.5 | 10 | 32.375 | 1.1778 | 1.4131 | 5.2126 | 100.79 | 734.28 | 414.63 | 234.55 |

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| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 10 | 5.7975 | 1.0715 | 1.1490 | 1.7456 | 95.272 | 1563.7 | 835.48 | 303.17 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 15 | 12.515 | 1.0929 | 1.2226 | 2.3791 | 100.87 | 1144.0 | 620.84 | 255.0 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 20 | 22.105 | 1.1392 | 1.3128 | 3.4894 | 103.00 | 893.17 | 494.30 | 237.36 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 25 | 34.631 | 1.1796 | 1.4201 | 5.4795 | 103.95 | 732.86 | 414.50 | 239.82 |
| 0.1 | 15 | 6 | 7 | 8 | 9 | 0.1 | 0.2 | 0.7 | 0.2 | 30 | 50.087 | 1.2237 | 1.5459 | 9.1778 | 104.44 | 623.57 | 361.33 | 259.29 |

## VII. SENSITIVITY ANALYSIS

In this section the sensitivity analysis of model with the values of parameters as t=0.1, $\lambda$=15, $\mu_1$=12, $\mu_2$=14, $\mu_3$=11, $\mu_4$=13, $\theta_1$=0.3 , $\theta_2$=0.2 , $p = 0.5$ $and$ $A = 20$ are considered and sensitivity of the model is studies. The effect on performance measures $L_1, L_2, L_3, L_4, L, W_1, W_2, W_3$ and $W_4$ with varying the parameters by ∓15%,∓10% and ∓5% was computed and are tabulated Table 5.

From Table 5 it is observed that as time t increases the values of $L_1, L_2, L_3, L_4, L, W_1, W_2,$ $W_3$ and $W_4$ are increase. The same phenomenon is observed by varying the arrival rate $\lambda$ .It is observed that as $\mu_1$ increases $L_1, L, W_1$ decrease while $L_2, L_3, L_4, W_2, W_3$ and $W_4$ increase. It is also observed that when $\mu_2$ increases $L_2, L$ and $W_2$ decrease whereas $L_3, L_4, W_3$ and $W_4$ remain constant. When $\mu_3$ increases $L_3, L, W_3$ decrease whereas $L_1, L_2, L_4, W_1, W_2$ and $W_4$ remain constant. When $\mu_4$ increases $L_4$ , $L, W_4$ decrease whereas $L_1, L_2, L_3, W_1, W_2$ and $W_3$ remain constant and a similar phenomenon is observed with $\mu_4$. We also observed that with increase in $\theta_1$ the performance measures $L_2, L, W_2$ are increasing $L_4, W_4$ are decreasing whereas $L_1, L_3, W_1$ and $W_3$ remain constant .Similarly with increase in $\theta_2$ the performance measures $L_3, L, W_3$ are increasing $L_4, W_4$ are decreasing whereas $L_1, L_2, W_1$ and $W_2$ remain constant. We also observed that with increase in batch size distribution parameters 'p' and ' A' the performance measures $L_1, L_2, L_3, L_4, L, W_1, W_2, W_3, W_4$ increase

**Table .5.Values of $L_1, L_2, L_3, L_4, L, W_1, W_2, W_3$ $and$ $W_4$ for different Values of $t, \lambda, \mu_1, \mu_2, \mu_3, \mu_4, p, A, \theta_1, \theta_2 and \theta_3$.**

**(SENSITIVITY ANALYSIS)**

| Variation Parameter | Performance Measure | Percentage Change in Parameter | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | -15% | -10% | -5% | 0 | 5% | 10% | 15% |
| t = 0.1 | $L_1(t)$ | 2.2378 | 2.3372 | 2.4337 | 2.5273 | 2.6182 | 2.7063 | 2.7919 |
| | $L_2(t)$ | 0.0508 | 0.0558 | 0.0610 | 0.0662 | 0.0715 | 0.0769 | 0.0823 |
| | $L_3(t)$ | 0.099 | 0.1086 | 0.1184 | 0.1283 | 0.1384 | 0.1487 | 0.159 |
| | $L_4(t)$ | 0.3377 | 0.3698 | 0.4025 | 0.4358 | 0.4694 | 0.5035 | 0.5378 |
| | $L(t)$ | 2.7253 | 2.8714 | 3.0156 | 3.1576 | 3.2975 | 3.4354 | 3.571 |
| | $W_1(t)$ | 0.5434 | 0.5527 | 0.5620 | 0.5711 | 0.5802 | 0.5891 | 0.5979 |
| | $W_2(t)$ | 0.1493 | 0.1496 | 0.1502 | 0.1508 | 0.1513 | 0.1517 | 0.1521 |
| | $W_3(t)$ | 0.1360 | 0.1368 | 0.1377 | 0.1384 | 0.1392 | 0.1400 | 0.1408 |
| | $W_4(t)$ | 0.1458 | 0.1482 | 0.1506 | 0.1531 | 0.1556 | 0.1581 | 0.1606 |
| $\lambda$=15 | $L_1(t)$ | 2.1482 | 2.2746 | 2.4009 | 2.5273 | 2.6537 | 2.7800 | 2.9064 |
| | $L_2(t)$ | 0.0562 | 0.0596 | 0.0629 | 0.0662 | 0.0695 | 0.0728 | 0.0761 |
| | $L_3(t)$ | 0.1091 | 0.1155 | 0.1219 | 0.1283 | 0.1348 | 0.1412 | 0.1476 |
| | $L_4(t)$ | 0.3704 | 0.3922 | 0.4140 | 0.4358 | 0.4576 | 0.4793 | 0.5011 |
| | $L(t)$ | 2.6839 | 2.8419 | 2.9997 | 3.1576 | 3.3156 | 3.4733 | 3.6312 |
| | $W_1(t)$ | 0.5271 | 0.5416 | 0.5563 | 0.5711 | 0.5862 | 0.6015 | 0.6169 |
| | $W_2(t)$ | 0.1498 | 0.1504 | 0.1505 | 0.1508 | 0.1511 | 0.1512 | 0.1514 |
| | $W_3(t)$ | 0.1372 | 0.1376 | 0.1380 | 0.1384 | 0.1389 | 0.1393 | 0.1397 |
| | $W_4(t)$ | 0.1491 | 0.1504 | 0.1518 | 0.1531 | 0.1545 | 0.1559 | 0.1572 |
| $\mu_1$=12 | $L_1(t)$ | 2.1068 | 2.0551 | 2.0053 | 1.9572 | 1.9108 | 1.8660 | 1.8227 |
| | $L_2(t)$ | 0.0989 | 0.1029 | 0.1067 | 0.1104 | 0.1139 | 0.1173 | 0.1206 |
| | $L_3(t)$ | 0.1916 | 0.1993 | 0.2066 | 0.2138 | 0.2206 | 0.2242 | 0.2335 |
| | $L_4(t)$ | 0.6499 | 0.6759 | 0.7009 | 0.7249 | 0.748 | 0.7702 | 0.7916 |
| | $L(t)$ | 3.0472 | 3.0332 | 3.0195 | 3.0063 | 2.9933 | 2.9777 | 2.9684 |
| | $W_1(t)$ | 0.2505 | 0.2464 | 0.2425 | 0.2387 | 0.2351 | 0.2316 | 0.2283 |
| | $W_2(t)$ | 0.1546 | 0.1551 | 0.1555 | 0.1560 | 0.1563 | 0.1566 | 0.1571 |
| | $W_3(t)$ | 0.2279 | 0.1459 | 0.1466 | 0.1473 | 0.1480 | 0.1467 | 0.1494 |
| | $W_4(t)$ | 0.1746 | 0.1772 | 0.1798 | 0.1822 | 0.1846 | 0.1869 | 0.1890 |
| $\mu_2$=14 | $L_1(t)$ | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 |
| | $L_2(t)$ | 0.0572 | 0.0561 | 0.055 | 0.0539 | 0.0529 | 0.0519 | 0.0509 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $L_3(t)$ | 0.1283 | 0.1283 | 0.1283 | 0.1283 | 0.1283 | 0.1283 | 0.1283 |
| | $L_4(t)$ | 0.4358 | 0.4358 | 0.4358 | 0.4358 | 0.4358 | 0.4358 | 0.4358 |
| | $L(t)$ | 3.1486 | 3.1475 | 3.1464 | 3.1453 | 3.1443 | 3.1433 | 3.1423 |
| | $W_1(t)$ | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 |
| | $W_2(t)$ | 0.0748 | 0.0748 | 0.0747 | 0.0746 | 0.0745 | 0.0744 | 0.0743 |
| | $W_3(t)$ | 0.1446 | 0.1446 | 0.1446 | 0.1446 | 0.1446 | 0.1446 | 0.1446 |
| | $W_4(t)$ | 0.1531 | 0.1531 | 0.1531 | 0.1531 | 0.1531 | 0.1531 | 0.1531 |
| $\mu_3$=11 | $L_1(t)$ | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 |
| | $L_2(t)$ | 0.0662 | 0.0662 | 0.0662 | 0.0662 | 0.0662 | 0.0662 | 0.0662 |
| | $L_3(t)$ | 0.1232 | 0.1212 | 0.1193 | 0.1174 | 0.1155 | 0.1137 | 0.1119 |
| | $L_4(t)$ | 0.4358 | 0.4358 | 0.4358 | 0.4358 | 0.4358 | 0.4358 | 0.4358 |
| | $L(t)$ | 3.1525 | 3.1505 | 3.1486 | 3.1467 | 3.1448 | 3.143 | 3.1412 |
| | $W_1(t)$ | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 |
| | $W_2(t)$ | 0.1508 | 0.1508 | 0.1508 | 0.1508 | 0.1508 | 0.1508 | 0.1508 |
| | $W_3(t)$ | 0.1003 | 0.1001 | 0.1028 | 0.0998 | 0.0996 | 0.0995 | 0.0992 |
| | $W_4(t)$ | 0.1531 | 0.1531 | 0.1531 | 0.1531 | 0.1531 | 0.1531 | 0.1531 |
| $\mu_4$=13 | $L_1(t)$ | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 |
| | $L_2(t)$ | 0.0662 | 0.0662 | 0.0662 | 0.0662 | 0.0662 | 0.0662 | 0.0662 |
| | $L_3(t)$ | 0.1283 | 0.1283 | 0.1283 | 0.1283 | 0.1283 | 0.1283 | 0.1283 |
| | $L_4(t)$ | 0.4102 | 0.4025 | 0.3951 | 0.3880 | 0.3810 | 0.3742 | 0.3676 |
| | $L(t)$ | 3.1320 | 3.1243 | 3.1169 | 3.1098 | 3.1028 | 3.0960 | 3.0894 |
| | $W_1(t)$ | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 |
| | $W_2(t)$ | 0.1508 | 0.1508 | 0.1508 | 0.1508 | 0.1508 | 0.1508 | 0.1508 |
| | $W_3(t)$ | 0.1384 | 0.1384 | 0.1384 | 0.1384 | 0.1384 | 0.1384 | 0.1384 |
| | $W_4(t)$ | 0.1041 | 0.1035 | 0.1030 | 0.1025 | 0.1019 | 0.1014 | 0.1010 |
| $\theta_1$ = 0.3 | $L_1(t)$ | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 |
| | $L_2(t)$ | 0.1687 | 0.1787 | 0.1886 | 0.1985 | 0.2085 | 0.2184 | 0.2283 |
| | $L_3(t)$ | 0.1283 | 0.1283 | 0.1283 | 0.1283 | 0.1283 | 0.1283 | 0.1283 |
| | $L_4(t)$ | 0.3393 | 0.3299 | 0.3206 | 0.3113 | 0.3019 | 0.2926 | 0.2832 |
| | $L(t)$ | 3.1636 | 3.1642 | 3.1648 | 3.1654 | 3.1660 | 3.1666 | 3.1671 |
| | $W_1(t)$ | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 |
| | $W_2(t)$ | 0.1632 | 0.1645 | 0.1656 | 0.1668 | 0.1682 | 0.1694 | 0.1706 |
| | $W_3(t)$ | 0.1384 | 0.1384 | 0.1384 | 0.1384 | 0.1384 | 0.1384 | 0.1384 |
| | $W_4(t)$ | 0.1435 | 0.1425 | 0.1416 | 0.1407 | 0.1398 | 0.1388 | 0.1379 |
| $\theta_2$ = 0.2 | $L_1(t)$ | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 | 2.5273 |
| | $L_2(t)$ | 0.0662 | 0.0662 | 0.0662 | 0.0662 | 0.0662 | 0.0662 | 0.0662 |
| | $L_3(t)$ | 0.1091 | 0.1155 | 0.1219 | 0.1283 | 0.1348 | 0.1412 | 0.1476 |
| | $L_4(t)$ | 0.4544 | 0.4482 | 0.442 | 0.4358 | 0.4295 | 0.4233 | 0.4171 |
| | $L(t)$ | 3.1570 | 3.1572 | 3.1574 | 3.1576 | 3.1578 | 3.1580 | 3.1582 |
| | $W_1(t)$ | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 | 0.5711 |
| | $W_2(t)$ | 0.1508 | 0.1508 | 0.1508 | 0.1508 | 0.1508 | 0.1508 | 0.1508 |
| | $W_3(t)$ | 0.1364 | 0.1371 | 0.1378 | 0.1384 | 0.1391 | 0.1399 | 0.1405 |
| | $W_4(t)$ | 0.1550 | 0.1544 | 0.1538 | 0.1531 | 0.1525 | 0.1519 | 0.1512 |
| P = 0.5 | $L_1(t)$ | 4.8129 | 5.0888 | 5.3664 | 5.6454 | 5.9253 | 6.206 | 6.4871 |
| | $L_2(t)$ | 0.126 | 0.1332 | 0.1405 | 0.1478 | 0.1551 | 0.1625 | 0.1699 |
| | $L_3(t)$ | 0.2444 | 0.2584 | 0.2725 | 0.2867 | 0.3009 | 0.3151 | 0.3294 |
| | $L_4(t)$ | 0.8299 | 0.8774 | 0.9253 | 0.9734 | 1.0217 | 1.0707 | 1.1185 |
| | $L(t)$ | 6.0132 | 6.3578 | 6.7047 | 7.0533 | 7.4030 | 7.7543 | 8.1049 |
| | $W_1(t)$ | 1.0434 | 1.1010 | 1.1607 | 1.2178 | 1.2767 | 1.3360 | 1.3956 |
| | $W_2(t)$ | 0.1586 | 0.1595 | 0.1604 | 0.1614 | 0.1623 | 0.1634 | 0.1643 |
| | $W_3(t)$ | 0.1521 | 0.1538 | 0.1554 | 0.1571 | 0.1587 | 0.1604 | 0.1620 |
| | $W_4(t)$ | 0.1984 | 0.2039 | 0.2094 | 0.2150 | 0.2206 | 0.2264 | 0.2320 |
| A = 20 | $L_1(t)$ | 3.9234 | 4.1352 | 4.3490 | 4.5645 | 4.7816 | 5.000 | 5.2195 |
| | $L_2(t)$ | 0.1027 | 0.1083 | 0.1139 | 0.1195 | 0.1252 | 0.1309 | 0.1367 |
| | $L_3(t)$ | 0.1992 | 0.2100 | 0.2208 | 0.2318 | 0.2428 | 0.2539 | 0.2650 |
| | $L_4(t)$ | 0.6765 | 0.7130 | 0.7499 | 0.787 | 0.8245 | 0.8621 | 0.9000 |
| | $L(t)$ | 4.9018 | 5.1665 | 5.4336 | 5.7028 | 5.9741 | 6.2469 | 6.5212 |
| | $W_1(t)$ | 0.8606 | 0.9047 | 0.9492 | 0.9942 | 1.0397 | 1.0854 | 1.1314 |
| | $W_2(t)$ | 0.1557 | 0.1566 | 0.1574 | 0.1581 | 0.1588 | 0.1597 | 0.1605 |
| | $W_3(t)$ | 0.1473 | 0.1486 | 0.1498 | 0.1512 | 0.1525 | 0.1539 | 0.1552 |
| | $W_4(t)$ | 0.1817 | 0.1861 | 0.1905 | 0.1948 | 0.1993 | 0.2038 | 0.2083 |

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## VIII . STEADY STATE ANALYSIS

In this section we study the steady-state analysis of queuing model .The Joint Probability generating function of number of customers in first, second,….k$^{th}$ queues respectively in steady state is $\lim_{t\to\infty} P(Z_1, Z_2, \ldots Z_k ; t) = P(Z_1, Z_2, \ldots Z_k) =$

$$\exp\left[\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\ldots\sum_{r_k=0}^{r_{k-1}}(-1)^{r_2}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\ldots\binom{r_{k-1}}{r_k}C_m \left\{(z_1-1)+\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}+\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}+\ldots+\frac{\theta_{k-1}\mu_1(z_k-1)}{\mu_k-\mu_1}\right\}^{r_1-r_2}\left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_2-r_3}\left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_3-r_4}\ldots\left\{\frac{\theta_{k-2}\mu_1(z_{k-1}-1)}{\mu_{k-1}-\mu_1}\right\}^{r_{k-1}-r_k}\left\{\frac{\theta_{k-1}\mu_1(z_k-1)}{\mu_k-\mu_1}\right\}^{r_k}\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\ldots+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k}\right\}\right] \quad (115)$$

## VIII.A.CHARACTERISTICS OF THE MODEL UNDER EQUILIBRIUM

Putting $z_1 = 0, z_2 = 0, \ldots, z_k = 0$ in (115) and expanding we get the probability that the k-server system is empty in steady state as $P(0,0,..0) =$

$$\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\ldots\sum_{r_k=0}^{r_{k-1}}(-1)^{r_2+r_1}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\ldots\binom{r_{k-1}}{r_k}C_m\left(1+\frac{\theta_1\mu_1}{\mu_2-\mu_1}+\frac{\theta_2\mu_1}{\mu_3-\mu_1}+\ldots+\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_1-r_2}\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^{r_2-r_3}\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_3-r_4}\ldots\left(\frac{\theta_{k-2}\mu_1}{\mu_{k-1}-\mu_1}\right)^{r_{k-1}-r_k}\left(\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_k}\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\ldots+\mu_{k-1}(r_{k-1}-r_k)+\mu_k r_k}\right\}\right] \quad (116)$$

## VIII.B.PERFORMANCE ANALYSIS OF FIRST QUEUE

Putting $z_2 = 1, z_3 = 1, \ldots, z_k = 1$ in (115) we get probability generating function of first queue size as

$$P(Z_1) = \exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m} C_m\binom{m}{r_1}(z_1-1)^{r_1}\left\{\frac{1}{\mu_1 r_1}\right\}\right] \quad (117)$$

Mean number of customers in first queue is

$$E(N_1) = L_1 = \frac{\lambda}{\mu_1}E(X) \quad (118)$$

Where E($X$) is the mean of batch size arrivals to first queue and is given by $E(X) = \sum_{m=1}^{\infty} m.C_m$

Putting $Z_1 = 0$ in (117) we get the probability that the first queue is empty as

$$P(0,\ldots,.) = \exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m} C_m\binom{m}{r_1}\left\{\frac{(-1)^{r_1}}{\mu_1 r_1}\right\}\right] \quad (119)$$

Utilization of first server is

$$U_1 = 1 - P(0,\ldots,.) = 1 - \exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m} C_m\binom{m}{r_1}\left\{\frac{(-1)^{r_1}}{\mu_1 r_1}\right\}\right] \quad (120)$$

Throughput of first server is

$$\text{Thp1} = \mu_1.U_1 = \mu_1\left\{1-\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m} C_m\binom{m}{r_1}\left\{\frac{(-1)^{r_1}}{\mu_1 r_1}\right\}\right]\right\} \quad (121)$$

Average waiting time of a customer in first queue is

$$W_1 = \frac{L_1}{Thp_1} = \frac{\left(\frac{\lambda}{\mu_1}\right)E(X)}{\left[1-\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m} C_m\binom{m}{r_1}\left\{\frac{(-1)^{r_1}}{\mu_1 r_1}\right\}\right]\right]} \quad (122)$$

Variance of the number of customers in first queue is

$$V(Z_1) = V_1 = \left(\frac{2\lambda}{\mu_1}\right)\sum_{m=2}^{\infty} C_m\binom{m}{2} \quad (123)$$

Coefficient of variation of the number of customers in first queue

$$CV_1 = \frac{\sqrt{V_1}}{L_1}x100 = \frac{\sqrt{\left(\frac{2\lambda}{\mu_1}\right)\sum_{m=2}^{\infty} C_m\binom{m}{2}}}{\frac{\lambda}{\mu_1}E(X)}x100 \quad (124)$$

## VIII.C. PERFORMANCE ANALYSIS OF i$^{th}$ QUEUE FOR i =2,3,..k.

Putting $z_1 = 1, z_2 = 1, z_3 = 1, \ldots, z_{i-1} = 1$ in (115) we get probability generating function of i$^{th}$ queue size distribution as

$P(Z_i)=$

$$\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_1}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1(z_i-1)}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (125)$$

Mean number of customers in i$^{th}$ queue is

$$E(N_i) = L_i = \left(\frac{\lambda\theta_{i-1}}{\mu_i}\right)E(X) \quad (126)$$

Where $E(X)$ is the mean of batch size arrivals at i$^{th}$ queue and $E(X) = \sum_m m.C_m$

Putting $z_i = 0$ in (125) we get the probability that the i$^{th}$ queue is empty as

$$P(.,.,0,..) = \exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (127)$$

Utilization of i$^{th}$ server is $U_i = 1 - P(.,.,..,0;t) =$

$$1-\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right] \quad (128)$$

Throughput of i$^{th}$ server is $Thp_i = \mu_i.U_i =$

$$\mu_i\left|1-\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right]\right| \quad (129)$$

Average waiting time of a customer in i$^{th}$ queue is $W_i = \frac{L_i}{Thp_i} =$

$$\frac{\left(\frac{\lambda\theta_{i-1}}{\mu_i}\right)E(X)}{\left|1-\exp\left[\lambda\sum_{m=1}^{\infty}\sum_{r_1=1}^{m}\sum_{r_i=0}^{r_1}(-1)^{r_i+r_1}C_m\binom{m}{r_1}\binom{r_1}{r_i}\left\{\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right\}^{r_i}\left\{\frac{1}{\mu_1(r_1-r_i)+\mu_i r_i}\right\}\right]\right|} \quad (130)$$

Variance of the number of customers in i$^{th}$ queue is $V(z_i) = V_i =$

$$\lambda\left[\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)^2\sum_{m=1}^{\infty}\binom{m}{2}.C_m\left\{\left(\frac{1}{\mu_1}\right)-\left(\frac{4}{\mu_1+\mu_i}\right)+\left(\frac{1}{\mu_i}\right)\right\}+\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)\left\{\left(\frac{1}{\mu_1}\right)+\left(\frac{1}{\mu_i}\right)\right\}E(X)\right] \quad (131)$$

Coefficient of varition in number of the number of customers in i$^{th}$ queue $CV_i = \frac{\sqrt{V_i}}{L_i}x100 =$

$$\frac{\sqrt{\lambda\left[\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)^2\sum_{m=1}^{\infty}\binom{m}{2}C_m\left\{\left(\frac{1}{\mu_1}\right)-\left(\frac{4}{\mu_1+\mu_i}\right)+\left(\frac{1}{\mu_i}\right)\right\}+\left(\frac{\theta_{i-1}\mu_1}{\mu_i-\mu_1}\right)\left\{\left(\frac{1}{\mu_1}\right)+\left(\frac{1}{\mu_i}\right)\right\}E(X)\right]}}{\left(\frac{\lambda\theta_{i-1}}{\mu_i}\right)E(X)}x100 \quad (132)$$

## VIII.D.PERFORMANCE MEASURES OF THE STAEDY STATE 4-SERVER MODEL WHEN BATCH SIZE DISTRIBUTION IS BINOMIAL

The Joint Probability generating function of number of customers in first, second, third and fourth queues respectively in steady state is $P(Z_1, Z_2, Z_3, Z_4) =$

$$\exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\sum_{r_4=0}^{r_3}(-1)^{r_2}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\binom{r_3}{r_4}\left(\frac{q^{m-1}p}{1-q^A}\right)\left\{(z_1-1)+\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}+\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}+\frac{\theta_3\mu_1(z_4-1)}{\mu_4-\mu_1}\right\}^{r_1-r_2}\left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_2-r_3}\left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_3-r_4}\left\{\frac{\theta_3\mu_1(z_4-1)}{\mu_4-\mu_1}\right\}^{r_4}\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4}\right\}\right] \quad (133)$$

## VIII.E.CHARACTERISTICS OF THE MODEL

Putting $z_1 = 0, z_2 = 0, \ldots, z_k = 0$ in ( 133 ) and expanding we get the probability that the 4-server system is empty in steady state $P(0,0,0,0)$=

$$\exp\left[\lambda\sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_2=0}^{r_1}\sum_{r_3=0}^{r_2}\sum_{r_4=0}^{r_3}(-1)^{r_2+r_1}\binom{m}{r_1}\binom{r_1}{r_2}\binom{r_2}{r_3}\binom{r_3}{r_4}\left(\frac{q^{m-1}p}{1-q^A}\right)\left(1+\frac{\theta_1\mu_1}{\mu_2-\mu_1}+\frac{\theta_2\mu_1}{\mu_3-\mu_1}+\ldots+\frac{\theta_{k-1}\mu_1}{\mu_k-\mu_1}\right)^{r_1-r_2}\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^{r_2-r_3}\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_3-r_4}\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^{r_4}\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2(r_2-r_3)+\mu_3(r_3-r_4)+\mu_4 r_4}\right\}\right]$$

(134)

### VIII.F. PERFORMANCE ANALYSIS OF FIRST QUEUE

Putting $z_2 = 1, z_3 = 1, z_4 = 1$ in ( 133 ) we get probability generating function of first queue size as

$$P(Z_1) = exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \binom{q^{m-1}p}{1-q^A}\binom{m}{r_1}(z_1-1)^{r_1}\left\{\frac{1}{\mu_1 r_1}\right\}\right] \quad (138)$$

Mean number of customers in first queue is

$$L_1 = \left[\frac{\lambda}{\mu_1}\right]\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\} \quad (139)$$

Putting $Z_1 = 0$ in (138 ) we get the probability that the first queue is empty as

$$P(0,.,.,.) = exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} (-1)^{r_1}\binom{m}{r_1}\binom{q^{m-1}p}{1-q^A}\left\{\frac{1}{\mu_1 r_1}\right\}\right] \quad (140)$$

Utilization of first server is

$$U_1 = 1 - P(0,.,.,.) = 1 - exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} (-1)^{r_1}\binom{m}{r_1}\binom{q^{m-1}p}{1-q^A}\left\{\frac{1}{\mu_1 r_1}\right\}\right] \quad (141)$$

Throughput of first server is

$$\text{Thp1} = \mu_1 . U_1 = \mu_1 . \left\{1 - exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} (-1)^{r_1}\binom{m}{r_1}\binom{q^{m-1}p}{1-q^A}\left\{\frac{1}{\mu_1 r_1}\right\}\right]\right\} \quad (142)$$

Average waiting time of a customer in first queue is

$$W_1 = \frac{L_1}{Thp_1} = \frac{\left[\frac{\lambda}{\mu_1^2}\right]\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}}{\left\{1-exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m}(-1)^{r_1}\binom{m}{r_1}\binom{q^{m-1}p}{1-q^A}\left(\frac{1}{\mu_1 r_1}\right)\right]\right\}} \quad (143)$$

Variance of number of customers in first queue is

$$V_1 = \lambda \sum_{m=1}^{A} \left[\binom{m}{2}\binom{q^{m-1}p}{1-q^A}\left(\frac{2}{\mu_1}\right)\right] \quad (144)$$

Coefficient of variation of number of customers in first queue is

$$CV_1 = \frac{\sqrt{V_1}}{L_1} x100 = \frac{\sqrt{\lambda \sum_{m=1}^{A} \left[\binom{m}{2}\binom{q^{m-1}p}{1-q^A}\left(\frac{2}{\mu_1}\right)\right]}}{\left[\frac{\lambda}{\mu_1}\right]\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}} x100 \quad (145)$$

### VIII.G.PERFORMANCE ANALYSIS OF SECOND QUEUE

Putting $z_1 = 1, z_3 = 1, z_4 = 1$ in (133 )we get probability generating function of second queue size distribution as

$$P(Z_2) = exp\left[\frac{\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2}\binom{m}{r_1}\binom{r_1}{r_2}\binom{q^{m-1}p}{1-q^A}\left\{\frac{\theta_1\mu_1(z_2-1)}{\mu_2-\mu_1}\right\}^{r_1}}{\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}}\right] \quad (146)$$

Mean number of customers in second queue is

$$L_2 = \left[\left(\frac{\lambda\theta_1}{\mu_2}\right)\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\right] \quad (147)$$

Putting $Z_2 = 0$ in (146) we get the probability that the second queue is empty as $P(.,0,.,.) =$

$$exp\left[\frac{\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1}\binom{m}{r_1}\binom{r_1}{r_2}\binom{q^{m-1}p}{1-q^A}\left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1}}{\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}}\right] \quad (148)$$

Utilization of second server is $U_2 = 1 - P(.,0,.,.) = 1 -$

$$exp\left[\frac{\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1}\binom{m}{r_1}\binom{r_1}{r_2}\binom{q^{m-1}p}{1-q^A}\left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1}}{\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}}\right] \quad (149)$$

Throughput of second server is $Thp_2 = \mu_2 . U_2 = \mu_2 .$

$$\left[1 - exp\left[\frac{\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1} (-1)^{r_2+r_1}\binom{m}{r_1}\binom{r_1}{r_2}\binom{q^{m-1}p}{1-q^A}\left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}^{r_1}}{\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}}\right]\right] \quad (150)$$

Average waiting time of a customer in second queue

$$W_2 = \frac{L_2}{Thp_2} = \frac{\left[\left(\frac{\lambda\theta_1}{\mu_2^2}\right)\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\right]}{\left[1-exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_2=0}^{r_1}(-1)^{r_2+r_1}\binom{m}{r_1}\binom{r_1}{r_2}\binom{q^{m-1}p}{1-q^A}\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^{r_1}\left\{\frac{1}{\mu_1(r_1-r_2)+\mu_2 r_2}\right\}\right]\right]} \quad (151)$$

Variance of number of customers in second queue is $V_2 =$

$$\lambda\left[\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^2 \sum_{m=1}^{A}\binom{m}{2}\binom{q^{m-1}p}{1-q^A}.\left\{\frac{1}{\mu_1}-\frac{4}{\mu_2+\mu_1}+\frac{1}{\mu_2}\right\}+\left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\left\{\frac{1}{\mu_1}-\frac{1}{\mu_2}\right\}\right] \quad (152)$$

Coefficient of variation of number of customers in second queue is

$$CV_2 = \frac{\sqrt{V_2}}{L_2} x100 = \frac{\sqrt{\lambda\left[\left(\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right)^2 \sum_{m=1}^{A}\binom{m}{2}\binom{q^{m-1}p}{1-q^A}\left\{\frac{1}{\mu_1}-\frac{4}{\mu_2+\mu_1}+\frac{1}{\mu_2}\right\}+\left\{\frac{\theta_1\mu_1}{\mu_2-\mu_1}\right\}\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\left\{\frac{1}{\mu_1}-\frac{1}{\mu_2}\right\}\right]}}{\left[\left(\frac{\lambda\theta_1}{\mu_2}\right)\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\right]} x100 \quad (153)$$

### VIII.H.PERFORMANCE ANALYSIS OF THIRD QUEUE

Putting $z_1 = 1, z_2 = 1, z_4 = 1$ in (136 ) we get probability generating function of third queue size distribution as $P(Z_3) =$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3}\binom{m}{r_1}\binom{r_1}{r_3}\binom{q^{m-1}p}{1-q^A}\left\{\frac{\theta_2\mu_1(z_3-1)}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right] \quad (154)$$

Mean number of customers in third queue is

$$L_3 = \left[\left(\frac{\lambda\theta_2}{\mu_3}\right)\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\right] \quad (155)$$

Putting $Z_3 = 0$ in (154 ) we get the probability that the third queue is empty as $P(.,.,0,.) =$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1}\binom{m}{r_1}\binom{r_1}{r_3}\binom{q^{m-1}p}{1-q^A}\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right] \quad (156)$$

Utilization of third server is $U_3 = 1 - P(.,.,0,.) = 1 -$

$$exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1}\binom{q^{m-1}p}{1-q^A}\binom{m}{r_1}\binom{r_1}{r_3}\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right] \quad (157)$$

Throughput of third server is $Thp_3 = \mu_3 . U_3 = \mu_3 .$

$$\left|1 - exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1} (-1)^{r_3+r_1}\binom{m}{r_1}\binom{r_1}{r_3}\binom{q^{m-1}p}{1-q^A}\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]\right] \quad (158)$$

Average waiting time of a customer in third queue is

$$W_3 = \frac{L_3}{Thp_3} = \frac{\left[\left(\frac{\lambda\theta_2}{\mu_3^2}\right)\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\right]}{\left[1-exp\left[\lambda \sum_{m=1}^{A} \sum_{r_1=1}^{m} \sum_{r_3=0}^{r_1}(-1)^{r_3+r_1}\binom{m}{r_1}\binom{r_1}{r_3}\binom{q^{m-1}p}{1-q^A}\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^{r_1}\left\{\frac{1}{\mu_1(r_1-r_3)+\mu_3 r_3}\right\}\right]\right]} \quad (159)$$

Variance of number of customers in third queue is $V_3(t) =$

$$\lambda\left[\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^2 \sum_{m=1}^{A}\binom{m}{2}\binom{q^{m-1}p}{1-q^A}.\left\{\frac{1}{\mu_1}-\frac{4}{\mu_3+\mu_1}+\frac{1}{\mu_3}\right\}+\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\left\{\frac{1}{\mu_1}-\frac{1}{\mu_3}\right\}\right] \quad (160)$$

Coefficient of variation of number of customers in third queue is

$$CV_3 = \frac{\sqrt{V_3}}{L_3} x100 = \frac{\sqrt{\lambda\left[\left(\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right)^2 \sum_{m=1}^{A}\binom{m}{2}\binom{q^{m-1}p}{1-q^A}\left\{\frac{1}{\mu_1}-\frac{4}{\mu_3+\mu_1}+\frac{1}{\mu_3}\right\}+\left\{\frac{\theta_2\mu_1}{\mu_3-\mu_1}\right\}\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\left\{\frac{1}{\mu_1}-\frac{1}{\mu_3}\right\}\right]}}{\left[\left(\frac{\lambda\theta_2}{\mu_3}\right)\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\right]} x100 \quad (161)$$

### VIII.I. PERFORMANCE ANALYSIS OF FOURTH QUEUE

Putting $z_1 = 1, z_2 = 1, z_3 = 1$ in (136 )we get probability generating function of fourth queue size distribution as

$P(Z_4) =$

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$$exp\left[\frac{\lambda \sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4}\binom{m}{r_1}\binom{r_1}{r_4}\left(\frac{q^{m-1}p}{1-q^A}\right)\left\{\frac{\theta_3\mu_1(z_4-1)}{\mu_4-\mu_1}\right\}^{r_1}}{\left\{\frac{1}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}}\right] \quad (162)$$

Mean number of customers in fourth queue is $L_4 = \left[\left(\frac{\lambda\theta_3}{\mu_4}\right)\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\right]$ (163)

Putting $z_4 = 0$ in (162) we get the probability that the fourth queue is empty as $P(.,.,.,.,0) =$

$$exp\left[\frac{\lambda \sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}\binom{m}{r_1}\binom{r_1}{r_4}\left(\frac{q^{m-1}p}{1-q^A}\right)\left\{\frac{\theta_3\mu_4}{\mu_4-\mu_1}\right\}^{r_1}}{\left\{\frac{1}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}}\right] \quad (164)$$

Utilization of fourth server is $U_4 = 1 - P(.,.,.,.,0) =$

$$1 - exp\left[\frac{\lambda \sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}\binom{m}{r_1}\binom{r_1}{r_4}\left(\frac{q^{m-1}p}{1-q^A}\right)\left\{\frac{\theta_3\mu_4}{\mu_4-\mu_1}\right\}^{r_1}}{\left\{\frac{1}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}}\right] \quad (165)$$

Throughput of fourth server is $Thp_4 = \mu_4.U_4 = \mu_4.$

$$\left[1 - exp\left[\frac{\lambda \sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}\left(\frac{q^{m-1}p}{1-q^A}\right)\binom{m}{r_1}\binom{r_1}{r_4}\left\{\frac{\theta_3\mu_4}{\mu_4-\mu_1}\right\}^{r_1}}{\left\{\frac{1}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}}\right]\right] \quad (166)$$

Average waiting time of a customer in fourth queue is $W_4 = \frac{L_4}{Thp_4} =$

$$\frac{\left[\left(\frac{\lambda\theta_3}{\mu_4^2}\right)\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\right]}{\left[1-exp\left[\lambda \sum_{m=1}^{A}\sum_{r_1=1}^{m}\sum_{r_4=0}^{r_1}(-1)^{r_4+r_1}\binom{m}{r_1}\binom{r_1}{r_4}\left(\frac{q^{m-1}p}{1-q^A}\right)\left\{\frac{\theta_3\mu_4}{\mu_4-\mu_1}\right\}^{r_1}\left\{\frac{1}{\mu_1(r_1-r_4)+\mu_4 r_4}\right\}\right]\right]} \quad (167)$$

Variance of number of customers in fourth queue is $V_4 =$

$$\lambda\left[\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^2\sum_{m=1}^{A}\binom{m}{2}\left(\frac{q^{m-1}p}{1-q^A}\right).\left\{\frac{1}{\mu_1}-\frac{4}{\mu_4+\mu_1}+\frac{1}{\mu_4}\right\}+\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\left\{\frac{1}{\mu_1}-\frac{1}{\mu_4}\right\}\right] \quad (168)$$

Coefficient of variation of number of customers in fourth queue is $CV_4 = \frac{\sqrt{V_4}}{L_4}x100 =$

$$\frac{\sqrt{\lambda\left[\left(\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right)^2\sum_{m=1}^{A}\binom{m}{2}\left(\frac{q^{m-1}p}{1-q^A}\right)\left\{\frac{1}{\mu_1}-\frac{4}{\mu_4+\mu_1}+\frac{1}{\mu_4}\right\}+\left\{\frac{\theta_3\mu_1}{\mu_4-\mu_1}\right\}\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\left\{\frac{1}{\mu_1}-\frac{1}{\mu_4}\right\}\right]}}{\left[\left(\frac{\lambda\theta_3}{\mu_4}\right)\left\{\frac{1-q^A-pAq^A}{p(1-q^A)}\right\}\right]}x100 \quad (169)$$

## IX.COMPARITIVE STUDY

A comparative study between transient and steady state of the system is carried with values of time t = 0.1, 0.2, 0.3, 0.4,0.5,1,2,3,4 and 5 is carried.

The difference and percentage of variation in all performance measures are calculated and presented in Table.6.

FromTable.6 it is observed that there is high significant difference between transient behavior and steady state behavior of the model. At t=0.1 the variation and percentage of variation in measures is highly significant which can be observed in last two columns of Table.6. and as we move through the values of t=0.2 to 0.5 the variation narrows down. At t=1 this percentage of variation is reduced further and some of the measures differ very closely. This explains that as t increases the difference between transient and steady state behaviour become negligible and from t=3 onwards we find that there is no difference between them. This indicates that the system attains equilibrium after time t=3 units.

## X. CONCLUSION

This paper focuses on the development and analysis of a K-node tandem Queuing model with bulk arrivals and state dependent service rates. It is assumed that the K servers are connected in tandem where the customers arrive to the first Queue in batches and after getting service at first server they may join at any one of the (K-1) Queues which are in parallel with certain probability. The service rate of each service station is dependent on the content of the buffers connected to it. The system characteristics such as average number of customers in the Queue, probability of idleness of each service station, throughput of the nodes, average waiting time customers in each Queue, utilisation of each server are discussed . The sensitivity of the model with respect to the changes in parameters is explained. The influence of bulk size distribution parameters on system performance measures is discussed. It is shown that the congestion in the queues and the mean waiting time or delay in service can be reduced by regulating the bulk size distribution parameters. A comparative study of the model between transient and steady state revealed that the time t has significant influence on the performance measures. The proposed model is very useful for scheduling the Communication networks at LAN,VAN and MAN, Production systems, Road traffic, Operation of dams, Packet switching communication systems . This model also includes some of the earlier models as particular cases for specific values of the parameters.

**Table6.Comparative tables of performance measures for values of t=0.1,0.2,0.3,0.4,0.5,1,2,3,4 and5**

$(\lambda = 15, \mu_1 = 6, \mu_2 = 7, \mu_3 = 8, \mu_4 = 9, \theta_1 = 0.1, \theta_2 = 0.2, p = 0.2, A = 10)$

| Time t | Performance | Transient State | Steady State | Difference between Transient and Steady state | % of variation |
|---|---|---|---|---|---|
| t = 0.1 | $L_1$ | 2.5273 | 5.6015 | -3.074 | -54.882 |
| | $L_2$ | 0.0662 | 0.4801 | -0.414 | -86.211 |
| | $L_3$ | 0.1283 | 0.8402 | -0.712 | -84.730 |
| | $L_4$ | 0.4358 | 2.6140 | -2.178 | -83.328 |
| | $L$ | 3.1576 | 9.5358 | -6.378 | -66.887 |
| | $P_{0000}$ | 0.2301 | 0.0028 | 0.227 | 8117.857 |
| | $P_{0...}$ | 0.2625 | 0.0228 | 0.240 | 1051.316 |
| | $P_{.0..}$ | 0.9373 | 0.6248 | 0.313 | 50.016 |
| | $P_{..0.}$ | 0.8841 | 0.4455 | 0.439 | 98.451 |
| | $P_{...0}$ | 0.6838 | 0.0991 | 0.585 | 590.010 |
| | $U_1$ | 0.7375 | 0.9772 | -0.240 | -24.529 |

| | | | | | |
|---|---|---|---|---|---|
| | $U_2$ | 0.0627 | 0.3752 | -0.313 | -83.289 |
| | $U_3$ | 0.1159 | 0.5545 | -0.439 | -79.098 |
| | $U_4$ | 0.3162 | 0.9009 | -0.585 | -64.902 |
| | $Thp_1$ | 4.4250 | 5.8632 | -1.438 | -24.529 |
| | $Thp_2$ | 0.4389 | 2.6264 | -2.188 | -83.289 |
| | $Thp_3$ | 0.9272 | 4.4360 | -3.509 | -79.098 |
| | $Thp_4$ | 2.8458 | 8.1081 | -5.262 | -64.902 |
| | $W_1$ | 0.5711 | 0.9554 | -0.384 | -40.218 |
| | $W_2$ | 0.1508 | 0.1828 | -0.032 | -17.487 |
| | $W_3$ | 0.1384 | 0.1894 | -0.051 | -26.943 |
| | $W_4$ | 0.1531 | 0.3224 | -0.169 | -52.500 |

| Time t | Performance | Transient State | Steady State | Difference between Transient and Steady state | % of variation |
|---|---|---|---|---|---|
| | $L_1$ | 3.9143 | 5.6015 | -1.687 | -43.103 |
| | $L_2$ | 0.1782 | 0.4801 | -0.302 | -169.416 |
| | $L_3$ | 0.3369 | 0.8402 | -0.503 | -149.392 |
| | $L_4$ | 1.1162 | 2.6140 | -1.498 | -134.187 |
| | $L$ | 5.5456 | 9.5358 | -3.990 | -71.953 |
| | $P_{0000}$ | 0.0619 | 0.0028 | 0.059 | 95.477 |
| | $P_{0...}$ | 0.1002 | 0.0228 | 0.077 | 77.246 |
| | $P_{.0..}$ | 0.8407 | 0.6248 | 0.216 | 25.681 |
| | $P_{..0.}$ | 0.7258 | 0.4455 | 0.280 | 38.619 |
| | $P_{...0}$ | 0.3871 | 0.0991 | 0.288 | 74.399 |
| | $U_1$ | 0.8998 | 0.9772 | -0.077 | -8.602 |
| t = 0.2 | $U_2$ | 0.1593 | 0.3752 | -0.216 | -135.530 |
| | $U_3$ | 0.2742 | 0.5545 | -0.280 | -102.225 |
| | $U_4$ | 0.6129 | 0.9009 | -0.288 | -46.990 |
| | $Thp_1$ | 5.3988 | 5.8632 | -0.464 | -8.602 |
| | $Thp_2$ | 1.1151 | 2.6264 | -1.511 | -135.530 |
| | $Thp_3$ | 2.1936 | 4.4360 | -2.242 | -102.225 |
| | $Thp_4$ | 5.5161 | 8.1081 | -2.592 | -46.990 |
| | $W_1$ | 0.7250 | 0.9554 | -0.230 | -31.769 |
| | $W_2$ | 0.1598 | 0.1828 | -0.023 | -14.387 |
| | $W_3$ | 0.1536 | 0.1894 | -0.036 | -23.324 |
| | $W_4$ | 0.2024 | 0.3224 | -0.120 | -59.322 |

| Time t | Performance | Transient State | Steady State | Difference between Transient and Steady state | % of variation |
|---|---|---|---|---|---|
| | $L_1$ | 4.6755 | 5.6015 | -0.926 | -19.805 |
| | $L_2$ | 0.2773 | 0.4801 | -0.203 | -73.134 |
| | $L_3$ | 0.5133 | 0.8402 | -0.327 | -63.686 |
| | $L_4$ | 1.6691 | 2.6140 | -0.945 | -56.611 |
| | $L$ | 7.1352 | 9.5358 | -2.401 | -33.644 |
| | $P_{0000}$ | 0.0216 | 0.0028 | 0.019 | 87.037 |
| | $P_{0...}$ | 0.0539 | 0.0228 | 0.031 | 57.699 |
| | $P_{.0..}$ | 0.7634 | 0.6248 | 0.139 | 18.156 |
| | $P_{..0.}$ | 0.6134 | 0.4455 | 0.168 | 27.372 |
| t = 0.3 | $P_{...0}$ | 0.2402 | 0.0991 | 0.141 | 58.743 |
| | $U_1$ | 0.9461 | 0.9772 | -0.031 | -3.287 |
| | $U_2$ | 0.2366 | 0.3752 | -0.139 | -58.580 |
| | $U_3$ | 0.3866 | 0.5545 | -0.168 | -43.430 |
| | $U_4$ | 0.7598 | 0.9009 | -0.141 | -18.571 |
| | $Thp_1$ | 5.6766 | 5.8632 | -0.187 | -3.287 |
| | $Thp_2$ | 1.6562 | 2.6264 | -0.970 | -58.580 |
| | $Thp_3$ | 3.0928 | 4.4360 | -1.343 | -43.430 |
| | $Thp_4$ | 6.8382 | 8.1081 | -1.270 | -18.571 |

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|---|---|---|---|---|---|
| | $W_1$ | 0.8236 | 0.9554 | -0.132 | -15.992 |
| | $W_2$ | 0.1674 | 0.1828 | -0.015 | -9.178 |
| | $W_3$ | 0.1660 | 0.1894 | -0.023 | -14.123 |
| | $W_4$ | 0.2441 | 0.3224 | -0.078 | -32.083 |

| Time t | Performance | Transient State | Steady State | Difference between Transient and Steady state | % of variation |
|---|---|---|---|---|---|
| t = 0.4 | $L_1$ | 5.0933 | 5.6015 | -0.508 | -9.978 |
| | $L_2$ | 0.3504 | 0.4801 | -0.130 | -37.015 |
| | $L_3$ | 0.6381 | 0.8402 | -0.202 | -31.672 |
| | $L_4$ | 2.0454 | 2.6140 | -0.569 | -27.799 |
| | $L$ | 8.1272 | 9.5358 | -1.409 | -17.332 |
| | $P_{0000}$ | 0.0101 | 0.0028 | 0.007 | 72.277 |
| | $P_{0...}$ | 0.0371 | 0.0228 | 0.014 | 38.544 |
| | $P_{.0..}$ | 0.7106 | 0.6248 | 0.086 | 12.074 |
| | $P_{..0.}$ | 0.5437 | 0.4455 | 0.098 | 18.061 |
| | $P_{...0}$ | 0.1710 | 0.0991 | 0.072 | 42.047 |
| | $U_1$ | 0.9629 | 0.9772 | -0.014 | -1.485 |
| | $U_2$ | 0.2894 | 0.3752 | -0.086 | -29.648 |
| | $U_3$ | 0.4563 | 0.5545 | -0.098 | -21.521 |
| | $U_4$ | 0.8290 | 0.9009 | -0.072 | -8.673 |
| | $Thp_1$ | 5.7774 | 5.8632 | -0.086 | -1.485 |
| | $Thp_2$ | 2.0258 | 2.6264 | -0.601 | -29.648 |
| | $Thp_3$ | 3.6504 | 4.4360 | -0.786 | -21.521 |
| | $Thp_4$ | 7.4610 | 8.1081 | -0.647 | -8.673 |
| | $W_1$ | 0.8816 | 0.9554 | -0.074 | -8.368 |
| | $W_2$ | 0.1730 | 0.1828 | -0.010 | -5.683 |
| | $W_3$ | 0.1748 | 0.1894 | -0.015 | -8.353 |
| | $W_4$ | 0.2741 | 0.3224 | -0.048 | -17.599 |

| Time t | Performance | Transient State | Steady State | Difference between Transient and Steady state | % of variation |
|---|---|---|---|---|---|
| t = 0.5 | $L_1$ | 5.3226 | 5.6015 | -0.2789 | -5.2399 |
| | $L_2$ | 0.3998 | 0.4801 | -0.0803 | -20.0850 |
| | $L_3$ | 0.7191 | 0.8402 | -0.1211 | -16.8405 |
| | $L_4$ | 2.2817 | 2.6140 | -0.3323 | -14.5637 |
| | $L$ | 8.7232 | 9.5358 | -0.8126 | -9.3154 |
| | $P_{0000}$ | 0.0061 | 0.0028 | 0.0033 | 54.0984 |
| | $P_{0...}$ | 0.0299 | 0.0228 | 0.0071 | 23.7458 |
| | $P_{.0..}$ | 0.6768 | 0.6248 | 0.0520 | 7.6832 |
| | $P_{..0.}$ | 0.5023 | 0.4455 | 0.0568 | 11.3080 |
| | $P_{...0}$ | 0.1370 | 0.0991 | 0.0379 | 27.6642 |
| | $U_1$ | 0.9701 | 0.9772 | -0.0071 | -0.7319 |
| | $U_2$ | 0.3232 | 0.3752 | -0.0520 | -16.0891 |
| | $U_3$ | 0.4977 | 0.5545 | -0.0568 | -11.4125 |
| | $U_4$ | 0.8630 | 0.9009 | -0.0379 | -4.3917 |
| | $Thp_1$ | 5.8206 | 5.8632 | -0.0426 | -0.7319 |
| | $Thp_2$ | 2.2624 | 2.6264 | -0.3640 | -16.0891 |
| | $Thp_3$ | 3.9816 | 4.4360 | -0.4544 | -11.4125 |
| | $Thp_4$ | 7.7670 | 8.1081 | -0.3411 | -4.3917 |
| | $W_1$ | 0.9144 | 0.9554 | -0.0409 | -4.4753 |
| | $W_2$ | 0.1767 | 0.1828 | -0.0061 | -3.4421 |
| | $W_3$ | 0.1806 | 0.1894 | -0.0088 | -4.8720 |
| | $W_4$ | 0.2938 | 0.3224 | -0.0286 | -9.7441 |

| Time t | Performance | Transient State | Steady State | Difference between Transient and Steady state | % of variation |
|---|---|---|---|---|---|
| t = 1 | $L_1$ | 5.5876 | 5.6015 | -0.0139 | -0.2488 |

| | | | | | |
|---|---|---|---|---|---|
| | $L_2$ | 0.4744 | 0.4801 | -0.0057 | -1.2015 |
| | $L_3$ | 0.8327 | 0.8402 | -0.0075 | -0.9007 |
| | $L_4$ | 2.5952 | 2.6140 | -0.0188 | -0.7244 |

| | | | | | |
|---|---|---|---|---|---|
| | $L$ | 9.4899 | 9.5358 | -0.0459 | -0.4837 |
| | $P_{0000}$ | 0.0030 | 0.0028 | 0.0002 | 6.6667 |
| | $P_{0...}$ | 0.0231 | 0.0228 | 0.0003 | 1.3414 |
| | $P_{.0..}$ | 0.6284 | 0.6248 | 0.0036 | 0.5729 |
| | $P_{..0.}$ | 0.4488 | 0.4455 | 0.0033 | 0.7353 |
| | $P_{...0}$ | 0.1009 | 0.0991 | 0.0018 | 1.7839 |
| | $U_1$ | 0.9769 | 0.9772 | -0.0003 | -0.0317 |
| | $U_2$ | 0.3716 | 0.3752 | -0.0036 | -0.9688 |
| | $U_3$ | 0.5512 | 0.5545 | -0.0033 | -0.5987 |
| | $U_4$ | 0.8991 | 0.9009 | -0.0018 | -0.2002 |
| | $Thp_1$ | 5.8613 | 5.8632 | -0.0019 | -0.0317 |
| | $Thp_2$ | 2.6012 | 2.6264 | -0.0252 | -0.9688 |
| | $Thp_3$ | 4.4096 | 4.4360 | -0.0264 | -0.5987 |
| | $Thp_4$ | 8.0919 | 8.1081 | -0.0162 | -0.2002 |
| | $W_1$ | 0.9533 | 0.9554 | -0.0021 | -0.2170 |
| | $W_2$ | 0.1824 | 0.1828 | -0.0004 | -0.2305 |
| | $W_3$ | 0.1888 | 0.1894 | -0.0006 | -0.3002 |
| | $W_4$ | 0.3207 | 0.3224 | -0.0017 | -0.5232 |

| Time t | Performance | Transient State | Steady State | Difference between Transient and Steady state | % of variation |
|---|---|---|---|---|---|
| t = 2 | $L_1$ | 5.6014 | 5.6015 | -0.0001 | -0.0018 |
| | $L_2$ | 0.4801 | 0.4801 | 0.0000 | 0.0000 |
| | $L_3$ | 0.8402 | 0.8402 | 0.0000 | 0.0000 |
| | $L_4$ | 2.6140 | 2.6140 | 0.0000 | 0.0000 |
| | $L$ | 9.5357 | 9.5358 | -0.0001 | -0.0010 |
| | $P_{0000}$ | 0.0028 | 0.0028 | 0.0000 | 0.0000 |
| | $P_{0...}$ | 0.0228 | 0.0228 | 0.0000 | 0.0000 |
| | $P_{.0..}$ | 0.6248 | 0.6248 | 0.0000 | 0.0000 |
| | $P_{..0.}$ | 0.4455 | 0.4455 | 0.0000 | 0.0000 |
| | $P_{...0}$ | 0.0991 | 0.0991 | 0.0000 | 0.0000 |
| | $U_1$ | 0.9772 | 0.9772 | 0.0000 | 0.0000 |
| | $U_2$ | 0.3752 | 0.3752 | 0.0000 | 0.0000 |
| | $U_3$ | 0.5545 | 0.5545 | 0.0000 | 0.0000 |
| | $U_4$ | 0.9009 | 0.9009 | 0.0000 | 0.0000 |
| | $Thp_1$ | 5.8632 | 5.8632 | 0.0000 | 0.0000 |
| | $Thp_2$ | 2.6264 | 2.6264 | 0.0000 | 0.0000 |
| | $Thp_3$ | 4.4360 | 4.4360 | 0.0000 | 0.0000 |
| | $Thp_4$ | 8.1081 | 8.1081 | 0.0000 | 0.0000 |
| | $W_1$ | 0.9553 | 0.9554 | 0.0000 | -0.0018 |
| | $W_2$ | 0.1828 | 0.1828 | 0.0000 | 0.0000 |
| | $W_3$ | 0.1894 | 0.1894 | 0.0000 | 0.0000 |
| | $W_4$ | 0.3224 | 0.3224 | 0.0000 | 0.0000 |

| Time t | Performance | Transient State | Steady State | Difference between Transient and Steady state | % of variation |
|---|---|---|---|---|---|
| t = 3 | $L_1$ | 5.6015 | 5.6015 | 0.000 | 0.000 |
| | $L_2$ | 0.4801 | 0.4801 | 0.000 | 0.000 |
| | $L_3$ | 0.8402 | 0.8402 | 0.000 | 0.000 |
| | $L_4$ | 2.6140 | 2.6140 | 0.000 | 0.000 |
| | $L$ | 9.5358 | 9.5358 | 0.000 | 0.000 |
| | $P_{0000}$ | 0.0028 | 0.0028 | 0.000 | 0.000 |
| | $P_{0...}$ | 0.0228 | 0.0228 | 0.000 | 0.000 |
| | $P_{.0..}$ | 0.6248 | 0.6248 | 0.000 | 0.000 |
| | $P_{..0.}$ | 0.4455 | 0.4455 | 0.000 | 0.000 |

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|---|---|---|---|---|---|
| | $P_{...0}$ | 0.0991 | 0.0991 | 0.000 | 0.000 |
| | $U_1$ | 0.9772 | 0.9772 | 0.000 | 0.000 |
| | $U_2$ | 0.3752 | 0.3752 | 0.000 | 0.000 |
| | $U_3$ | 0.5545 | 0.5545 | 0.000 | 0.000 |
| | $U_4$ | 0.9009 | 0.9009 | 0.000 | 0.000 |
| | $Thp_1$ | 5.8632 | 5.8632 | 0.000 | 0.000 |
| | $Thp_2$ | 2.6264 | 2.6264 | 0.000 | 0.000 |
| | $Thp_3$ | 4.4360 | 4.4360 | 0.000 | 0.000 |
| | $Thp_4$ | 8.1081 | 8.1081 | 0.000 | 0.000 |
| | $W_1$ | 0.9554 | 0.9554 | 0.000 | 0.000 |
| | $W_2$ | 0.1828 | 0.1828 | 0.000 | 0.000 |
| | $W_3$ | 0.1894 | 0.1894 | 0.000 | 0.000 |
| | $W_4$ | 0.3224 | 0.3224 | 0.000 | 0.000 |

| Time t | Performance | Transient State | Steady State | Difference between Transient and Steady state | % of variation |
|---|---|---|---|---|---|
| t = 4 | $L_1$ | 5.6015 | 5.6015 | 0.0000 | 0.0000 |
| | $L_2$ | 0.4801 | 0.4801 | 0.0000 | 0.0000 |
| | $L_3$ | 0.8402 | 0.8402 | 0.0000 | 0.0000 |
| | $L_4$ | 2.6140 | 2.6140 | 0.0000 | 0.0000 |
| | $L$ | 9.5358 | 9.5358 | 0.0000 | 0.0000 |
| | $P_{0000}$ | 0.0028 | 0.0028 | 0.0000 | 0.0000 |
| | $P_{0...}$ | 0.0228 | 0.0228 | 0.0000 | 0.0000 |
| | $P_{.0..}$ | 0.6248 | 0.6248 | 0.0000 | 0.0000 |
| | $P_{..0.}$ | 0.4455 | 0.4455 | 0.0000 | 0.0000 |
| | $P_{...0}$ | 0.0991 | 0.0991 | 0.0000 | 0.0000 |
| | $U_1$ | 0.9772 | 0.9772 | 0.0000 | 0.0000 |
| | $U_2$ | 0.3752 | 0.3752 | 0.0000 | 0.0000 |
| | $U_3$ | 0.5545 | 0.5545 | 0.0000 | 0.0000 |
| | $U_4$ | 0.9009 | 0.9009 | 0.0000 | 0.0000 |
| | $Thp_1$ | 5.8632 | 5.8632 | 0.0000 | 0.0000 |
| | $Thp_2$ | 2.6264 | 2.6264 | 0.0000 | 0.0000 |
| | $Thp_3$ | 4.4360 | 4.4360 | 0.0000 | 0.0000 |
| | $Thp_4$ | 8.1081 | 8.1081 | 0.0000 | 0.0000 |
| | $W_1$ | 0.9554 | 0.9554 | 0.0000 | 0.0000 |
| | $W_2$ | 0.1828 | 0.1828 | 0.0000 | 0.0000 |
| | $W_3$ | 0.1894 | 0.1894 | 0.0000 | 0.0000 |
| | $W_4$ | 0.3224 | 0.3224 | 0.0000 | 0.0000 |

| Time t | Performance | Transient State | Steady State | Difference between Transient and Steady state | % of variation |
|---|---|---|---|---|---|
| t = 5 | $L_1$ | 5.6015 | 5.6015 | 0.000 | 0.000 |
| | $L_2$ | 0.4801 | 0.4801 | 0.000 | 0.000 |
| | $L_3$ | 0.8402 | 0.8402 | 0.000 | 0.000 |
| | $L_4$ | 2.6140 | 2.6140 | 0.000 | 0.000 |
| | $L$ | 9.5358 | 9.5358 | 0.000 | 0.000 |
| | $P_{0000}$ | 0.0028 | 0.0028 | 0.000 | 0.000 |
| | $P_{0...}$ | 0.0228 | 0.0228 | 0.000 | 0.000 |
| | $P_{.0..}$ | 0.6248 | 0.6248 | 0.000 | 0.000 |
| | $P_{..0.}$ | 0.4455 | 0.4455 | 0.000 | 0.000 |
| | $P_{...0}$ | 0.0991 | 0.0991 | 0.000 | 0.000 |
| | $U_1$ | 0.9772 | 0.9772 | 0.000 | 0.000 |
| | $U_2$ | 0.3752 | 0.3752 | 0.000 | 0.000 |
| | $U_3$ | 0.5545 | 0.5545 | 0.000 | 0.000 |
| | $U_4$ | 0.9009 | 0.9009 | 0.000 | 0.000 |
| | $Thp_1$ | 5.8632 | 5.8632 | 0.000 | 0.000 |
| | $Thp_2$ | 2.6264 | 2.6264 | 0.000 | 0.000 |

| | | | | | |
|---|---|---|---|---|---|
| | $Thp_3$ | 4.4360 | 4.4360 | 0.000 | 0.000 |
| | $Thp_4$ | 8.1081 | 8.1081 | 0.000 | 0.000 |
| | $W_1$ | 0.9554 | 0.9554 | 0.000 | 0.000 |

| | | | | | |
|---|---|---|---|---|---|
| | $W_2$ | 0.1828 | 0.1828 | 0.000 | 0.000 |
| | $W_3$ | 0.1894 | 0.1894 | 0.000 | 0.000 |
| | $W_4$ | 0.3224 | 0.3224 | 0.000 | 0.000 |

| Performance | t=0.1 | t=0.2 | t=0.3 | t=0.4 | t=0.5 | Steady State |
|---|---|---|---|---|---|---|
| $L_1$ | 2.5273 | 3.9143 | 4.6755 | 5.0933 | 5.3226 | 5.6015 |
| $L_2$ | 0.0662 | 0.1782 | 0.2773 | 0.3504 | 0.3998 | 0.4801 |
| $L_3$ | 0.1283 | 0.3369 | 0.5133 | 0.6381 | 0.7191 | 0.8402 |
| $L_4$ | 0.4358 | 1.1162 | 1.6691 | 2.0454 | 2.2817 | 2.6140 |
| $L$ | 3.1576 | 5.5456 | 7.1352 | 8.1272 | 8.7232 | 9.5358 |
| $P_{0000}$ | 0.2301 | 0.0619 | 0.0216 | 0.0101 | 0.0061 | 0.0028 |
| $P_{0...}$ | 0.2625 | 0.1002 | 0.0539 | 0.0371 | 0.0299 | 0.0228 |
| $P_{.0..}$ | 0.9373 | 0.8407 | 0.7634 | 0.7106 | 0.6768 | 0.6248 |
| $P_{..0.}$ | 0.8841 | 0.7258 | 0.6134 | 0.5437 | 0.5023 | 0.4455 |
| $P_{...0}$ | 0.6838 | 0.3871 | 0.2402 | 0.1710 | 0.1370 | 0.0991 |
| $U_1$ | 0.7375 | 0.8998 | 0.9461 | 0.9629 | 0.9701 | 0.9772 |
| $U_2$ | 0.0627 | 0.1593 | 0.2366 | 0.2894 | 0.3232 | 0.3752 |
| $U_3$ | 0.1159 | 0.2742 | 0.3866 | 0.4563 | 0.4977 | 0.5545 |
| $U_4$ | 0.3162 | 0.6129 | 0.7598 | 0.8290 | 0.8630 | 0.9009 |
| $Thp_1$ | 4.4250 | 5.3988 | 5.6766 | 5.7774 | 5.8206 | 5.8632 |
| $Thp_2$ | 0.4389 | 1.1151 | 1.6562 | 2.0258 | 2.2624 | 2.6264 |
| $Thp_3$ | 0.9272 | 2.1936 | 3.0928 | 3.6504 | 3.9816 | 4.4360 |
| $Thp_4$ | 2.8458 | 5.5161 | 6.8382 | 7.4610 | 7.7670 | 8.1081 |
| $W_1$ | 0.5711 | 0.7250 | 0.8236 | 0.8816 | 0.9144 | 0.9554 |
| $W_2$ | 0.1508 | 0.1598 | 0.1674 | 0.1730 | 0.1767 | 0.1828 |
| $W_3$ | 0.1384 | 0.1536 | 0.1660 | 0.1748 | 0.1806 | 0.1894 |
| $W_4$ | 0.1531 | 0.2024 | 0.2441 | 0.2741 | 0.2938 | 0.3224 |

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