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Onpairwise Compactness in Fuzzy Finebi-Topological Spaces

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Abstract: *In this paper the concepts of Pairwise (T_{if}, T_{jf}) fuzzy fine dually open cover, Pairwise (T_{if}, T_{jf}) fuzzy fine dually compact spacesand Pairwise (T_{if}, T_{jf}) fuzzy fine connected spaces are introduced and also an equivalent statement on Pairwise (T_{if}, T_{jf}) fuzzy fine dually compact spaces is established.*

Keywords: *Fuzzy fine bi-topological spaces, Pairwise (T_{if}, T_{jf}) fuzzy fine dually open cover, Pairwise (T_{if}, T_{jf}) fuzzy fine dually compact spacesand Pairwise (T_{if}, T_{jf}) fuzzy fine connected spaces*

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I. INTRODUCTION

Zadeh [9] presented the idea of Fuzzy set in 1965. In 1968, Chang [5] considered the idea of fuzzy topological space. In 1963 Kelly [6], first characterized the idea of bitopological spaces. Powar and Rajak [8] have presented a space namely fine space which is a generalization of a topological space. The idea of Fine fuzzy topological space was characterized by Amudhambigai and Rowthri [4]. Fletcher [8] set up the idea of strong Pairwise compactness In this paper the ideas of fuzzy fine bi-topological space, Pairwise (T_{if}, T_{jf}) fuzzy fine dually open cover, Pairwise (T_{if}, T_{jf}) fuzzy fine dually compact spacesare presented and furthermore some equivalent statements on Pairwise (T_{if}, T_{jf}) fuzzy fine dually compact spaces is established.

II. PRELIMINARIES

In this section the fundamental and necessary definitions required for this paper are discussed.

Definition 2.1[5]A fuzzy topology on a set X is a collection δ of fuzzy sets in X satisfying:

(1) $0 \in \delta$ and $1 \in \delta$

(2) If μ and γ belongs to δ, then does $\mu \wedge \gamma$ and

(3) If μ_i belongs to δ for each $i \in J$, then so does $\vee \mu_i$.

If δ is a fuzzy topology on X, then the pair (X, δ) is called a fuzzy topological space. The members of δ arecalled fuzzy open sets. Fuzzy sets of the form $(1-\mu)$, where μ is a fuzzy open set, are called fuzzy closed sets.

Definition 2.2[7]Let (X, T) be a topological space. We define $T(A_\alpha) = T_\alpha$ (say) $= \{ G_\alpha (\neq X) : G_\alpha \cap A_\alpha \neq \phi$, for $A_\alpha \in T$ and $A_\alpha \neq \phi, X$ for some $\alpha \in J$, where J is the indexed set $\}$. Now, we define $T_f = \{ \phi, X, \cup_{\alpha \in J} \{T_\alpha\}\}$. The above collection T_f of subsets of X is called the fine collection of subsets of X and (X, T, T_f) is said to be the fine spaceX generated by the topology T onX.

Definition 2.3[3] A fuzzy bitopological space is a triple (X, T_1, T_2) where X is a set T_1 and T_2 are any two fuzzy topologies on X.

Definition 2.4[8]A bi-topological space $(X, \mathcal{P}_1, \mathcal{P}_2)$ is said to be Pairwise dually compact if each Pairwise open cover $\mathcal{U}$ of X has a finite subcollection $\mathcal{V}$ of $\mathcal{U}$ such that $\{ \mathcal{P}_j\text{-}int(V) : V \in \mathcal{V} \cap \mathcal{P}_i, i \in \{1, 2\} \}$ covers X.

Definition 2.5[4] Let (X, T) be a fuzzy topological space. Let $T(\lambda_\alpha) = T_\alpha = \{ \mu_\alpha \in I^X, \mu_\alpha \neq 1_X : \mu_\alpha q \lambda_\alpha$ for each $\lambda_\alpha \in T$ and $\lambda_\alpha \neq 0_X, 1_X$ for some $\alpha \in J$, where J is an indexed set $\}$. Then the collection $T_f = \{ 0_X, 1_X \} \cup_\alpha \{T_\alpha\}$ of fuzzy sets is called the fine fuzzy collection and (X, T, T_f) is said to be the fine fuzzy topological space. The members of T_f are called fine fuzzy open sets and the complement of a fine fuzzy open set is called a fine fuzzy closed set.

III. PAIRWISE (T_{if}, T_{jf}) FUZZYFINE DUALLY COMPACT

In this section the concepts of fuzzy fine bi-topological spaces, Pairwise (T_{if}, T_{jf}) fuzzy fine dually open cover, Pairwise (T_{if}, T_{jf}) fuzzy fine dually compact spacesare discussed. Also some of its interesting properties are studied.

Definition 3.1 Let (X, T_1, T_2) be a fuzzy bi-topological space. Let $T_i(\lambda_\alpha) = T_{i\alpha}$ (say) $= \{ \mu_\alpha \in I^X, \mu_\alpha \neq 1_X : \mu_\alpha q \lambda_\alpha$ for each $\lambda_\alpha \in T_i$ and $\lambda_\alpha \neq 0_X, 1_X$ for some $\alpha \in J$, where J is an indexed set $\}$, i = 1, 2. Then the collection $T_{if} = \{ 0_X, 1_X \} \cup_\alpha \{T_{i\alpha}\}$ for i = 1, 2 of fuzzy sets is called the fuzzy fine bi-collection and $(X, T_1, T_2, T_{1f}, T_{2f})$ is said to be the fuzzy finebi-topological space. Each member of T_{1f} is called a T_1-fuzzy fine open set and each member of T_{2f} is called a T_2-fuzzy fine open set.

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Example 3.1 Let X = { a, b }. Let$\mu_1, \mu_2, \mu_3, \mu_4, \mu_5 \in I^X$ be defined as follows: μ_1(a) = 0, μ_1(b) = 1; μ_2(a) = 1, μ_2(b) = 0; μ_3(a) = 0.5, μ_3(b) = 0; μ_4(a) = 0, μ_4(b) = 0.5 and μ_5(a) = 0.5, μ_5(b) = 0.5.Define T_1 = { 0_X, 1_X, $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$ } and T_2 = { 0_X, 1_X, μ_3, μ_4, μ_5 }. Clearly, T_1 and T_2 are fuzzy topologies and(X, T_1, T_2) is called a fuzzy bi-topological space.Then T_{if} = { 0_X, 1_X } $\Upsilon_\alpha \{T_{i\alpha}\}$ for i = 1, 2 of fuzzy sets is a fuzzy fine bi-collection and hence $(X, T_1, T_2, T_{1f}, T_{2f})$ is afuzzy fine bi-topological space.

Definition 3.2 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ be a fuzzy finebi-topological space. For any $\lambda \in I^X$, the fuzzy fine bi-interior and fuzzy fine bi-closure are respectively defined as follows:

Fint$_{T_{if}}(\lambda) = \vee\{\mu$:each $\mu \in I^X$ is a T_{if}-fuzzy fine open set for i = 1, 2 and $\mu \le \lambda$ } and

Fcl$_{T_{if}}(\lambda) = \wedge\{\mu$:each $\mu \in I^X$ is a T_{if}-fuzzy fine closed set for i = 1, 2 and $\mu \ge \lambda$ }.

Definition 3.3 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ be a fuzzy finebi-topological space. Any $\lambda \in I^X$ is said to be a (T_{if}, T_{jf}) fuzzy finedually open setif there exists a T_{jf}-fuzzy fine open set $\mu \in I^X$ such that λ = Fint$_{T_{if}}(\mu)$, where i, j = 1, 2 andi $\neq$ j. The complement of a ((T_{if}, T_{jf}) fuzzy fine dually open set is called a (T_{if}, T_{jf}) fuzzy fine dually closed set.

Definition 3.4 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ be a fuzzy finebi-topological space. Any $\mu \in I^X$ is said to be a Pairwise (T_{if}, T_{jf}) fuzzy finedually open set if μ is a (T_{if}, T_{jf}) fuzzy fine dually open set for i, j = 1, 2 and foreach $\lambda \in T_{if}$, $\mu\, q\, \lambda$. The complement of aPairwise (T_{if}, T_{jf}) fuzzy fine dually open set is called a Pairwise (T_{if}, T_{jf}) fuzzy fine duallyclosed set.

Definition 3.5 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ be a fuzzy finebi-topological space. A Pairwise (T_{if}, T_{jf}) fuzzy finedually open cover of $(X, T_1, T_2, T_{1f}, T_{2f})$ is the collection $\mathcal{PO}$ = { μ_α : each $\mu_\alpha \in I^X$ is aPairwise (T_{if}, T_{jf}) fuzzy fine duallyopen set and α ∈ J is an indexed set } such that $\vee_{\alpha\in J}\, \mu_\alpha = 1_X$.

Also $\mathcal{PC}$ = { γ_α : each $\gamma_\alpha \in I^X$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine duallyclosed set and α ∈ J is an indexed set } is said to be aPairwise (T_{if}, T_{jf}) fuzzy fine duallyclosed cover of $(X, T_1, T_2, T_{1f}, T_{2f})$ if $\wedge_{\alpha\in J}\, \gamma_\alpha = 0_X$.

Definition 3.6A fuzzy finebi-topological space $(X, T_1, T_2, T_{1f}, T_{2f})$ is calleda Pairwise (T_{if}, T_{jf}) fuzzy finedually compact spaceif for eachPairwise (T_{if}, T_{jf}) fuzzy fine duallyopen cover{ μ_α : each $\mu_\alpha \in I^X$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine dually open set and α ∈ J is an indexed set } of $(X, T_1, T_2, T_{1f}, T_{2f})$, there exists a finite subset J_0 of J such that $\vee_{\alpha\in J_0}\, \mu_\alpha = 1_X$.

Definition 3.7 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ be a fuzzy finebi-topological space. A cover $\mathcal{PO}_1$ = { $\lambda \in I^X$: $\lambda \in T_{1f}$ or $\lambda \in T_{2f}$ } is said to be a fuzzy fine step refinement of a Pairwise (T_{if}, T_{jf}) fuzzy fine dually open cover $\mathcal{PO}$ of $(X, T_1, T_2, T_{1f}, T_{2f})$ if for each λ of $\mathcal{PO}_1$ there exists a μ of$\mathcal{PO}$ such that $\lambda \le \mu$.

Also $\mathcal{PC}_1$ = { $1_X - \lambda \in I^X$: $\lambda \in T_{1f}$ or $\lambda \in T_{2f}$ } is said to be a fuzzy fine step refinement of a Pairwise (T_{if}, T_{jf}) fuzzy fine dually closed cover $\mathcal{PC}$ of $(X, T_1, T_2, T_{1f}, T_{2f})$ if for each $1_X - \lambda$ of $\mathcal{PO}_1$ there exists a$1_X - \mu$ of $\mathcal{PC}$ such that $1_X - \lambda \le 1_X - \mu$.

Proposition 3.1 For anyfuzzy finebi-topological space $(X, T_1, T_2, T_{1f}, T_{2f})$, the following statements are equivalent:

(1) $(X, T_1, T_2, T_{1f}, T_{2f})$ isa Pairwise (T_{if}, T_{jf}) fuzzy fine dually compact space.

(2) EachPairwise (T_{if}, T_{jf}) fuzzy fine duallyopen cover of $(X, T_1, T_2, T_{1f}, T_{2f})$ has afinite fuzzy fine step refinement.

(3) EachPairwise (T_{if}, T_{jf}) fuzzy fine duallyclosed cover$\mathcal{PC}$ = { μ_α: each $\mu_\alpha \in I^X$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine dually closed set }of $(X, T_1, T_2, T_{1f}, T_{2f})$ there exists a finite subset J_0 of J such that $\wedge_{\alpha\in J_0} \mathrm{Fcl}_{T_{if}}(\mu_\alpha) = 0_X$ whenever $1_X - \mu_\alpha \in T_{if}$, i ∈ { 1, 2 }.

Proof (1)$\Rightarrow$(2) The proof is obvious, since on a Pairwise (T_{if}, T_{jf}) fuzzy fine dually compact space, eachPairwise (T_{if}, T_{jf}) fuzzy fine duallyopen cover has a finite step refinement.

(2) $\Rightarrow$ (3) Let $\mathcal{PC}$ = { μ_α : each $\mu_\alpha \in I^X$ is aPairwise (T_{if}, T_{jf}) fuzzy fine duallyclosed set }be aPairwise (T_{if}, T_{jf}) fuzzy fine duallyclosed cover of $(X, T_1, T_2, T_{1f}, T_{2f})$ such that $\wedge_{\alpha\in J}\mu_\alpha = 0_X$. Then $\mathcal{PO}$ = { $1_X - \mu_\alpha$: each $1_X - \mu_\alpha \in I^X$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine dually open set} is a Pairwise (T_{if}, T_{jf}) fuzzy fine duallyopen cover of $(X, T_1, T_2, T_{1f}, T_{2f})$. By (2), there is a finite fuzzy step refinement $\mathcal{PO}_1$ of $\mathcal{PO}$and there exists $\gamma_\alpha \in I^X$ of$\mathcal{PO}_1$ and $1_X - \mu_\alpha \in T_{if}$ of $\mathcal{PO}$ such that $\gamma_\alpha \le 1_X - \mu_\alpha$. Thus $\mathrm{Fcl}_{T_{if}}(\mu_\alpha) \le 1_X - \gamma_\alpha$ .Since $\wedge_{\alpha\in J_0}(1_X - \gamma_\alpha) = 0_X$ which in turn implies $\wedge_{\alpha\in J_0}(\mathrm{Fcl}_{T_{if}}(\mu_\alpha)) = 0_X$.

(3)$\Rightarrow$(1): Let $\mathcal{PO}$ = { λ_α : each $\lambda_\alpha \in I^X$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine dually open set } is a Pairwise (T_{if}, T_{jf}) fuzzy fine dually open cover of $(X, T_1, T_2, T_{1f}, T_{2f})$. So$\mathcal{PC}$ ={ $1_X - \lambda_\alpha$: each $\lambda_\alpha \in I^X$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine dually closed set} is a

Pairwise (T_{if}, T_{jf}) fuzzy fine dually closed cover of $(X, T_1, T_2, T_{1f}, T_{2f})$. By (3), there exists a finite subset J_0 of J such that $\wedge_{\alpha \in J_0} Fcl_{T_{if}}(1_X - \lambda_\alpha) = 0_X$ whenever $\lambda_\alpha \in T_{if}$, i ∈ { 1, 2 }.

Now $1_X - \wedge_{\alpha\in J_0} Fcl_{T_{if}}(1_X - \lambda_\alpha) = 1_X$. Then $\vee_{\alpha\in J_0} Fint_{T_{if}}(\lambda_\alpha) = 1_X$. Thus $(X, T_1, T_2, T_{1f}, T_{2f})$ is a Pairwise (T_{if}, T_{jf}) fuzzy finedually compact space.

Definition 3.8 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ and $(Y, S_1, S_2, S_{1f}, S_{2f})$ be any two fuzzy fine bi-topological spaces. A functionf : $(X, T_1, T_2, T_{1f}, T_{2f})$ → $(Y, S_1, S_2, S_{1f}, S_{2f})$ is called Pairwise (T_{if}, T_{jf}) fuzzy fine dually continuous if for each fuzzy fine open set λ in $(Y, S_1, S_2, S_{1f}, S_{2f})$, the inverse image $f^{-1}(\lambda)$ is Pairwise (T_{if}, T_{jf}) fuzzy fine dually open in $(X, T_1, T_2, T_{1f}, T_{2f})$.

Definition 3.9 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ and $(Y, S_1, S_2, S_{1f}, S_{2f})$ beany two fuzzy fine bi-topological spaces. A functionf : $(X, T_1, T_2, T_{1f}, T_{2f})$ → $(Y, S_1, S_2, S_{1f}, S_{2f})$ is called Pairwise (T_{if}, T_{jf}) fuzzy fine dually homeomorphism if f is bijective andPairwise (T_{if}, T_{jf}) fuzzy fine dually bicontinuous.

Proposition 3.2 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ and $(Y, S_1, S_2, S_{1f}, S_{2f})$ be any two fuzzy fine bi-topological spaces. Iff : $(X, T_1, T_2, T_{1f}, T_{2f}) \to (Y, S_1, S_2, S_{1f}, S_{2f})$ is Pairwise (T_{if}, T_{jf}) fuzzy fine dually homeomorphism and $(X, T_1, T_2, T_{1f}, T_{2f})$ be a Pairwise (T_{if}, T_{jf}) fuzzy fine dually compact space then $(Y, S_1, S_2, S_{1f}, S_{2f})$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine dually compact space.

ProofLet{ μ_α: each $\mu_\alpha \in I^Y$ is aPairwise (T_{if}, T_{jf}) fuzzy fine duallyopen set and $\alpha \in J$ is an indexed set } be a fuzzy fine open covering of $(Y, S_1, S_2, S_{1f}, S_{2f})$ such that $\vee_{\alpha\in J}\mu_\alpha$ = 1Y. Since f is Pairwise (T_{if}, T_{jf}) fuzzy fine dually homeomorphism, $f^{-1}(\mu_\alpha) \in I^X$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine dually open set in $(X, T_1, T_2, T_{1f}, T_{2f})$ .Since $(X, T_1, T_2, T_{1f}, T_{2f})$ is Pairwise (T_{if}, T_{jf}) fuzzy finedually compact, $\vee_{\alpha\in J}\mu_\alpha = 1_X$, there exists a finite subset J_0 of J such that $\vee_{\alpha\in J_0} f^{-1}(\mu_\alpha) = 1_X$. Since f is onto, $f(\vee_{\alpha\in J_0} f^{-1}(\mu_\alpha)) = f(1_X)$ which gives $\vee_{\alpha\in J_0}\mu_\alpha = 1_Y$. Hence $(Y, S_1, S_2, S_{1f}, S_{2f})$ is a Pairwise (T_{if}, T_{jf}) fuzzyfine dually compact space.

IV. PAIRWISE (T_{if}, T_{jf}) FUZZY FINE CONNECTED SPACES

In this section the concepts of Pairwise (T_{if}, T_{jf}) fuzzy fine connected space and Pairwise (T_{if}, T_{jf}) fuzzy fine c-bicontinuous function are introduced. Also some of their interesting Properties are discussed.

Definition 4.1 A fuzzy fine bi-topological space $(X, T_1, T_2, T_{1f}, T_{2f})$ is said to be a Pairwise (T_{if}, T_{jf}) fuzzy fine connected space if for $\lambda, \mu \in I^X$, $(\lambda \wedge cl_{T_{if}}(\mu)) \vee (cl_{T_{jf}}(\lambda) \wedge \mu) = 0_X$ where i, j = 1, 2 and $i \neq j$. Then λ, μ and $1_X - \lambda, 1_X - \mu$ are said to be a Pairwise (T_{if}, T_{jf}) fuzzy fineseparated sets and are said to be a Pairwise (T_{if}, T_{jf}) fuzzy fine connected sets respectively.

A non-Pairwise (T_{if}, T_{jf}) fuzzy fine connected Space is called a Pairwise (T_{if}, T_{jf}) fuzzy fine disconnected space.

Proposition 4.1 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ be any fuzzy fine bi-topological space. Then thefollowing conditions are equivalent:

(1) $(X, T_1, T_2, T_{1f}, T_{2f})$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine disconnected space.

(2) $1_X = \lambda \vee \mu$ where $\lambda \tilde{q} \mu$, λ is a T_1-fuzzy open set and μ is a T_2-fuzzy open set.

(3) $1_X = \lambda \vee \mu$ where $\lambda \tilde{q} \mu$, λ is a T_1-fuzzy closed set and μ is a T_2-fuzzy closed set.

ProofObvious from the definition.

Proposition 4.2 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ be a fuzzy fine bi-topological space. Then forany Pairwise (T_{if}, T_{jf}) fuzzy fine connected set $\gamma \in I^X$, there exists a Pairwise (T_{if}, T_{jf}) fuzzy fine separated sets λ and μ in I^X such that $\gamma \leq \lambda$ or $\gamma \leq \mu$

Proof Let $\lambda, \mu \in I^X$ be any two Pairwise (T_{if}, T_{jf}) fuzzy fine separated sets. Then by Definition 4.1, $(\lambda \wedge cl_{T_{if}}(\mu)) \vee (cl_{T_{jf}}(\lambda) \wedge \mu) = 0_X$ where i, j = 1, 2 and $i \neq j$. Now Consider,

$$((\gamma \wedge \lambda) \wedge cl_{T_{if}}(\gamma \wedge \mu)) \vee (cl_{T_{jf}}(\gamma \wedge \lambda) \wedge (\gamma \wedge \mu)) \leq$$
$$(\lambda \wedge cl_{T_{if}}(\mu)) \vee (cl_{T_{jf}}(\lambda) \wedge \mu) = 0_X$$
$$\Rightarrow ((\gamma \wedge \lambda) \wedge cl_{T_{if}}(\gamma \wedge \mu)) \vee (cl_{T_{jf}}(\gamma \wedge \lambda) \wedge (\gamma \wedge \mu)) = 0_X$$
$$\Rightarrow ((\gamma \wedge \lambda) \wedge cl_{T_{if}}(\gamma \wedge \mu)) = 0_X \ or \ (cl_{T_{jf}}(\gamma \wedge \lambda) \wedge (\gamma \wedge \mu)) = 0_X$$
$$\Rightarrow \gamma \wedge \lambda = 0_X \quad or \quad \gamma \wedge \mu = 0_X$$
$$\Rightarrow \gamma \leq 1_X - \lambda \quad or \quad \gamma \leq 1_X - \mu$$

Since

$$\lambda + \mu \, \phi \, 1_X, 1_X - \lambda \, \pi \, \mu \ and \ 1_X - \mu \, \pi \, \lambda$$
$$\Rightarrow \gamma \leq \lambda \quad or \quad \gamma \leq \mu$$

Proposition 4.3 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ be a fuzzy fine bi-topological space if $\lambda \in I^X$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine connected set and $\lambda \leq \mu \leq cl_{T_{if}}(\lambda) \wedge cl_{T_{jf}}(\lambda)$ then μ is aPairwise (T_{if}, T_{jf}) fuzzy fine connected set.

ProofSuppose $\mu \in I^X$ is not a Pairwise (T_{if}, T_{jf}) fuzzy fine connected set. Then $\mu = \gamma \wedge \delta$, where $\gamma, \delta \in I^X$ and $\mu + \lambda \, \phi \, 1_X$ such that $(\gamma \wedge cl_{T_{if}}(\delta)) \vee (cl_{T_{jf}}(\gamma) \wedge \delta) = 0_X$

Since $\lambda \in I^X$ is Pairwise (T_{if}, T_{jf}) fuzzy fine connected, ByProposition 4.1, $\lambda \leq \gamma$ or $\lambda \leq \delta$. Suppose $\lambda \leq \gamma$ T.hen $\delta \leq \delta \wedge (\gamma \vee \delta) = \delta \wedge \mu \leq \delta \wedge cl_{T_{if}}(\lambda) = 0_X$. Therefore, $\delta = 0_X$. Similarly we can prove $\gamma = 0_X$. But thisis the contradiction to the fact that $\mu + \lambda \phi 1_X$. Hence $\mu \in I^X$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine connected set.

Definition 4.2 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ and $(Y, S_1, S_2, S_{1f}, S_{2f})$ be any two fuzzy fine bi-topological spaces. A function f : $(X, T_1, T_2, T_{1f}, T_{2f}) \rightarrow (Y, S_1, S_2, S_{1f}, S_{2f})$ is called a Pairwise (T_{if}, T_{jf}) fuzzy fine c-continuous if for each fuzzy fine connected open set λ in $(Y, S_1, S_2, S_{1f}, S_{2f})$, the inverse image $f^{-1}(\lambda)$ is Pairwise (T_{if}, T_{jf}) fuzzy fine connected open in $(X, T_1, T_2, T_{1f}, T_{2f})$.

Definition 4.3 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ and $(Y, S_1, S_2, S_{1f}, S_{2f})$ be any two fuzzy fine bi-topological spaces. A function f : $(X, T_1, T_2, T_{1f}, T_{2f}) \rightarrow (Y, S_1, S_2, S_{1f}, S_{2f})$ is called a Pairwise (T_{if}, T_{jf}) fuzzy fine c-homeomorphism if f is bijective and Pairwise (T_{if}, T_{jf}) fuzzy fine c-bicontinuous.

Proposition 4.4 Let $(X, T_1, T_2, T_{1f}, T_{2f})$ and $(Y, S_1, S_2, S_{1f}, S_{2f})$ be any two fuzzy fine bi-topological spaces. A function f : $(X, T_1, T_2, T_{1f}, T_{2f}) \rightarrow (Y, S_1, S_2, S_{1f}, S_{2f})$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine c-homeomorphism and $(X, T_1, T_2, T_{1f}, T_{2f})$ be a Pairwise (T_{if}, T_{jf}) fuzzy fine connected space then $(Y, S_1, S_2, S_{1f}, S_{2f})$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine connectedspace.

Proof Let f : $(X, T_1, T_2, T_{1f}, T_{2f}) \rightarrow (Y, S_1, S_2, S_{1f}, S_{2f})$ be a Pairwise (T_{if}, T_{jf}) fuzzy fine c-homeomorphism and let $(X, T_1, T_2, T_{1f}, T_{2f})$ be a Pairwise (T_{if}, T_{jf}) fuzzy fine connected space. Assume that $(Y, S_1, S_2, S_{1f}, S_{2f})$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine disconnected space.Then $1_Y = \gamma \vee \delta$ where γ is a T_1-fuzzy fine open and δ is a T_2 fuzzy fine open in $(Y, S_1, S_2, S_{1f}, S_{2f})$ .Since f isPairwise (T_{if}, T_{jf}) fuzzy fine dually homeomorphism, $f^{-1}(\gamma)$ is T_1-fuzzy fine openand $f^{-1}(\delta)$ is T_2-fuzzy fine open in $(X, T_1, T_2, T_{1f}, T_{2f})$. Hence $1_X = f^{-1}(\gamma) \vee f^{-1}(\delta)$, where $f^{-1}(\gamma) \bar{q} f^{-1}(\delta)$. Hence $(X, T_1, T_2, T_{1f}, T_{2f})$ is a pairwise (T_{if}, T_{jf}) fuzzy fine disconnected space. But this leads to a contradiction. Therefore $(Y, S_1, S_2, S_{1f}, S_{2f})$ is a Pairwise (T_{if}, T_{jf}) fuzzy fine connected space.

V. CONCLUSION

In this paper, we discussed some properties on Pairwise (T_{if}, T_{jf}) fuzzy fine dually compact spaces. We can extend the work in fuzzy Peano space.

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