

# Effect of Variable Viscosity and Gravity Modulation on Linear and Non-Linear Rayleigh-Benard Convection in Viscoelastic Ferromagnetic Liquids

G. Roopa, D. Uma



**Abstract:** The combined effect of various parameters of gravity modulation on the onset of ferroconvection is studied for both linear and non-linear stability. The effect of various parameters of ferroconvection is studied for linear stability analysis. The resulting seven-mode generalized Lorenz model obtained in non-linear stability analysis is solved using Runge-Kutta-Felberg 45 method to analyze the heat transfer. Consequently the individual effect of gravity modulation on heat transport has been investigated. Further, the effect of physical parameters on heat transport has been analyzed and depicted graphically. The low-frequency gravity modulation is observed to get an effective influence on the stability of the system. Therefore ferro convection can be advanced or delayed by controlling different governing parameters. It shows that the influence of gravity modulation stabilizes system.

**Keywords:** Gravity modulation, Ferromagnetic liquids, Ferroconvection, Variable viscosity, Generalized Lorenz model, Heat transport.

## I. INTRODUCTION

Ferromagnetic fluids are magnetic liquids forming a stable colloidal suspension in a carrier liquid with dispersed magnetic nanoparticles. Without an external magnetic field applied, the orientations of the particle's magnetic moments are random, leading to a vanishing macroscopic magnetization. Neuringer and Rosensweig (1964) presented the first continuum description of magnetic fluid by applying a vertical magnetic field. Finlayson (1970) discussed the convective instability of a magnetic fluid for a liquid surface heated from below. More research has been dedicated in recent decades to the study of ferrofluid convection mechanisms. Moreover, due to its technical applications, heat transfer by magnetic fluids has been one of the leading areas of scientific study. For associated heat transfer applications, ferrofluids are useful as they can be regulated by an externally acting

magnetic field, because of the concept that the advantage of ferroconvection could significantly improve heat transfer. A few interesting applications of the ferrofluid namely engine cooling, loudspeakers, and transmission lines have shown that the magnetic strength drastically changes the critical values associated with ferroconvective instability. Many researchers studied nonlinear convection in different viscoelastic liquids. Rudraiah et al. (1989) and Kim et al. (2003) discussed thermal convection saturated with viscoelastic fluid in a porous layer. Bhadauria (2005) and Bhadauria et al. (2006) analyzed the modulation of gravity on a liquid surface. Malashetty et al. (2006) and Shivakumara et al. (2006) studied the influence of thermal instability on the onset of convection in a porous layer saturated with viscoelastic fluid. On the other hand, a lot of attention has been paid in recent times to the heat convection induced by gravitational forces. Saravanan et al. (2009) studied the effect of gravity modulation in porous media. The effect on the onset of thermal convection in a liquid and porous surface is reported by Malashetty et al. (2011). Siddheshwar et al. (2012) performed a Rayleigh-Bénard magneto convection of local non-linear stability study using the Ginzburg Landau equation for stationary temperature / gravity modulated convection mode in a rotating viscous fluid surface. Nisha et al. (2013) studied ferrofluid stability when the fluid layer is heated from below by a periodical body force. They used Darcy law modified to define the movement of fluids. Bhadauria et al. (2014) studied the oscillatory convection mode for a nonlinear case and calculated heat transport. The effect on heat transfer of viscoelastic fluid relaxation was discussed. The weakly nonlinear double-diffusive magneto convection under gravity modulation was studied by Bhadauria et al. (2015). In an electrically conducting two-component fluid surface, they studied that the gravity field varies with time in a sinusoidal way of thermo-convective instability. Sameena et al. (2016) discussed the influence of gravity fluctuating with a saturated porous surface for couple stress liquid. Vasudha et al. (2016) discussed the influence of gravity modulation and internal heat generation for micro-polar fluid. Maria et al. (2018) used a method called Maxwell-Cattaneo law to study the effect of internal heat generation and gravity modulation on natural convection of in a couple stress fluids. In our problem, we analyze the effect of variable viscosity and gravity modulation in viscoelastic ferromagnetic liquids using generalized Khayat-Lorenz model for flow, magnetic potential and amplitudes of temperature.

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\* Correspondence Author

G. Roopa\*, Department of Mathematics, Raja Rajeswari College of Engineering, Bengaluru, Karnataka, India

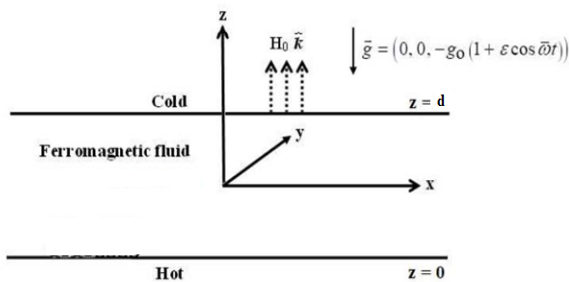
D. Uma, Department of Computer Science, Engineering, PES University, Bengaluru, Karnataka, India,

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The paper is organized as follows. Section II describes mathematical model, Section III, the governing equations, Section IV, the generalized Khayat-Lorenz method, Section V, linear stability analysis, Section VI, heat transport. Results and analysis are provided in section VII. Concluding remarks are given in section VIII.

## II. MATHEMATICAL MODEL

Consider an infinite horizontal layer of ferromagnetic liquid confined between two boundaries at  $z = 0$  and  $z = d$  subjected to an externally applied magnetic field  $H_0$  and a gravitational force,  $g = (0, 0, -g(t))$ , where  $g(t) = g_0(1 + \varepsilon \cos \omega t)$ ,  $g_0$ , gravity mean,  $\varepsilon$ , small amplitude,  $\omega$ , frequency and  $t$ , time. The lower and upper boundaries are held at constant temperatures  $T_0 + \Delta T$  and  $T_0$  respectively. (See figure 1).



## III. GOVERNING EQUATIONS

The governing equations of ferromagnetic liquids for gravity modulation are

$$\nabla \cdot \mathbf{v} = 0 \quad (1)$$

$$\frac{1}{Pr} d_t \mathbf{v} = -\nabla p_{eff} + Ra \sum (\theta, \Phi) + \nabla \cdot [\mu(H, T)(\nabla \vec{q} + \nabla \cdot \vec{q}^{Tr})] + \rho \vec{g} \quad (2)$$

Where  $\vec{g} = g_0(1 + \varepsilon \cos \Omega t)$

$$d_t \theta = \nabla^2 \theta + v_z \quad (3)$$

$$\frac{\partial^2 \Phi}{\partial z^2} + M_3 \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} \right) - \frac{\partial \theta}{\partial z} = 0 \quad (4)$$

Effective viscosity  $\mu_b$  and the  $g_1(z)$  are

$$\begin{aligned} \mu_b &= \mu_0 \left[ 1 + \delta_H (H_b - H_0)^2 - \delta_T (T_b - T_0)^2 \right] \\ &= \mu_0 \left[ 1 - (\delta_T (\Delta T)^2 (1 - z)^2 + \left( \frac{\delta_H k_l^2}{1 + \chi_m} \right) (\Delta T)^2) (1 - z)^2 \right] \\ g_1(z) &= [1 - V(1 - z)^2] , \\ \text{where } V &= \left( \delta_T - \frac{\delta_H k_l^2}{1 + \chi_m} \right) (\Delta T)^2. \end{aligned}$$

Where the velocity  $\mathbf{v}$ , temperature  $\theta$  and magnetic scalar potential  $\Phi$  are the non-dimensional perturbations. Here  $d_t f = \partial_t f + \mathbf{v} \cdot \nabla f$  is a material derivative,  $p_{eff}$ , effective pressure and magnetic force  $\sum \pi_1(\theta, \Phi) \hat{z} + M_1 \nabla \theta \cdot \hat{z}$

$$\begin{aligned} \text{The Rayleigh number } Ra &= \frac{\alpha \rho_0 g d^3 \Delta T}{\mu_0 \chi}, \text{ the Prandtl number,} \\ Pr &= \frac{\mu_0}{\rho_0 \chi}, \text{ the buoyancy magnetization parameter, } M_1 = \frac{\beta \chi_1^2 H_0^2}{\alpha T \rho_0 g (1 + \chi_m)}, \text{ the non-buoyancy magnetization parameter,} \\ M_3 &= \frac{1 + \chi}{(1 + \chi + \chi H_0^2)}. \end{aligned}$$

## IV. GENERALIZED KHAYAT-LORENZ MODEL

For simplicity, an analysis is restricted to two-dimensional flows. In specific, in the  $x$ -direction and periodic wave number  $k$  laterally, we suppose a two-dimensional system depicting parallel convection along the  $y$ -axis. Non-linear stability is performed to study the effect of various physical parameters of ferroconvection. The velocity field is expressed in terms of stream function  $\Psi$  and  $\mathbf{v} = \left( \frac{\partial \Psi}{\partial z}, 0, \frac{\partial \Psi}{\partial x} \right)$  and hence the set of Eqs. (2) – (4) on eliminating the pressure and nondimensionalizing as in Lorenz et al. [16].

$$\begin{aligned} \frac{1}{Pr} d_t \nabla^2 \Psi \left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) &= Ra \left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) g_0 (1 + \varepsilon \cos \Omega t) \\ &\left[ (1 + M_1) \frac{\partial \theta}{\partial x} - M_1 \left( \frac{\partial^2 \Phi}{\partial x \partial z} \right) \right] + \left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \\ &(1 + \varepsilon \cos \Omega t) Ra M_1 \left[ \frac{\partial \theta}{\partial x} \frac{\partial^2 \Phi}{\partial z^2} - \frac{\partial \theta}{\partial z} \frac{\partial^2 \Phi}{\partial x \partial z} \right] + \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) \\ &g_1(z) \nabla^4 \Psi + 2 D [g_1(z)] \nabla^2 \Psi + D^2 [g_1(z)] \left( \frac{\partial^2 \Psi}{\partial z^2} \right) \\ &- D^2 [g_1(z)] \left( \frac{\partial^2 \Psi}{\partial x^2} \right), \end{aligned} \quad (5)$$

$$d_t \theta = \nabla^2 \theta + \frac{\partial \Psi}{\partial x}, \quad (6)$$

$$\frac{\partial^2 \Phi}{\partial z^2} + M_3 \frac{\partial^2 \Phi}{\partial x^2} - \frac{\partial \theta}{\partial z} = 0, \quad (7)$$

$$\left( 1 + \lambda_1 \frac{\partial M}{\partial t} \right) = -M + (1 - A) \nabla^4 \Psi, \quad (8)$$

$$\text{Where } M \text{ is that } \frac{1}{Pr} d_t \nabla^2 \Psi = Ra (1 + \varepsilon \cos \Omega t) \frac{\partial T}{\partial \theta} + \Lambda \nabla^4 \Psi + M \quad (9)$$

Where

$$\begin{aligned} d_t f &= \frac{\partial f}{\partial t} + \frac{\partial \Psi}{\partial x} \frac{\partial f}{\partial z} - \frac{\partial \Psi}{\partial z} \frac{\partial f}{\partial x}, \quad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}, \\ \nabla^4 f &= \frac{\partial^4 f}{\partial x^2} + 2 \frac{\partial^4 f}{\partial x^2 \partial z^2} + \frac{\partial^4 f}{\partial z^2} \end{aligned}$$

We consider the following boundary condition for temperature, stream function, scalar magnetic potential and  $M$ .

$$\theta = \Psi = \nabla^2 \Psi = \frac{\partial \Phi}{\partial z} = M = 0, \quad z = (0, 1) \quad (10)$$

Finite amplitude convection in ferromagnetic liquid is studied by applying double Fourier series and is given in Eqs. (11) - (14),

$$\begin{aligned} \Psi &= A_1(\tau) \sin(kx) \sin(\pi z) + A_2(\tau) \cos(kx) \sin(\pi z) \\ &+ A(\tau) \sin 2\pi z, \end{aligned} \quad (11)$$

$$\theta = B_1(\tau) \cos(kx) \sin(\pi z) + B_2(\tau) \sin(kx) \sin(\pi z) + C(\tau) 2\pi z, \quad (12)$$

$$M = D_1(\tau) \sin(kx) \sin(\pi z) + D_2(\tau) \cos(kx) \sin(\pi z) + D(\tau) \sin 2\pi z, \quad (13)$$

$$\Phi = E_1(\tau) \cos(kx) \cos(\pi z) + E_2(\tau) \cos(kx) \sin(\pi z) + E(\tau) \cos 2\pi z. \quad (14)$$

Where M is determined by the form of  $\Psi$ . In Eq. (12), the term  $C(\tau)$  reflects a small change to the temperature field in the convection scale.

Projecting Eqs. (5) - (9) onto modes (11) - (14), we have the following seven-dimensional system of equations.

$$\dot{X}_1 = \frac{dX_1}{d\tau} = \text{Pr} \left[ \frac{[\pi^2 + k^2(1+M_1)M_3]}{(k^2 M_3 + \pi^2)} + \frac{Z}{R'} \frac{\pi^2 k^2 M_1 M_3}{(k^2 M_3 + \pi^2)} \right] Y_1 (1+\epsilon) \cos \Omega t - \Lambda f(V) X_1 - (1-\Lambda) N_1, \quad (15)$$

$$\dot{X}_2 = \frac{dX_2}{d\tau} = \text{Pr} \left[ \frac{[\pi^2 + k^2(1+M_1)M_3]}{(k^2 M_3 + \pi^2)} + \frac{Z}{R'} \frac{\pi^2 k^2 M_1 M_3}{(k^2 M_3 + \pi^2)} \right] Y_2 (1+\epsilon) \cos \Omega t - \Lambda f(V) X_2 - (1-\Lambda) N_2, \quad (16)$$

$$\dot{Y}_1 = R' X_1 - Y_1 - X_1 Z, \quad (17)$$

$$\dot{Y}_2 = R' X_2 - Y_2 - X_2 Z, \quad (18)$$

$$\dot{Z} = \left( \frac{X_1 Y_1 + X_2 Y_2}{2} \right) - bZ, \quad (19)$$

$$\dot{N}_1 = \frac{1}{\Gamma} (f(V) X_1 - N_1), \quad (20)$$

$$\dot{N}_2 = \frac{1}{\Gamma} (f(V) X_2 - N_2), \quad (21)$$

$$\text{Where } f(V) = 1 + \left( \frac{-1}{3} + \frac{1}{2\pi^2} \right) V + \left( \frac{2\pi^2 V}{\delta^4} \right) - \left( \frac{2k^2 V}{\delta^4} \right) - \frac{V}{\delta^2}.$$

Eqs. (15) - (21) are the generalized seven-mode Khayat-Lorenz system. The magnetic amplitudes are solved by  $E_1(\tau) = \frac{-\pi B_1(\tau)}{(k^2 M_3 + \pi^2)}$ ,  $E_2(\tau) = \frac{-\pi B_2(\tau)}{(k^2 M_3 + \pi^2)}$  and  $E(\tau) = \frac{-1}{2\pi} C(\tau)$ . Where  $X_1 = \frac{A_1 k \pi}{\delta^2}$ ,  $X_2 = \frac{A_2 k \pi}{\delta^2}$ ,  $Y_1 = B_1 \pi R'$ ,  $Y_2 = B_2 \pi R'$ ,  $Z = -C \pi R'$ ,  $N_1 = \frac{D_1 k \pi}{(1-\Lambda)\delta^6}$ ,  $N_2 = \frac{D_2 k \pi}{(1-\Lambda)\delta^6}$ ,  $\Lambda = \frac{A_2}{A_1}$ ,  $R' = \frac{Rk^2}{\delta^6}$ ,  $b = \frac{4k^2}{\delta^2}$ ,  $\delta^2 = \pi^2 + k^2$ ,  $\Gamma = \Lambda \delta^2$ .

## V. LINEAR STABILITY ANALYSIS

The linearized version of Eqs. (15) - (16) are considered to study linear stability theory.

$$-f(V) X_1 - \left( \frac{[\pi^2 + k^2(1+M_1)M_3]}{k^2 M_3 + \pi^2} \right) Y_1 = 0, \quad (22)$$

$$Ra_s X_1 - Y_1 = 0 \quad (23)$$

A non-trivial solution of the Eqs. (22) & (23), we require  $Ra_s$  taking the form,

$$Ra_s = \frac{q^6(k_c^2 M_3 + \pi^2)}{k_c^2(k_c^2 M_3 + k_c^2 M_1 M_3 + \pi^2)} f(v), \quad (24)$$

$$\text{Where } f(V) = 1 + \left( \frac{-1}{3} + \frac{1}{2\pi^2} \right) V + \left( \frac{2\pi^2 V}{\delta^4} \right) - \left( \frac{2k_c^2 V}{\delta^4} \right) - \frac{V}{\delta^2}$$

We are now discussing non-linear stability to determine the influence of physical parameters on the ferro convection of finite amplitude and to understand heat transport.

## VI. HEAT TRANSPORT

We now study heat transport for gravity modulation, using Nusselt number  $Nu$ ,

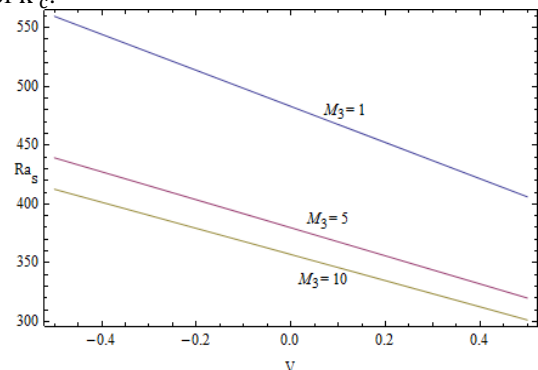
$$Nu(\tau) = \frac{\left[ \frac{k}{2\pi} \int_{x=0}^{2\pi} ((1-Z)+T)_z dx \right]_{x=0}}{\left[ \frac{k}{2\pi} \int_{x=0}^{2\pi} ((1-Z))_z dx \right]_{z=0}} \quad (25)$$

Replacing Eq. (12) with Eq. (25) and doing the integration, the phrase  $Nu(\tau)$  appears as follows

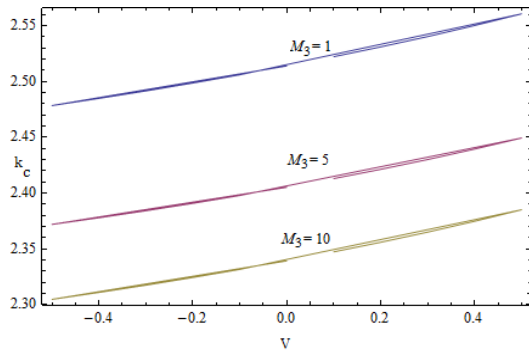
$$Nu(\tau) = 1 + \frac{2}{r} C(\tau) \quad (26)$$

## VII. RESULTS AND DISCUSSION

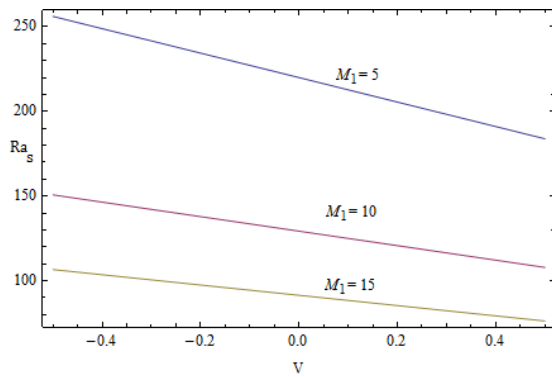
The convergence is obtained by using algebra package of Mathematica 12.0 to compute numerically, the minimum value of  $Ra_s$  corresponding  $k_c$ , for various parameters  $V$ ,  $M_1$  and  $M_3$ . In the case of linear stability analysis, Fig. 1(a) is the plot of  $Ra_s$  versus  $V$  for  $M_1 = 1$ , varying with  $M_3$ . From this graph, we can observe that when increasing the viscosity parameter  $V$  and non-buoyancy magnetization parameter  $M_3$  are to decrease  $Ra_s$ . Thus the system destabilizes. Fig. 1(b) is the plot of  $k_c$  versus  $V$  for  $M_1 = 1$ , varying with  $M_3$ . From the graph, we see that when the viscosity parameter  $V$  and non-buoyancy magnetization parameter  $M_3$  increases,  $k_c$  increases. Therefore, we conclude that the system stabilizes. Fig. 2(a) is the plot of  $Ra_s$  versus  $V$  for  $M_3 = 1$ , varying with  $M_1$ . Fig. 2(b) is the graph of  $k_c$  against  $V$  for  $M_3 = 1$ , varying with  $M_1$ . From these two graphs, we can observe the same effect as in the case of Fig. 1 (a) and (b). Thus Figs. 1 to 2 shows that increasing in magnetization parameter  $s$   $M_1$  and  $M_3$  with  $Ra_s$ , its destabilizing the system and stabilizes in the case of  $k_c$ .



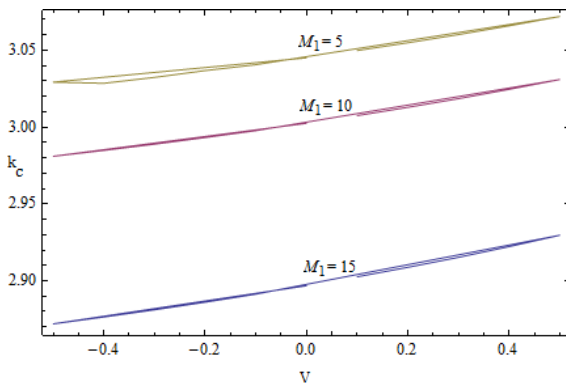
(a)



(b)  
Figure 1. Variation of  $Ra_s$  and  $k_c$  on  $V$  by varying  $M_3$  and with a fixed value of  $M_1 = 1$ .



(a)



(b)

Figure 2. Variation of  $Ra_s$  and  $k_c$  on  $V$  by varying  $M_1$  and with a fixed value of  $M_3 = 1$ .

Table 1: Critical values of  $k_c$  for various values of variable viscosity with different values of  $M_3$  with a fixed value of  $M_1 = 1$ .

	$M_1 = 1$ $M_3 = 1$	$M_1 = 1$ $M_3 = 10$	$M_1 = 10$ $M_3 = 1$
V	$k_c$	$k_c$	$k_c$
-0.1	2.50619	2.33166	2.99772
0	2.51404	2.33931	3.00248
0.1	2.52322	2.34741	3.00752

Table 2: Critical wave number  $k_c$ , for different values of buoyancy and non-buoyancy parameters.

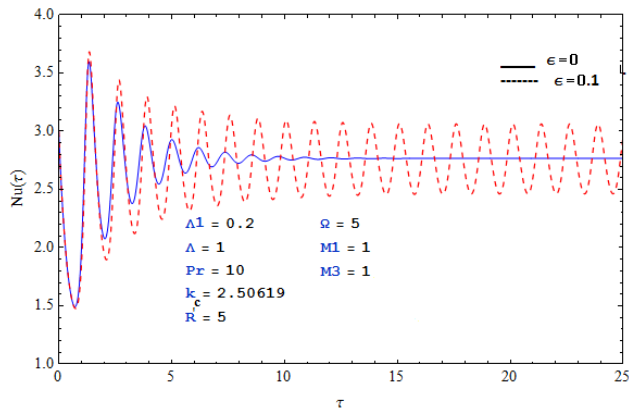
$M_1 = 1$						
	$M_3 = 1$		$M_3 = 5$		$M_3 = 10$	
V	$Ra_s$	$k_c$	$Ra_s$	$k_c$	$Ra_s$	$k_c$
-0.5	559.37	2.4785	439.27	2.3719	412.56	2.3048
-0.4	544.103	2.48495	427.424	2.37796	401.515	2.31102
-0.3	528.821	2.49167	415.560	2.38426	390.460	2.31754
-0.2	513.527	2.49874	403.686	2.39089	379.395	2.32441
-0.1	498.223	2.50611	391.803	2.39789	368.320	2.33166
0	482.903	2.51404	379.901	2.40527	357.235	2.33931
0.1	467.571	2.52232	368.004	2.41309	346.137	2.34741
0.2	452.223	2.53109	356.086	2.42136	335.026	2.35598
0.3	436.859	2.54037	344.154	2.43015	323.901	2.36509
0.4	421.477	2.55022	332.206	2.43949	312.760	2.37478
0.5	406.075	2.56070	320.235	2.44945	301.603	2.38511

For non-linear stability analysis, the minimal representation of the Fourier series provides a Lorenz model for variable viscosity using a chaotic problem system. In this paper, we discuss natural convection with gravity modulation in the ferromagnetic liquid by using a nonlinear stability analysis. is the non-dimensional stress relaxation parameter,  $\Lambda$ , is the elastic ratio and  $\gamma$ , control parameter, represents competing for temperature influences and instability elasticity. Runge - Kutta Fehlberg 45 adaptive step method- size is solved. The initial conditions are used to integrate the Eqs. (15) - (21) now become,

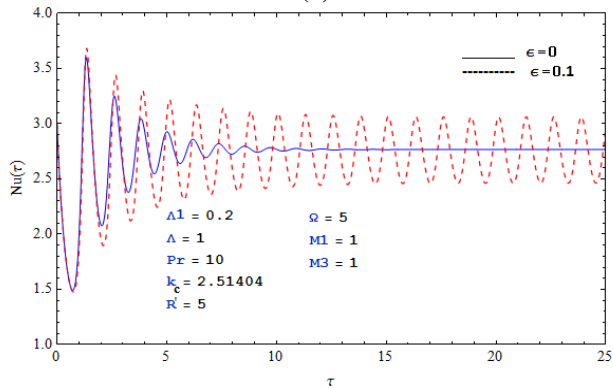
$$X_1(0) = X_2(0) = Y_1(0) = Y_2(0) = Z(0) = N_1(0) = N_2(0) = 5.$$

We consider seven modes of Khayat-Lorenz mode in the heat transport estimation. The distinct parameter of Nu versus  $\tau$  shown as a result of choices that were set for considering parameters. The corresponding results are shown in figs. 3 - 7. For viscoelastic liquids, Prandtl numbers can be much greater or equal to 10. Buoyancy and non - buoyancy parameter could be  $M_1 \sim 10^{-4} - 10^2$ ,  $M_3 \geq 1$ . Therefore in the present paper, the values of the parameters,  $M_1 = 10$ ,  $M_3 = 10$ ,  $Pr = 10$  and  $R' = 28$  are chosen for numerical calculations. Figs. 3 - 7 show the effects of  $M_1$ ,  $M_3$ ,  $Pr$  and  $R'$  on the Nusselt number with time with  $\tau$  for different variable viscosity  $V$ . Figure 3(a) to (c) are the Nu against time  $\tau$  plots for  $V = -0.1$ ,  $V = 0$  and  $V = 0.1$  with fixed values other parameters such as  $\Lambda_1 = 10$ ,  $\Lambda = 1$ ,  $Pr = 10$ ,  $M_1 = 1$ ,  $M_3 = 1$ ,  $R' = 5$  and  $\Omega = 5$ . From the graph it can be observed that, when increasing amplitude  $\varepsilon = 0$  to  $\varepsilon = 0.1$  and varying the viscosity  $V$ , the effect of the gravity modulation, is to decrease the heat transport. The results of Siddheshwar et al. [16] will be recovered for viscoelastic liquids. Fig. 4 (a) to (c) are the plots of Nu against  $\tau$  for various values for  $V$ , varying buoyancy magnetization parameter,  $M_1$  with fixed values of  $\Lambda_1 = 10$ ,  $\Lambda = 1$ ,  $Pr = 10$ ,  $M_3 = 1$ ,  $R' = 5$  and  $\Omega = 5$ . It can seen from the graph that when increasing the value of  $M_1 = 1$  to 10 and amplitude  $\varepsilon = 0$  to  $\varepsilon = 0.1$ , by varying the viscosity  $V$ , the effect of the gravity modulation, is to decrease the heat transport. Similarly figs. 5 - 7 are the plots for other varying parameters such as  $M_3$ ,  $Pr$  and  $R'$ . From these graphs, we can observe the same effect as in the case of fig. 3 & fig 4.

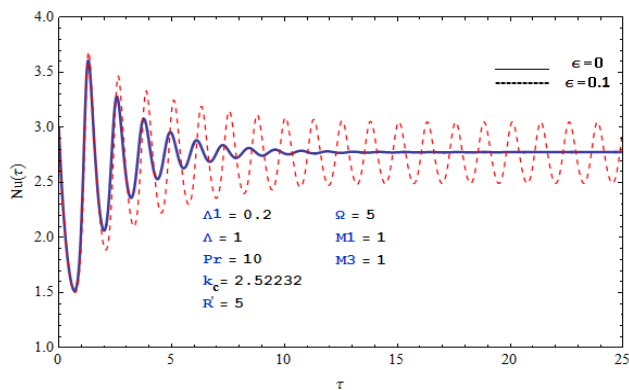




(a)

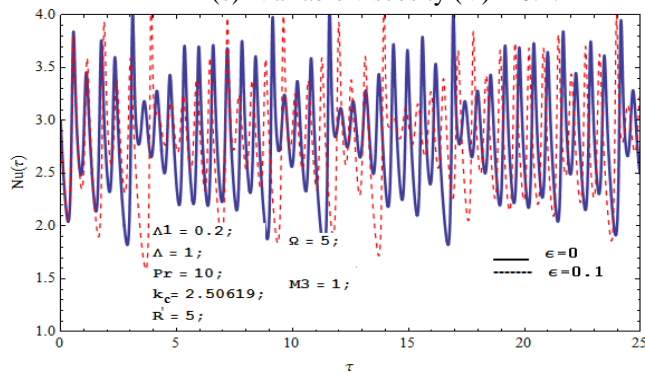


(b)

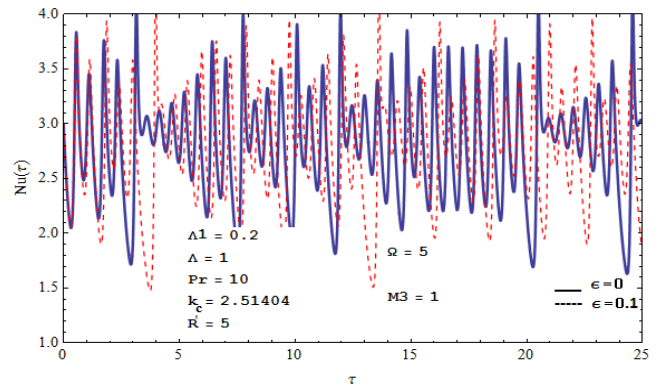


(c)

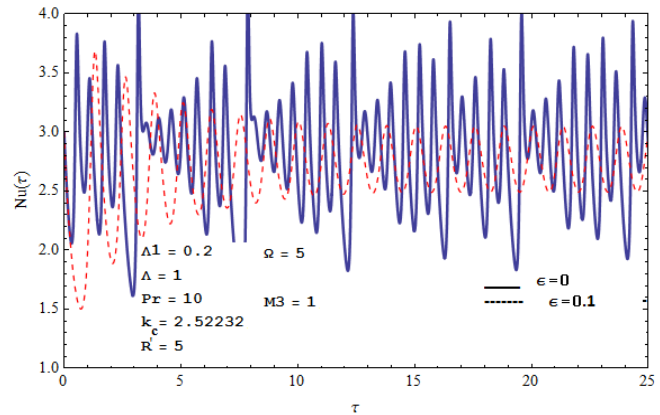
**Figure 3: Variation of  $Nu(\tau)$  on  $\tau$  for (a) Variable viscosity (V) = -0.1 (b) Variable viscosity (V) = 0 (c) Variable viscosity (V) = 0.1.**



(a)

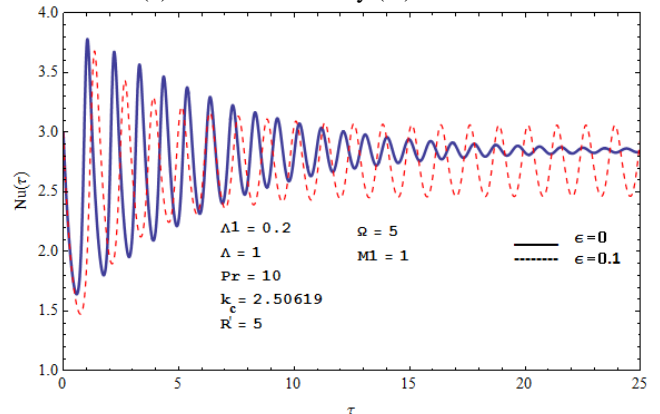


(b)

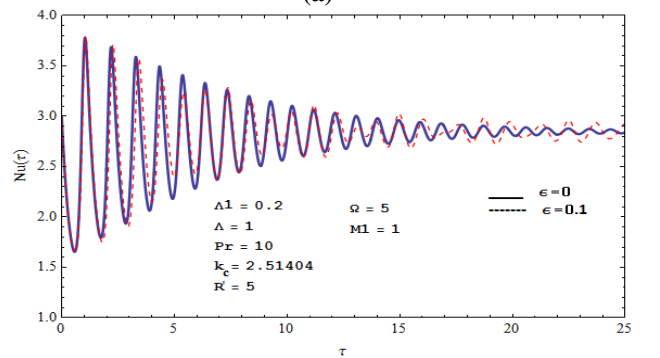


(c)

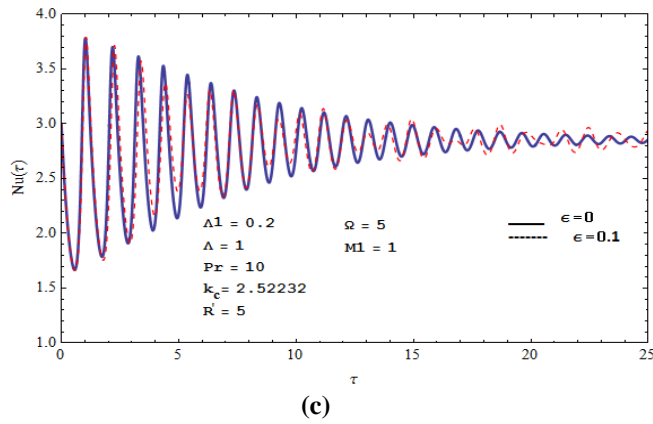
**Figure 4: Variation of  $Nu(\tau)$  on  $\tau$  by varying  $M_1 = 10$ , for (a) Variable viscosity (V) = -0.1 (b) Variable viscosity (V) = 0 (c) Variable viscosity (V) = 0.1.**



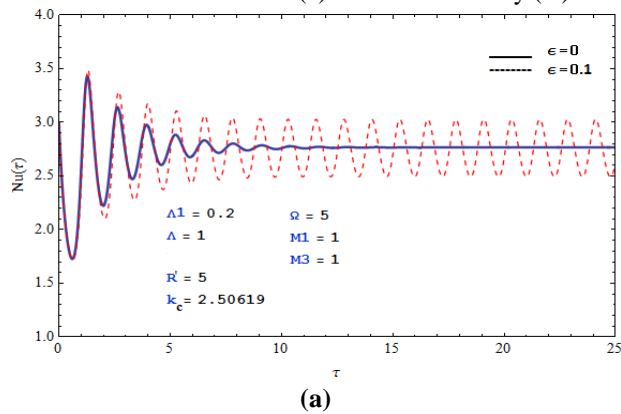
(a)



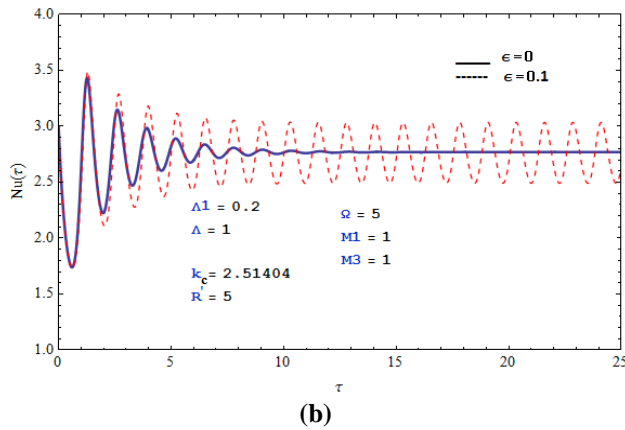
(b)



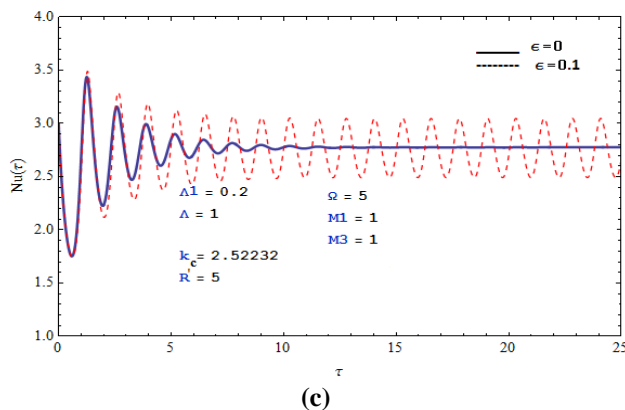
**Figure 5: Variation of  $Nu(\tau)$  on  $\tau$  by varying  $M_3 = 10$ , for (a) Variable viscosity ( $V$ ) = - 0.1 (b) Variable viscosity ( $V$ ) = 0 (c) Variable viscosity ( $V$ ) = 0.1**



(a)

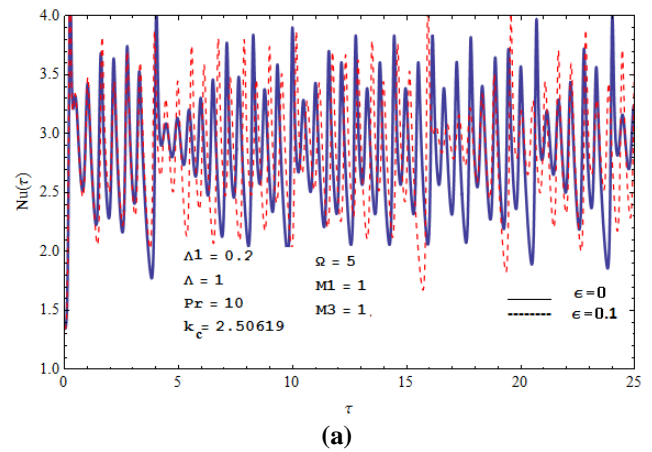


(b)

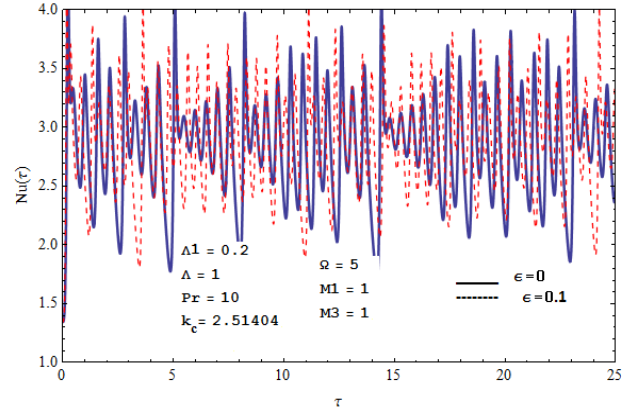


(c)

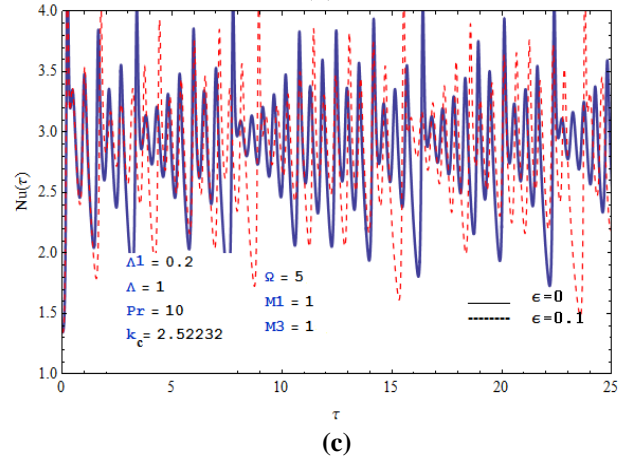
**Figure 6: Variation of  $Nu(\tau)$  on  $\tau$  by varying  $Pr = 15$ , for (a) Variable viscosity ( $V$ ) = - 0.1 (b) Variable viscosity ( $V$ ) = 0 (c) Variable viscosity ( $V$ ) = 0.1.**



(a)



(b)



(c)

**Figure 7: Variation of  $Nu(\tau)$  on  $\tau$  by varying  $R' = 28$ , for (a) Variable viscosity ( $V$ ) = - 0.1 (b) Variable viscosity ( $V$ ) = 0 (c) Variable viscosity ( $V$ ) = 0.1**

## VIII. CONCLUSION

The effect of gravity modulation in a ferromagnetic liquid on the onset of ferro convection has been analyzed by performing nonlinear stability analysis using Generalized Khayat-Lorenz Model. The conclusions are drawn as follows:

- Subcritical instability appears for low frequency due to the modulation of gravity.

- Variable viscosity parameter enhances the destabilizing effect of small frequency gravity modulation, while the effect is to augment the stabilizing effect of gravity modulation for moderate and large frequencies.
- Gravity modulation and the magnetic mechanism have mutually antagonistic effect on the ferroconvection provided the gravity modulation frequency is small as well as moderate.
- In the presence of variable viscosity, gravity modulation may advance or delay ferroconvection.
- The effect of applied gravity modulation will control the convective instability in a ferromagnetic liquid.

## REFERENCE

1. E. N. Lorenz, "Deterministic Non-Periodic Flow", J. Atmos. Sci., vol. 20, 1963, pp. 130 – 141.
2. J. L. Neuringer, R. E. Rosensweig, "Ferro hydrodynamics", Physics of fluids., vol. 7, 1964, pp. 1927 – 1937.
3. B. A. Finlayson, "Convective instability of ferromagnetic fluids", J. Fluid Mech., vol. 40, 1970, pp. 753 – 767.
4. N. Riahi, Nonlinear convection in a viscoelastic fluid, J. Math. Phys. Sci., vol. 26, 1976, pp. 211 – 220.
5. I. A. Eltayeb, "Nonlinear thermal convection in an elasticoviscous layer heated from below", Proc. R. Soc. Lond. A., vol. 356, 1977, pp. 161 – 176.
6. P. G. Siddheshwar, S. Pranesh, "An analytical study of linear and non-linear convection in Boussinesq-Stokes suspensions", Int. J. Non-Linear Mech., 2004, pp. 165-172.
7. B. S. Bhadauria, P. K. Bhatia and, L. Debnath, "Convection in Hele-Shaw Cell With Parametric Excitation," Int. J. Non-Linear Mech., vol. 40, 2005, pp. 475-484.
8. B. S. Bhadauria, "Gravitational Modulation of Rayleigh-Benard convection, Proc. Natl. Acad. Sci. India, vol. 76(A), 2006, pp. 61-67.
9. S. Saravanan, A. Purusothaman, "Floquet instability of a gravity modulated Rayleigh Bénard problem in an anisotropic porous media", Int. J. Therm. Sci., vol. 48, 2009, pp. 2085 - 2091.
10. P. G. Siddheshwar, G. N. Sekhar, G. Jayalatha, "Effect of time-periodic vertical oscillations of the Rayleigh-Bénard system on nonlinear convection in viscoelastic liquids", Journal of Non-Newtonian Fluid Mechanics, vol. 165, 2010, pp. 1412 – 1418.
11. P. G. Siddheshwar, G. N. Sekhar, G. Jayalatha, "Analytical study of convection in Jeffreys liquid with a heat source", Proceedings of the 37th International and 4th National Conference on Fluid Mechanics and Fluid Power, vol. 481, 2010, pp. 1 – 10.
12. P. G. Siddheshwar, "A Series solution for the Ginzburg-Landau equation with a time-periodic coefficient", Appl. Math., vol. 1, 2010, pp. 542 - 554.
13. M. S. Malashetty and M. Swamy, "Effect of gravity modulation on the onset of thermal convection in a rotating fluid and porous layer", Phys. of Fluids, vol. 23, 2011, pp. 064108.
14. P. G. Siddheshwar, B. S. Bhadauria, P. Mishra, A. K. Srivastava, "Study of heat transport by stationary magneto-convection in a Newtonian liquid under temperature or gravity modulation Using Ginzburg-Landau model", Int. J. Non-linear Mech., vol. 47, 2012, pp. 418 - 425.
15. T. Nisha Mary, S. Maruthamanikandan, "Effect of gravity modulation on the onset of ferroconvection in a densely packed porous layer", IOSR J. Appl. Phys., vol. 3, 2013, pp. 30 – 40.
16. D. Laroze, P. G. Siddheshwar, H. Pleiner, "Chaotic Convection in a Ferrofluid", Commun., Nonlinear Sci. Numer. Simul., vol. 18, 2013, pp. 2436 – 2447.
17. B. S. Bhadauria, P. Kiran, "Weak Nonlinear Oscillatory Convection in a Viscoelastic Fluid-Saturated Porous Medium Under Gravity Modulation", Article in Transport in Porous Media, vol. 3, 2014, pp. 451- 467.
18. S. Pranesh, "Linear and weakly non-linear analyses of gravity modulation and electric field on the onset of Rayleigh- Bénard convection in a micropolar Fluid", Journal of Advances in Mathematics., vol. 9, 2014, pp. 2057-2082.
19. B. S. Bhadauria, P. Kiran, "Chaotic and oscillatory magneto-convection in a binary viscoelastic fluid under G-jitter", Int J Heat Mass Transfer, vol. 84, 2015, pp. 610 - 624.
20. T. Sameena, S. Pranesh, "Effect of gravity modulation on the onset of Rayleigh-Bénard convection in a weak electrically conducting couple

stress fluid with saturated porous layer", Int J Eng Res., vol. 5, 2016, pp. 914 - 928.

22. Y. Vasudha, S. Pranesh, B. Shahnaz, "Effects of gravity modulation and internal heat generation on the onset of Rayleigh-Bénard convection in a micropolar fluid", J. Advances Math, vol. 12, 2016, pp. 6270 - 6285.
23. A. Vatani, P. L. Woodfield, N. Nam-Trung, D. V. Dao, "Onset of thermo magnetic convection around a vertically oriented hot-wire in ferrofluid", JMMM, vol. 456, 2018, pp. 300 – 310.
24. M. Thomas, G. Sangeetha, "Effect of gravity modulation and internal heat generation on Rayleigh-Bénard convection in couple stress fluid with maxwell-cattaneo law", International Journal of Applied Engineering Research, vol. 13, 2018, pp. 2688 – 2693.

## AUTHOR'S PROFILE



**G. Roopa** is a Assistant Professor working at Raja Rajeswari College of Engineering. She is currently pursuing her Ph.D degree under VTU. Her area of research is applied mathematics. She has around 17 years of teaching and 6 years of research experience. She holds a life-term ISTE membership.



**Dr. D. Uma** is currently working as a professor in the department of Computer Science and Engineering, PES University, Bengaluru. She has approximately 26 years of academic and 10 years of research experience. She has published her work in various National and International journals. She owns a life-term

ISTE membership.