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Application of Non-Additive Entropy in Questionnaire Theory

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Abstract: In present communication We proposed two new measure of average charge for miscellaneous set of questions under non-additivity and studied their properties These measures includes the earlier studied measures as limiting and particular cases .A relation between these measures have been established. it shows that the average charges are upper bounds of non-additive mean value entropies.

Keywords : Non-additive measures, Renyi's entropy, Holder's inequality, Kraft and Duncan inequalities

I. INTRODUCTION

Picard[7] established applications of information theory in questionnaire theory. Picard and Campbell[2] established relation between noiseless coding theory and information theory with a charging method supported the decision of queries. Duncan[4] have Generalized the Kraft's inequality for miscellaneous set of questions . Based on the charging method supported for a subject of decision, a generalized coding theorem has been proven by Duncan[4] for miscellaneous set of questions, according to this theorem Claude E. Shannon entropy is the lower bound for average charge . B.D. Sharma and Asha Garg[11] have established the charge of order t by using the additive property of measure of mean charge and they have proved that Renyi's measure of order α [8] is lower bound for this measure which they named charge of order t .The measures given by C.E Shannon and Renyi are both additive for independent probability distributions and Harvda and Charvat[6] stidied non-additive measures and which are later studied by Darcozy[3]. We can see some examples from biological and social sciences for non-additivity property of measures .Take into account 2 medicine and which will cure at completely different times the ailments and from that someone suffer. however the medicine and along might not cure someone plagued by each the ailments and . There will many samples of this sort taken from social systems. so we want to contemplate non-additive systems.

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With a system we are able to correlate a quantitative measure to get a number of its properties. The measure that we have a tendency to associate is also additive, non-additive and also the like. If μ could be a non-additive measure, then the non-additive system ought to think about the distinction

$$\mu(AB) = \mu(A) - \mu(B)$$

where A and B are two independent system. Define index η as

$$\eta = \frac{\mu(AB) - \mu(A) - \mu(B)}{\mu(A)\mu(B)}$$

This index is taken to describe the non-additive system. This index have two properties, first this index is symmetric in given systems and secondly it is lessening to zero for an additive system. Harvda and Charvat[6] considered this system. Alternative systems are also there however we tend to impound our interest to the current system. Sharma and Mittal[10] established two information measures which are non-additive and contain Renyi's measure of order a [8] as well as Harvda Charvat Darcozy measures of type b as particular cases.

In this paper, by taking the mean charge for miscellaneous set of questions to be non-additive, we proposed new measures of average charge which contain the average charge defined by Ducan[4] and the charge function given by Sharma and Garg[11] as limiting case and theorem which gives lower bounds for these new measures of charge have been proved.

II. METHODOLOGY

Let $S = (s_1, s_2, ..., s_k)$ be fixed state space having probability distribution $E = (e_1, e_2, ..., e_k)$ such that the probability of true state s_j is

e_j, $(j = 1, 2, ..., k)$ and

$$\sum_{j=1}^{k} e_j = 1\,;\; e_j \geq 0, \quad (j = 1, 2, ..., k)\,.$$

Let Q be set of questions on S and $n_{j\delta}$ indicate the number of questions of decision δ needed to find the state s_j .For $n_{j\delta}$ number of miscellaneous questions of decision $\delta(\delta = 1, 2, ..., k)$, following inequality is given by Duncan[4]

$$\sum_{j=1}^{k} \prod_{\delta=1}^{\infty} \delta^{-n_{j\delta}} \leq 1$$

If $\log_2(\delta)$ is the charge for each query of decision

δ, $R_C(Q)$ is the arbitrary charge then the estimated charge for Q is given by

$$AC_E(Q)=\sum_{j=1}^{k}\sum_{\delta=1}^{\infty} e_j n_{j\delta}\log_2\delta .$$

Random charge for Q is given by

$$R_C^f(Q)=f\left(\log_2\prod_{\delta=1}^{\infty} d^{n_{j\delta}}\right)$$

Where, $f:[1,\infty[\to R$, be continuous, strictly increasing function and the generalized mean charge for given set of questions Q may be expressed as

$$G_C^f(Q)=f^{-1}\left[\sum_{j=1}^{k} e_j f\left(\sum_{\delta=1}^{\infty} n_{j\delta}\log_2\delta\right)\right] \quad (1)$$

If $n_{1\delta}=n_{2\delta}=...=n_{m\delta}=n_\delta$ (say), then Equation (1) reduces to $AC_E(Q)$

$$G_C^f(Q)=\sum_{\delta=1}^{\infty} n_\delta\log_2\delta = AC_E(Q).$$

Also , if we take

$$f(y)=f_0(y)=py+q; \qquad p\neq 0, \qquad y\in[1,\infty[$$

then $G_C{}^f(Q)=\sum_{j=1}^{k}\sum_{\delta=1}^{\infty} p_j n_{j\delta}\log_2\delta = AC_p(Q)$.

Now consider two independent state space $S=(s_1,s_2,...,s_L)$ and $S^*=(s_1^*,s_2^*,...,s_T^*)$ with associated probability distributions $E=(e_1,e_2,...,e_L)$ and $W=(w_1,w_2,...,w_t)$ such that

$$e_l\geq 0,\quad \sum_{l=1}^{L} e_j=1,\quad (l=1,2,...,L) \quad \text{and}$$

$$w_t\geq 0,\quad \sum_{t=1}^{T} w_t=1,\quad (t=1,2,...,T) \quad .$$

Since S and S^* are independent the probability of the pair (s_l,s_t^*) is $e_l w_t (l=1,2,...,L;t=1,2,...,T)$.

Let PW be the probability distribution on

$$S\times S^*\{e_1w_1,e_1w_2,...,e_1w_T,e_2w_1,e_2w_2,...,e_2w_T,...,$$
$$e_Lw_1,e_Lw_2,...,e_Lw_T\}$$

and let the valid heterogeneous questionnaire Q_1 and Q_2 exist on S and S^*, which use precisely $m_{l\delta}(l=1,2,...,L)$ and $n_{t\delta}(t=1,2,...,T)$ questions of resolution δ respectively to determine s_l and s_t^*. A questionnaire say Q, may now be developed from the above two questionnaire on S and S^* in which $m_{l\delta}+n_{t\delta}(l=1,2,...,L;\ t=1,2,...,T)$ questions of resolution δ are required to determine the pair (s_l,s_t^*).

Now, because a questionnaire for (s_l,s_t^*) exists with $m_{l\delta}+n_{t\delta}$ questions of resolution δ $(\delta=1,2,...)$, we have the inequality

$$\sum_{l=1}^{L}\sum_{t=1}^{T}\prod_{\delta=1}^{\infty}\delta^{-(m_{l\delta}+n_{t\delta})}\leq 1.$$

Now if $A_{EW}^f C(Q)$ is a function of mean charge for the given set of questions Q, then following two case arises. Firstly,

$$A_{EW}^f C(Q)=A_E^f C(Q_1)+A_W^f C(Q_2) \quad (2)$$

Sharma and Garg [7] studied the mean charge

$$A_E C(Q)=\sum_{i=1}^{m}\sum_{\delta=1}^{\infty} e_i n_{i\delta}\log_2\delta \quad (3)$$

and also characterize the charge of order t

$$A_E{}^t C(Q)=\frac{1}{t}\log_2\left(\sum_{i=1}^{m} e_i\prod_{\delta=1}^{\infty}\delta^{tn_{i\delta}}\right),\quad 0<t<\infty . \quad (4)$$

Secondly, $A_{EW}^f C(Q)$ satisfies,

$$A_{EW}^f C(Q)=A_E^f C(Q_1)+A_W^f C(Q_2)+\eta A_E^f C(Q_1)A_W^f C(Q_2) \quad (5)$$

and the mean value property

$$A_E^f C(Q)=f^{-1}\left(\sum_{l=1}^{L} p_j f\left(g\left(\sum_{\delta=1}^{\infty} n_{l\delta}\log_2\delta\right)\right)\right) \quad (6)$$

where $g\left(\sum_{l=1}^{L} n_{j\delta}\log_2\delta\right)$ is η-non-additive function of random charge based on a $\log_2\delta$ charging scheme.

III. CHARACTERIZATION OF NON-ADDITIVE MEASURES OF AVERAGE CHARGE

Next we characterized two new measures of mean charge for miscellaneous set of questions by using the non-additivity condition of measure of mean charge. We also discuss special cases of these measures and a theorem which defines the lower bound for these functions is given

First of all we shall determine the non-additive charge function g of the charging method depends upon $\log_2\delta$ which uses exactly $n_{j\delta}$ questions of decision δ to find the state s_j which satisfies the non-additive condition.

$$g\left(\sum_{\delta=1}^{\infty} n_\delta\log_2\delta+\sum_{\delta=1}^{\infty} m_\delta\log_2\delta\right)=g\left(\sum_{\delta=1}^{\infty} n_\delta\log_2\delta\right)+g\left(\sum_{\delta=1}^{\infty} m_\delta\log_2\delta\right)$$
$$+\eta g\left(\sum_{\delta=1}^{\infty} n_\delta\log_2\delta\right)g\left(\sum_{\delta=1}^{\infty} m_\delta\log_2\delta\right),\quad \eta\neq 0 \quad (7)$$

taking

$$\sum_{\delta=1}^{\infty} n_\delta\log_2\delta = v, \qquad \sum_{\delta=1}^{\infty} m_\delta\log_2\delta = w.$$

$$\sum_{\delta=1}^{\infty} n_{i\delta}\log_2\delta = n_{i.}, \qquad \sum_{\delta=1}^{\infty} m_{k\delta}\log_2\delta = m_{k.},$$

then, (7) takes the form

$$g(v.+w)=g(v)+g(w)+\eta g(v)h(w)$$

Taking $\phi(v)=1+\eta g(v.)$, we get

$$\phi(v+w)=\phi(v)\phi(w) \tag{8}$$

The non-zero general solution of (8) , is given by

$$\phi(v)=2^{\left(\frac{1-b}{a}\right)v}$$

where $a,b\neq 1$ are arbitrary constants.

hence

$$g(v)=\frac{2^{\left(\frac{1-b}{a}\right)v.}-1}{\eta},\quad \eta\neq 0 \tag{9}$$

By taking

$$\eta=(2^{\left(\frac{1-b}{a}\right)}-1),\ b\neq 1 \ \text{ as } \eta\neq 0$$

when $b\to 1$, the function $g(v)$ reduces to additive one , and in general form it can taken as

$$g(v)=\frac{N^{\left(\frac{1-b}{a}\right)v.}-1}{N^{\left(\frac{1-b}{a}\right)}-1},\quad b\neq 1 \tag{10}$$

To find $A_E{}^f C(Q)$, put the value of $g(v)$ from (10) in (6) and t using (5) with $\eta=(N^{\left(\frac{1-b}{a}\right)}-1),\ b\neq 1$, we have

$$f^{-1}\left(\sum_{l=1}^{L}\sum_{t=1}^{T}e_l w_t f\left(\frac{2^{\left(\frac{1-b}{a}\right)(n_l+m_t)}-1}{2^{\left(\frac{1-b}{a}\right)}-1}\right)\right)$$

$$=f^{-1}\left(\sum_{l=1}^{L}p_l f\left(\frac{2^{\left(\frac{1-b}{a}\right)n_l}-1}{2^{\left(\frac{1-b}{a}\right)}-1}\right)\right)+$$

$$+f^{-1}\left(\sum_{t=1}^{T}w_t f\left(\frac{2^{\left(\frac{1-b}{a}\right)m_t}-1}{2^{\left(\frac{1-b}{a}\right)}-1}\right)\right)$$

$$+\left(2^{\left(\frac{1-b}{a}\right)}-1\right)f^{-1}\left(\sum_{l=1}^{L}p_l f\left(\frac{N^{\left(\frac{1-b}{a}\right)n_l}-1}{N^{\left(\frac{1-b}{a}\right)}-1}\right)\right)$$

$$f^{-1}\left(\sum_{t=1}^{T}w_l f\left(\frac{N^{\left(\frac{1-b}{a}\right)m_l}-1}{N^{\left(\frac{1-b}{a}\right)}-1}\right)\right) \tag{11}$$

Take

$W=\{w\},\ m_{t\delta}=m_\delta\ (t=1,2,\ldots,T)$,

$E=(e_1,e_2,\ldots,e_L),\ e_l\geq 0,\ \sum_{l=1}^{L}e_l=1$ in (11) to get

$$f^{-1}\left(\sum_{l=1}^{L}e_j f\left(\frac{2^{\left(\frac{1-b}{a}\right)(n_l+m_t)}-1}{2^{\left(\frac{1-b}{a}\right)}-1}\right)\right)$$

$$=f^{-1}\left(\sum_{l=1}^{L}e_l f\left(\frac{2^{\left(\frac{1-b}{a}\right)n_j}-1}{2^{\left(\frac{1-b}{a}\right)}-1}\right)\right)2^{\left(\frac{1-b}{a}\right)m_t}$$

$$+\frac{2^{\left(\frac{1-b}{a}\right)m_t}-1}{2^{\left(\frac{1-b}{a}\right)}-1}$$

or

$$h_{m_\delta}^{-1}\left(\sum_{l=1}^{L}e_l h_{m_\delta}\left(\frac{2^{\left(\frac{1-b}{a}\right)n_l}-1}{2^{\left(\frac{1-b}{a}\right)}-1}\right)\right)$$

$$=f^{-1}\left(\sum_{l=1}^{L}e_l f\left(\frac{2^{\left(\frac{1-b}{a}\right)n_l}-1}{2^{\left(\frac{1-b}{a}\right)}-1}\right)\right)2^{\left(\frac{1-b}{a}\right)m}+\frac{2^{\left(\frac{1-b}{a}\right)m}-1}{2^{\left(\frac{1-b}{a}\right)}-1} \tag{12}$$

Where

$$h_{m_\delta}\left(\frac{2^{\left(\frac{1-b}{a}\right)n_l}-1}{2^{\left(\frac{1-b}{a}\right)}-1}\right)=f\left(\frac{2^{\left(\frac{1-b}{a}\right)(n_l+m)}-1}{2^{\left(\frac{1-b}{a}\right)}-1}\right) \tag{13}$$

Hardy Littlewood and Polya[5], showed that h_{m_δ} and f , satisfies following relation

$$h_{m_\delta}(v)=f_1(w)f(v)+f_2(w) \tag{14}$$

where $f_1(w)$ and $f\ (w)$ are independent of v .

Making use of equation (13) and (14), we get

$$g(v+w)=f_1(w)g(v)+f_2(w) \tag{15}$$

where

$$g(v)=f\left(\frac{2^{\left(\frac{1-b}{a}\right)v}-1}{2^{\left(\frac{1-b}{a}\right)}-1}\right) \tag{16}$$

or

$$G(v+w)=f_1(w)G(v)+G(w) \tag{17}$$

where

$$G(v)=g(v)-c_1 \tag{18}$$

and c_1 is a constant.

By symmetry of (17), we have

$$f_1(w)G(v)+G(w)=f_1(v)G(w)+G(v) \tag{19}$$

There are two possibilities, $f_1(v)\equiv 1$ and $f_2(v)\neq 1$.

Case I. $f_1(v)\equiv 1$, then the general solution of (19) takes the form

$$G(v)=c_2 v. \tag{20}$$

where c_2 is an arbitrary constant.

Making use of equation (18) and (16), in equation(20) , we have

$$f\left(\frac{2^{\left(\frac{1-b}{a}\right)v}-1}{2^{\left(\frac{1-b}{a}\right)}-1}\right)=c_1+c_2 v$$

which implies

$$f(n_\delta \log_2 \delta)=$$

$$=c_1+\frac{c_2 a}{1-b}\log_2\left(1+(2^{\frac{1-b}{a}}-1)\sum_{\delta=1}^{\infty} n_\delta \log_2 \delta\right) \quad (21)$$

Case II If $f_1(v)\neq 1$, By Hardy Littlewood and Polya[5] the general continuous solutions are given by

$f_1(v)=0$ for all v, which we ignore

and $$f_1(v)=2^{tv} \quad (22)$$

where $t\neq 0$ is an arbitrary constant.

Making use of (22) and (18), (16) gives

$$f\left(\frac{2^{\left(\frac{1-b}{a}\right)v}-1}{2^{\left(\frac{1-b}{a}\right)}-1}\right)=c_1+\frac{2^{tv}-1}{K}$$

which gives

$$f(\sum_{\delta=1}^{\infty} n_\delta \log_2 \delta)=$$

$$=c_1+\frac{[(2^{\frac{1-b}{a}}-1)\sum_{\delta=1}^{\infty} n_\delta \log_2 \delta+1]^{ta/(1-b)}-1}{K}$$

$$b\neq 1,\ t\neq 0 \quad (23)$$

With the help of values of f given in (21) and (23) we define following two non-additive measures of charge

$$C_E^{(1,a,b)}(Q)=(2^{\frac{1-a}{b}}-1)^{-1}\left[2^{\frac{(1-b)}{a}\sum_{l=1}^{L}\sum_{\delta=1}^{\infty} e_l n_{l\delta}\log_2\delta}-1\right],$$

$$a\neq 0, b\neq 1 \quad (24)$$

$$C_E^{(t,a,b)}(Q)=(2^{\frac{1-b}{a}}-1)^{-1}\left[2^{\frac{(1-b)}{at}\log_2\sum_{l=1}^{L} e_l\prod_{\delta=1}^{\infty}\delta^{tn_{l\delta}}}-1\right],$$

$$a,t\neq 0, b\neq 1 \quad (25)$$

These functions for charging method depends upon $\log_2 \delta$ symbolized by $C_E^{(1,a,b)}(Q)$ and $C_E^{(t,a,b)}(Q)$ which we call non-additive charge of type (a,b) of order 1 and t respectively.

The above discussion can be stated in the form of following theorem.

Theorem 1. The mean charges defined in (4) of a set of questions Q (which uses exactly $n_{l\delta}$ questions of resolution δ to find the lth state) defined on the state space $S=(s_1,s_2,...,s_l)$ with probability distribution

$E=(e_1,e_2,...,e_l),\ e_l\geq 0,\ \sum_{l=1}^{L} e_l=1$ satisfying

$\sum_{l=1}^{L}\prod_{\delta=1}^{\infty}\delta^{-n_{l\delta}}\leq 1$ and non-additivity relation are defined in (24) and (25).

Theorem 2. Let $S=(s_1,s_2,...,s_m)$ be a finite state space and $E=(e_1,e_2,...,e_m)$ be a discrete probability distribution. If Q is a valid miscellaneous set of questions and $C(Q)$ is the random charge depends upon $\log_2\delta$ for each question of resolution δ, then

(i) $$C_E^{(1,a,b)}(Q)\geq Z(E;a,b) \quad (26)$$

here equal sign occur iff $n_{i\delta}=0,\ \forall\ \delta>m.$ and

$$e_i=\prod_{\delta=2}^{m}\delta^{-n_{i\delta}}$$

and where

$$Z(E;a,b)=\frac{2^{(\frac{b-1}{a})\sum_{i=1}^{m} e_i\log_2 e_i}-1}{2^{\frac{1-b}{a}}-1},$$

$$a\neq 0,\ b\neq 1$$

(ii) $$C_E^{(t,b)}(Q)\geq Z(E;a,b), \quad (27)$$

here equal sign occur iff

$n_{i\delta}=0,\ \forall\ \delta>m$ and

$$\frac{e_i^{\,a}}{\sum_{j=1}^{m} e_j^{\,a}}=\prod_{\delta=2}^{m}\delta^{-n_{i\delta}} \qquad (i=1,2,...,m)$$

where $a=(1+t)^{-1}$

and

$$Z(E;a,b)=\frac{\left(\sum_{i=1}^{m} e_i^{\,a}\right)^{(\frac{b-1}{a-1})}-1}{2^{\frac{1-b}{a}}-1},$$

$$a\neq 1,\ b\neq 1,\ a>0, b>0$$

Proof . (i) Ducan[4] has shown that

$$\sum_{i=1}^{m}\sum_{\delta=1}^{\infty} e_i n_{i\delta}\log_2\delta\geq -\sum_{i=1}^{m} e_i\log_2 e_i. \quad (28)$$

here equal sign holds iff

$n_{i\delta}=0,\forall\ \delta>m$ and

$$e_i = \prod_{\delta=2}^{m} \delta^{-n_{i\delta}}; \quad i = 1,2,...,m$$

Now $(2^{\left(\frac{1-b}{a}\right)} - 1) > 0$ if $b < 1$ and

$$(2^{\left(\frac{1-b}{a}\right)} - 1) < 0 \text{ if } b > 1$$

Hence by simple calculations, we have

$$\frac{2^{\frac{1-b}{a}\sum_{i=1}^{m}\sum_{\delta=1}^{\infty} p_i n_{i\delta}\log_2\delta} - 1}{2^{\frac{1-b}{a}} - 1} \geq \frac{2^{\frac{b-1}{a}\sum_{i=1}^{m} p_i \log_2 p_i} - 1}{N^{\frac{1-b}{a}} - 1}, \quad b \neq 1$$

which gives the required result.

Proof (ii)

If $t = 0$ and $a = 1$, the result can be proved in similar way as in part (i). For values other than 0 and 1 we use Holder's inequality

$$\left(\sum_{i=1}^{m} x_i^p\right)^{1/p} \cdot \left(\sum_{i=1}^{m} y_i^q\right) \leq \sum_{i=1}^{m} x_i y_i, \quad (29)$$

where $p^{-1} + q^{-1} = 1$ and $p < 1$.

choosing

$$p = -t, \quad q = (1-a), \; x_i = e_i^{-1/t} \prod_{\delta=1}^{\infty} \delta^{-n_{i\delta}}$$ and

$y_i = e_i^{1/t}$ in (29) and taking power $(1-b)/a$ on both sides and with the fact $(2^{\left(\frac{1-b}{a}\right)} - 1) > 0$ for $b < 1$ and $(2^{\left(\frac{1-b}{a}\right)} - 1) < 0$ for $b > 1$ according as $b < 1$ we get result (27) with simple calculations.

IV. RESULT ANALYSIS AND DISCUSSION

The proposed new measures of average charge depends upon parametric values in opposition to average charge defined by Ducan and the charge of order t studied by B.D, Sharma and Asha Garg. Therefore theorem 2 is generali- zation of results given by Ducan ,Sharma and Garg. The following points justify this.

$$1. \lim_{t \to 0} C_E^{(t,a,b)}(Q) = C_E^{(1,a,b)}(Q) \quad (30)$$

$$2. \lim_{b \to 1} C_E^{(1,a,b)}(Q) =$$

$$= \sum_{l=1}^{L}\sum_{\delta=1}^{\infty} e_l n_{l\delta} \log_2 \delta = AC_E(Q) \quad (31)$$

This is the average charge given by Ducan[4]

$$3. \lim_{b \to 1} C_E^{(t,a,b)}(Q) =$$

$$= \frac{1}{t}\log_2\left(\sum_{l=1}^{L} e_l \prod_{\delta=1}^{\infty} \delta^{t n_{l\delta}}\right) = C_E{}^t(Q) \quad (32)$$

This is the charge of order t given by B.D. Sharma and Asha Garg[11].

4. If $n_{1\delta} = n_{2\delta} = ... = n_{n\delta} = n_\delta$ (say), then equations (30) and (31) becomes

$$\left(2^{\left(\frac{1-b}{a}\right)} - 1\right)^{-1}\left[2^{\left(\frac{1-b}{a}\right)\sum_{\delta=1}^{\infty} n_\delta \log_2\delta} - 1\right]$$

when $b \to 1$ then above expression will becomes

$$\sum_{\delta=1}^{\infty} n_\delta \log_2 \delta.$$

Hence it is clear that $C_E^{(1,a,b)}(Q)$ and $C_E^{(t,a,b)}(Q)$ are type b generalizations of

$$C_E(Q) = \sum_{l=1}^{L}\sum_{\delta=1}^{\infty} e_j n_{l\delta} \log_2 \delta$$

and

$$C_E{}^t(Q) = \frac{1}{t}\log_2\left(\sum_{l=1}^{L} e_l \prod_{\delta=1}^{\infty} \delta^{t n_{l\delta}}\right)$$

respectively.

V. CONCLUSION

We have successfully introduced two new information measures of order 1 and type a and of order a and type b and gives their application in questionnaire theory by introducing a new measure of average charge, As a conclusion we remarked that these new measures of average charges are new measures for which it is worthwhile to consider further applications in fuzzy theory, intuistic fuzzy theory, soft fuzzy set theory etc .

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