Unsteady Flow of a Dusty Gas Through a Horizontal Pipe with Time Varying Pressure Gradient

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***Abstract:**In the present paper, the unsteady flow of a dusty gas through a pipe under the effect of the linear and exponential pressure gradient is established. Firstly the equation of motion of the fluid and dust particles is considered and then converts it into the Bessel equation by introducing two different parameters. Using the solution of the Bessel equation, the velocity of the gas and dust particles is obtained and shown graphical representation.It is found that the velocities increases as one move towards the axis of the pipe. Under the linear pressure gradient, the velocity of the gas is greater than the velocity of dust particles. As time progresses velocity increases and under exponential pressure gradient the velocity of the dust particles is greater than the velocity of the gas. As time progresses velocity decreases.*

***Keywords:** Bessel equation, Dust particles, Fluid, Pressure gradient, Velocity.*

I. INTRODUCTION

M.H. Hamdan and R.M. Barron [4] examined a dusty gas flow model in porous media. This model has direct applicability to irrigation problems. P.G.Saffman [6] proposed the first phenomenology if the dust is fine then the relaxation time would be relatively small and the gas flow would get destabilized whereas if the dust is coarse then the relaxation time would be relatively large and the gas flow would be stabilized. In an alternate phenomenology, P.Samba Siva Rao [7] obtained analytical expressions for the velocities of the fluid and dust particles in two different cases. M.S.Abu Zaytoon and M.H.Hamdan [5] analyzed a Saffman's dusty gas flow through porous media. Praveen Sharma and C.L. Varshney [8] observed thevariations of velocity of gas and dust particles for different values of parameters such as magnetic field, thermal dispersion and volume fraction. M.Allan Fathi, Qatanani Naji, Barghouthi Imad and M. Takatka Khaled [3] described the influence of the Reynold number, the permeability and drag coefficient on the horizontal velocity. A. Damesh Rebhi [1] focused the flow of a viscous incompressible gas with uniformly distributed dust particles across an isothermal cylinder. R.K.Gupta [9] expressed that the movement of gas is faster than the particles through a channel under a constant pressure gradient.

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G.Narsimlu, L.Anand Babu and P.Raji Reddy [2] observed that the velocity decreases with the decrease of n and velocity of the dust particle is greater than the clean dust particle. In the present paper unsteady flow of a dusty gas under the effect of linear and exponential pressure gradient with uniform distribution has been obtained and shown in figures graphically.

II. MATHEMATICAL FORMULATION OF A PROBLEM

The equations of unsteady motion of a dusty, viscous incompressible fluid are

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = -\frac{\nabla P}{\rho} + \nu\, \nabla^2\, \mathbf{u} + \frac{KN}{\rho}(\mathbf{v}-\mathbf{u}) \quad (1)$$

$$\text{div}\, \mathbf{u} = \mathbf{0} \quad (2)$$

$$m\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v}\right] = K\,(\mathbf{u}-\mathbf{v}) \quad (3)$$

$$\frac{\partial \mathbf{N}}{\partial t} + \text{div}(N\mathbf{v}) = 0 \quad (4)$$

Where **u** and **v** denote the velocity of fluid and dust particles respectively, P is the fluid pressure**,** m is the mass of dust particles, N is the number density, K is the Stoke's resistance coefficient**,** ρ is the density and ν is the kinematic viscosity.Consider a horizontal pipe of radius r_0 . Under the effect of linear and exponential pressure gradient, the unsteady flow of gas with uniform distribution is investigated. The velocity distribution of gas and dust particles are defined respectively as

$$u_x = 0,\ u_y = 0\ ,\ u_z = U(r,t) \quad (5)$$

$$v_x = 0\ ,\quad v_y = 0\ ,\quad v_z = V(r,\ t) \quad (6)$$

In terms of cylindrical coordinates (r, θ ,z) the equations (1) and (3) reduce to

$$\frac{\partial \mathbf{U}}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \nu\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r}\right) + \frac{KN}{\rho}(\mathbf{V}-\mathbf{U}) \quad (7)$$

$$m\,\frac{\partial \mathbf{V}}{\partial t} = K\,(\mathbf{U}-\mathbf{V}) \quad (8)$$

Differentiating partially equation (7) w.r.t 't' and eliminating V from (7) and (8), we get

$$\frac{\partial^2 U}{\partial t^2} = \frac{\partial}{\partial t}\left(-\frac{1}{\rho}\frac{\partial P}{\partial z}\right) + \nu\,\frac{\partial}{\partial t}\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r}\right) - \left(\frac{KN}{\rho} + \frac{K}{m}\right)\frac{\partial \mathbf{U}}{\partial t} - \frac{K}{m}\left[\frac{1}{\rho}\frac{\partial P}{\partial z} - \nu\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r}\right)\right] \quad (9)$$

CaseI : Assuming $-\frac{1}{\rho}\frac{\partial P}{\partial z} = a + bt\ ,\ R = \frac{r}{r_0},$ (10)

$U(R, t) = f(R)(a + bt)$, (11)

and $V(R, t) = g(R)(a + bt)$ (12)

Initial and Boundary conditions are

$f(0) = 0\,, g(0) = 0 \quad ; \; r = 0 \;\; , t = 0$ (13)
$f(1) = 0, g(1) = 0 \quad ; \; r = r_0\,, t > 0$ (14)

Using equations (10) (11) (12) and (13) then (8) and (9) becomes $f = g\left(1 + \frac{\tau b}{a}\right)$ (15)
$\frac{\nu}{{r_0}^2}\left(\frac{d^2f}{dR^2} + \frac{1}{R}\frac{df}{dR}\right)b - fb\left(\frac{KN}{\rho} + \frac{K}{m}\right) + \frac{K(a)}{m}\left[1 + \right.$
𝜈r02 𝑑2f𝑑R2+ 1R 𝑑f𝑑R+b=0 (16)

After simplification equation (16) reduce to Bessel equation as

$$\frac{d^2f}{dR^2} + \frac{1}{R}\frac{df}{dR} + n^2\,(\varepsilon - f) = 0 \quad (17)$$

Where $n^2 = \frac{b{r_0}^2(l+1)}{\nu(a+\tau b)}$ and $\varepsilon = \frac{a+\tau b}{b(l+1)}$ are two dimensionless parameters; $\tau = m/k$ is the relaxation time of dust particles; $l = \frac{mN}{\rho}$ is the mass concentration of the dust particles.
The solution of Bessel equation (17) is given by
$f(R) = C_1J_0(nR) + C_2Y_0(nR) - \varepsilon$ (18)
Where C_1 and C_2 are arbitrary constant and $J_0(nR)$ and $Y_0(nR)$ are Bessel functions of first and second kind of order zero.
Consider function which satisfy the boundary conditions,
$f(R) = \varepsilon\,(\frac{J_0(nR)}{J_0(n)} - 1)$ (19)
Where $J_0(nR) \approx 1 - \frac{(nR)^2}{4}$ (20) Using (20), the equation (19) reduce to $f(R) = \frac{\epsilon\, n^2(1-R^2)}{(4-n^2)}$ (21)
On simplifying equations (11) and (12) are given by
$U(R,t) = \frac{\epsilon\, n^2(1-R^2)}{(4-n^2)}\;(a + bt)$ (22)
$V(R,t) = \frac{\epsilon\, a\, n^2(1-R^2)(a+bt)}{(4-n^2)(a+\tau b)}$ (23)
Case II: Assuming $-\frac{1}{\rho}\frac{\partial P}{\partial z} = \frac{1}{\beta}\, e^{-t/\beta}$; $\beta > 0$ is a parameter. (24) $W_1(R, t) = \emptyset_1(R)\, e^{-t/\beta}$, (25) $W_2(R,t) = \emptyset_2(R)\, e^{-t/\beta}$ (26) Initial and Boundary conditions are
$\emptyset_1(0) = 0, \emptyset_2(0) = 0\, ;\, r = 0, t = 0$ (27)
$\emptyset_1(1) = 0, \emptyset_2(1) = 0\, ;\, r = r_0\,, t > 0$ (28)
Using equations (24) (25) (26)and (27) then (8) and (9) becomes $\emptyset_1 =$
∅2 1−τβ (29)
$\frac{\emptyset_1}{\beta^2} = -\frac{1}{\beta^2} - \frac{\nu}{{r_0}^2\beta}\left(\frac{d^2\emptyset_1}{dR^2} + \frac{1}{R}\frac{d\emptyset_1}{dR}\right) + \frac{\emptyset_1}{\beta}\left(\frac{KN}{\rho} + \frac{K}{m}\right) + \frac{K}{m}\,[\frac{1}{\beta} + \frac{\nu}{{r_0}^2}\left(\frac{d^2\emptyset_1}{dR^2} + \frac{1}{R}\frac{d\emptyset_1}{dR}\right)]$ (30)

After simplification equation (30) reduce to Bessel equation as $\frac{d^2\emptyset_1}{dR^2} + \frac{1}{R}\frac{d\emptyset_1}{dR} + n^2\,(\emptyset_1 + \Omega) = 0$ (31)

Where $n^2 = \frac{{r_0}^2}{\nu\beta}(1\text{-}\frac{l\beta}{\tau-\beta})$ and $\Omega = \frac{(\tau-\beta)}{(\tau-l\beta-\beta)}$ are two dimensionless parameters.
The solution of Bessel equation (31) is given by $\emptyset_1(R) = C_1J_0(nR) + C_2Y_0(nR) - \Omega$ (32) Where C_1 and C_2 are arbitrary constant and $J_0(nR)$ and $Y_0(nR)$ are Bessel functions of first and second kind of order zero.
Consider a function which satisfy the boundaryconditions
$\emptyset_1(R) = \Omega\,(\frac{J_0(nR)}{J_0(n)} - 1)$ (33)
Where $J_0(nR) \approx 1 - \frac{(nR)^2}{4}$ (34)
Using (34), the equation (33) reduce to
$\emptyset_1(R) = \frac{\Omega\, n^2(1-R^2)}{(4-n^2)}$ (35)
On simplifying equations (25) and (26) are given by
$W_1\,(R,t) = \frac{\Omega\, n^2(1-R^2)}{(4-n^2)}\; e^{-t/\beta}$ (36)
$W_2\,(R,t) = \frac{\Omega\, n^2(1-R^2)}{(4-n^2)}\,\frac{\beta}{(\beta-\tau)}\; e^{-t/\beta}$ (37)

III. RESULT AND DISCUSSION

Represntative results for velocity of gas and velocity of dust particles as a function of radial coordinate and time under the effect of linear and exponential pressure gradient along a pipe are obtained by taking a=1, b=1. Additionally we have computed the velocities of gas and dust particles separately at time t = 0.1 and t=0.2 . In Table I, for the case linear pressure gradient, velocity of the gas U is more than the velocity of dust particles. In Table II and Table III, as the value of R increases velocities of gas and dust particles decreases. From Fig. 1 it is observed that velocity of gas is more than the velocity of dust particles. As the value of R increases, velocity of the gas or dust particles decreases i.e, velocity increases as one move towards the axis of pipe. If we take multiple cylinders of different radius it is observed that the velocity of gas or dust particles is greater in cylinder with greater radius for same value of r i.e., as the value of n decreases, velocity also decreases. Fig. 2 and 3 depict that as the time progresses, the velocity increases. In Table IV, for the case exponential pressure gradient, velocity of the dust particles W_2 is more than the velocity of gas W_1. In Table V and Table VI, as the value of R increases velocities of gas and dust particles decreases. Fig. 4 reveals that velocity of dust particles is more than the velocity of gas. As the value of n decreases velocity decreases and velocity increases as one move towards the axis of pipe. Fig. 5 and 6 depict that as the time progresses the velocity decreases.

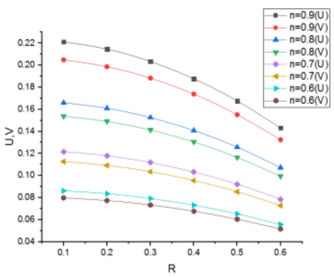

Fig. 1. Velocity of gas and dust particles for distinct values of n under linear pressure gradient.

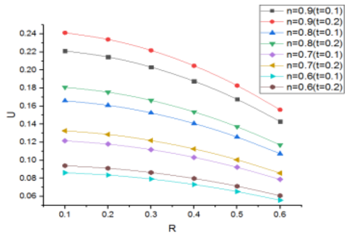

Fig. 2. Velocity of gas for distinct values of n w.r.t. time t.

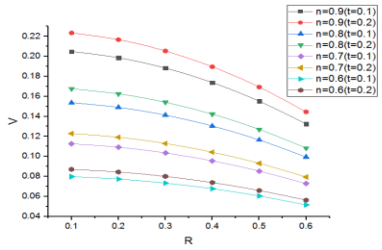

Fig.3.Velocity of dust particles for distinct values of n w.r.t. time t.

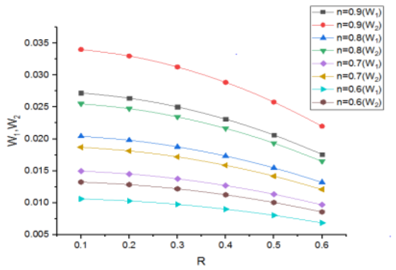

Fig.4. Velocity of gas and dust particles for distinct values of n under exponential pressure gradient.

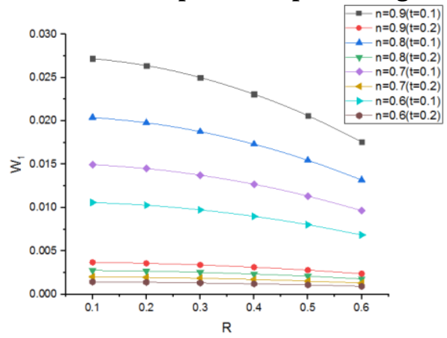

Fig.5. Velocity of gas for distinct values of n w.r.t. time t.

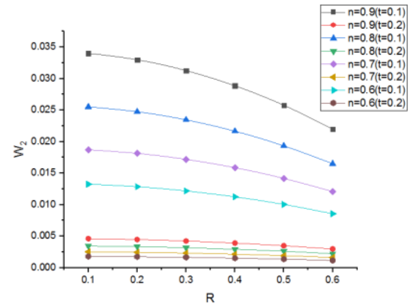

Fig.6. Velocity of dust particles for distinct values of n w.r.t. time t.

Table I: Velocities of gas and dust particles under linear pressure gradient

	U				V			
R	n=0.9	n=0.8	n=0.7	n=0.6	n=0.9	n=0.8	n=0.7	n=0.6
0.1	0.221214	0.165943	0.121621	0.086163	0.204828	0.153651	0.112612	0.079781
0.2	0.21451	0.160914	0.117935	0.083552	0.19862	0.148949	0.109199	0.077363
0.3	0.203338	0.152533	0.111793	0.0792	0.188276	0.141234	0.103512	0.073333
0.4	0.187696	0.140799	0.103193	0.073108	0.173793	0.130369	0.095549	0.067693
0.5	0.167586	0.125714	0.092137	0.065275	0.155172	0.116402	0.085312	0.060444
0.6	0.143007	0.107276	0.078623	0.055701	0.132414	0.09933	0.072799	0.051575

Table II: Velocity of gas at t=0.1, 0.2

	U							
	n=0.9		n=0.8		n=0.7		n=0.6	
R	t=0.1	t=0.2	t=0.1	t=0.2	t=0.1	t=0.2	t=0.1	t=0.2
0.1	0.221214	0.241324	0.165943	0.181029	0.121621	0.132677	0.086163	0.093996
0.2	0.21451	0.234012	0.160914	0.175543	0.117935	0.128656	0.083552	0.091147
0.3	0.203338	0.221823	0.152533	0.166399	0.111793	0.121955	0.0792	0.086399
0.4	0.187696	0.20476	0.140799	0.153599	0.103193	0.112574	0.073108	0.079754
0.5	0.167586	0.182822	0.125714	0.137143	0.092137	0.100513	0.065275	0.071209
0.6	0.143007	0.156008	0.107276	0.117028	0.078623	0.085771	0.055701	0.060765

Table III: Velocity of dust particles at t= 0.1,0.2

	V							
	n=0.9		n=0.8		n=0.7		n=0.6	
R	t=0.1	t=0.2	t=0.1	t=0.2	t=0.1	t=0.2	t=0.1	t=0.2
0.1	0.204828	0.223448	0.153651	0.167619	0.112612	0.122849	0.79981	0.087033
0.2	0.19862	0.216678	0.148994	0.16254	0.109799	0.119126	0.077363	0.084395
0.3	0.188276	0.205392	0.141234	0.154073	0.103512	0.112921	0.073333	0.079999
0.4	0.173793	0.189593	0.130369	0.142221	0.095549	0.104235	0.067693	0.073846
0.5	0.155172	0.16928	0.116402	0.126984	0.085312	0.093068	0.06044	0.065934
0.6	0.132414	0.144452	0.09933	0.108359	0.072799	0.079418	0.051575	0.056264

Table IV : Velocities of gas and dust particles under exponential pressure gradient

	W_1				W_2			
R	n=0.9	n=0.8	n=0.7	n=0.6	n=0.9	n=0.8	n=0.7	n=0.6
0.1	0.027217	0.020417	0.014963	0.010601	0.034021	0.025521	0.018704	0.013251
0.2	0.026393	0.019798	0.014509	0.010279	0.032991	0.024748	0.018136	0.012849
0.3	0.025017	0.018767	0.013754	0.009744	0.031271	0.023459	0.017193	0.01218
0.4	0.023093	0.017323	0.012696	0.008995	0.028866	0.021654	0.01587	0.011244
0.5	0.020619	0.015467	0.011336	0.008031	0.025774	0.091334	0.01417	0.010039
0.6	0.017594	0.013199	0.009673	0.006853	0.021993	0.016499	0.012091	0.008566

Table V: Velocity of gas at t = 0.1,0.2

	W_1							
	n=0.9		n=0.8		n=0.7		n=0.6	
R	t=0.1	t=0.2	t=0.1	t=0.2	t=0.1	t=0.2	t=0.1	t=0.2
0.1	0.027217	0.003684	0.020417	0.002763	0.014963	0.002025	0.010601	0.001435
0.2	0.026393	0.003572	0.019798	0.002679	0.014509	0.001964	0.010279	0.001391
0.3	0.025017	0.003386	0.018767	0.002539	0.013754	0.001862	0.009744	0.001319
0.4	0.023093	0.003126	0.017323	0.002344	0.012696	0.001719	0.008995	0.001217
0.5	0.020619	0.002791	0.015467	0.002093	0.011336	0.001535	0.008031	0.001087
0.6	0.017594	0.002381	0.013199	0.001786	0.009673	0.001309	0.006853	0.000927

Table VI: Velocity of dust particles at t = 0.1,0.2

	W_2							
	n=0.9		n=0.8		n=0.7		n=0.6	
R	t=0.1	t=0.2	t=0.1	t=0.2	t=0.1	t=0.2	t=0.1	t=0.2
0.1	0.034021	0.004605	0.025521	0.003454	0.018704	0.002531	0.013251	0.001794
0.2	0.032991	0.004465	0.024748	0.003349	0.018136	0.002455	0.012849	0.001739
0.3	0.031271	0.004233	0.023459	0.003174	0.017193	0.002328	0.01218	0.001649
0.4	0.028866	0.003908	0.021654	0.00293	0.01587	0.002149	0.011244	0.001521
0.5	0.025774	0.003489	0.019334	0.002616	0.01417	0.001919	0.010039	0.001359
0.6	0.021993	0.002976	0.016499	0.002233	0.012091	0.001636	0.008566	0.001159

IV. CONCLUSION

In this paper, the velocity of gas and dust particles is obtained for different values of parameter n at time t. Equations (22),(23) and (36),(37) represent the velocity of gas and dust particles under the effect of linear and exponential pressure gradient respectively shown in Figs. 1-6 which concluded that velocities increases as one move towards the axis of the pipe. Under linear pressure gradient velocity of the gas is greater than the velocity of the dust particles. Under the exponential pressure gradient velocity of the dust particles is greater than the velocity of the gas.

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