

# Statistical Analysis of $0^+$ Excitations in Even-Even Nuclei



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**Abstract:** In past breathing mode and compressibility in even-even nuclide have been investigated using microscopic and macroscopic approaches but present paper, we statistically analyzed experimental data for first breathing mode excitations ( $0^+$ ) of the even-even nuclei lying in the mass region  $12 \leq A \leq 250$ . Statistical relationship between mass number ( $A$ ) and first breathing mode excitations is presented by dividing this mass region  $12 \leq A \leq 250$  into five sub-regions. The correlation coefficients are calculated for the above said five sub-regions. We also present the statistical correlation between the mass number ( $A$ ) and compression modulus ( $k$ ). It is clear that the phase of correlation, between mass number ( $A$ ) and breathing mode excitation energies, and, between mass number and compression modulus ( $k$ ), is same except the mass regions  $96 \leq A \leq 144$  and  $144 \leq A \leq 192$  which indicates some peculiar behavior of nuclear structure in these regions and need rigorous theoretical calculations. The role of  $N/Z$  ratio in observed staggering pattern exhibited by first breathing mode excitation energies with mass number ( $A$ ) is also discussed.

**Keywords:** Breathing mode, Correlation coefficients, Nuclear Excitations, Even-Even Nuclei.

## I. INTRODUCTION

The advent of large gamma-ray detector arrays, such as GAMMASPHERE, EUROBALL and highly sophisticated gamma-ray analysis techniques for discovering band structures have given a push to the observation of rotational as well as vibrational band structures. Two types of collective motions, namely the vibrations of the nucleus as a whole and a rotational motion of a non-spherical nucleus, are observed. In the present paper, we focus our study only on collective vibrations of atomic nuclei, especially on the first breathing mode excitations. When a high energy charged particle passes nearby an atomic nucleus, the given nucleus may acquire excess of energy due to Coulomb excitations and can vibrate about its equilibrium shape. This mode of excitations involves the change in nuclear density without changing nuclear shape and such a change in density is analogous to the density

variations involved in respiration, hence this mode of vibration is known as breathing mode [1]. In past breathing mode and compressibility in even-even nuclide using Microscopic and macroscopic approaches have been investigated by several research groups [2-5]. Additionally, effect of pairing on breathing mode and nuclear matter compressibility has also been studied [6]. But in this paper, we present the statistical analysis of the first breathing mode excitations of even-even nuclei lying in the mass region  $12 \leq A \leq 250$ . The experimental data has been extracted from Table of Isotopes [7]. In order to establish a statistical relationship between the mass number ( $A$ ) and first breathing mode excitations, we divided the mass region  $12 \leq A \leq 250$  into five sub regions and calculated the co-relation coefficients for these sub-regions.

## II. FORMULATION

In order to study vibrations and rotations, the nuclear surface is parameterized generally by an expansion of nuclear radius vector  $R(\theta, \phi, t)$  in terms of spherical harmonics ( $Y_\lambda^\mu(\theta, \phi)$ ) with time dependent shape parameters ( $\alpha_{\lambda\mu}(t)$ ) as expansion coefficients. Thus, the radius vector  $R(\theta, \phi, t)$  is given as [1]

$$R(\theta, \phi, t) = R_0 \left[ 1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}(t) Y_\lambda^\mu(\theta, \phi) \right] \quad (1)$$

where  $R_0$  is the radius of spherical nucleus when all time dependent shape parameters vanishes. The time dependent parameters or amplitudes  $\alpha_{\lambda\mu}(t)$ , describes the distortions or vibrations of the nucleus with different multipolarity around the ground state and thus serve as collective coordinates. The general expansion of the nuclear surface described by equation (1) allows for arbitrary distortions or vibrations. The nuclear surface exhibits different types of multiple deformations, corresponding to each arbitrary value of  $\lambda$ . Each deformation has characteristic frequency  $\omega_\lambda$ , which depends on the amount of material in motion and strength of the restoring force. In the present paper we focused our analysis only on the monopole mode ( $\lambda=0$ ) of deformation. Spherical harmonics associated with this mode is constant and given by  $Y_0^0(\theta, \phi) = \frac{1}{4\pi}$ , where as the expansion coefficient

$\alpha_{00}(t)$  has a non-vanishing value corresponds to a change of the radius of sphere. The associated excitation is the so called breathing mode of the nucleus. If the nucleus is incompressible,  $\alpha_{00}(t)$  is constant and this mode is forbidden.

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Actually, nuclear matter is not completely incompressible but the restoring force is very high which means that the frequency and energy of this  $0^+$  state will be high. Thus first breathing mode excitations ( $0^+$ ) are found at excitation energies of the order of several hundred MeV and has been attributed to the excitation of the monopole vibration modes. The amount of energy involved is related to the nuclear shape and, as a result, the shape parameters become the appropriate canonical variables to describe the motion rather than, the coordinates specifying the position of each nucleon in the nucleus

### III. RESULTS AND DISCUSSIONS

Due to the availability of highly sophisticated gamma ray-detection facilities, now we have large amount of experimental data for rotational as well as vibrational states. In the present paper, we present the statistical analysis of experimental data of first breathing mode excitations ( $0^+$ ) of even-even nuclei lying in the mass region  $12 \leq A \leq 250$ . We extracted total 212 confirmed first breathing mode excitations ( $0^+$ ) from the Table of Isotopes [7]. On the basis of statistical correlation between mass number (A) and first breathing mode excitation energies, we divide the entire mass region,  $12 \leq A \leq 250$ , into five sub mass regions  $12 \leq A \leq 48$ ,  $48 \leq A \leq 96$ ,  $96 \leq A \leq 144$ ,  $144 \leq A \leq 192$ , and  $192 \leq A \leq 250$ . The correlation coefficients (r) are calculated for all the five sub regions and are presented in Table I. From this table it is clear that the nuclides lying in the lower mass region ( $12 \leq A \leq 48$ ) have overall negative linear correlation between the mass number (A) and first breathing mode excitation energies. Thus, in this mass region, the overall compressibility of nuclei decreases almost linearly with increase in mass number (A). Similarly in other mass regions, except ( $96 \leq A \leq 144$ ), there is a overall negative correlation between mass number (A) and first breathing mode excitation energies. But in these mass regions, the correlation is not as much liner as observed in the first mass region i.e. in  $12 \leq A \leq 48$ . For the mass region  $96 \leq A \leq 144$ , there is a overall positive correlation between mass number (A) and first breathing mode excitation energies, which means in this mass region the overall compressibility increases with increase in mass number (A) but this correlation is not linear. In Table I, we also present the statistical correlation between the mass number (A) and compression modulus (k) [2]. From this table it is clear that phase of correlation, between mass number (A) and breathing mode excitation energies, and, between mass number and compression modulus (k), is same except the mass regions  $96 \leq A \leq 144$  and  $144 \leq A \leq 192$ .

TABLE I: Correlation Analysis of different mass regions

Mass Number Range	Corr. Coeff. (r)	Corr. Coeff. (k)
$12 \leq A \leq 48$	-0.76	-0.34
$48 \leq A \leq 96$	-0.24	-0.49
$96 \leq A \leq 144$	+0.52	-0.03
$144 \leq A \leq 192$	-0.43	0.07
$192 \leq A \leq 250$	-0.12	-0.73
$12 \leq A \leq 250$	-0.53	-0.17

More detailed calculations are required for the explanation of out of phase behavior of both the co-relationships in these

mass regions. From the present analysis, we also suggest that the neutron proton ratio (N/Z) ratio plays a major role in explaining the variation of first breathing mode excitation energies ( $0^+$ ) with mass number (A). In our present compilation of experimental data, we also pointed out 6 nuclei namely  $^{12}\text{C}$ ,  $^{16}\text{O}$ ,  $^{24}\text{Mg}$ ,  $^{28}\text{Si}$ ,  $^{32}\text{S}$ ,  $^{40}\text{Ca}$  having N/Z ratio as unity. In all these cases (except Mg), their first breathing mode excitation energies decreases with increase in mass number. Thus N/Z ratio plays a major role in the observed staggering pattern exhibited by first breathing mode excitation energies with mass number (A).

### IV. CONCLUSIONS

A statistical analysis of first breathing mode excitation energies of even-even nuclei lying in the mass region  $12 \leq A \leq 250$  is presented. On the basis of correlation coefficient, we divided the whole mass region into five sub-regions. The variation of nuclear compressibility with mass number A is explored on the basis of correlation coefficients calculations. For nuclides having N/Z ratio as unity, the first breathing mode excitation energies decrease with increases in mass number except  $^{24}\text{Mg}$  nuclide.

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