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Variable Density Sampling Trajectories for Compressive Sensed MRI

Kavitha S, B. Aziz Musthafa

Abstract*: *Diagnosis of diseases require high resolution images of human body parts. Magnetic Resonance Imaging (MRI) is a popular technology commonly used for this purpose. In addition to having several benefits, this technology has few shortcomings also. One of them is its high scanning time. In MRI acquisition of image is based on the principle of traditional sampling theorem. The novel sampling theory called as Compressive Sensing (CS) which allows the reconstruction of sparse signals from undersampled data. The application of CS onto MRI will drastically reduce the acquisition time and hence scanning time. In this manuscript analysis and application of CS on to MRI is demonstrated. Simulations are carried out using Variable Density Sampling trajectories (VDS). Then a comparative study is made in terms of Signal to Noise Ratio (SNR) and execution time based on the result obtained.

Keywords* :*Magnetic Resonance Imaging, Compressive Sensing, Variable Density Sampling, Signal to Noise Ratio, execution time.

I. INTRODUCTION

MRI is a popular technology used to produce detailed pictures of human body organs and tissues. It plays a vital role in diagnosing of diseases in the medical world. During the scanning process patients have to be still since movement creates motion artifacts in the image. Some cases even patient have to hold breath for producing good quality images. This creates discomfort in patients since the scanning time may vary from 20 to 60 minutes. All these things will become insignificant if the scanning time is reduced [1]. There are different approaches proposed to reduce this time. But there is no significant reduction in the acquisition time because all these techniques are based on Nyquists sampling theorem. Compressive sensing [2] is the novel undersampling theory which allows the sparse signal reconstruction using few measurements. The key requirements for this criterion are sparsity of signals, possibility of random undersampling and non linear reconstruction. Most of the MR images are sparse in nature either in its domain or in the transform domain.

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***** Correspondence Author
Kavitha S*, Department, of Electonics and Communication, NMAMIT Nitte, affiliated VTU Belagavi, Udupi, India. Email: hereiskavitha@gmail.com
Dr. B. Aziz Musthafa, Department of Computer Science, BIT Mangalore, Mangalore, India. Email: azizmusthafa@gmail.com

In MRI process the prior knowledge about the testing organ is available therefore one can easily identify the sparsity domain of the image. In MRI scanner Fourier coefficients of the test image are collected and stored in a space called as K space [3]. Data acquisition in MRI is nothing but sampling of K space. Energy distribution in MR images is not uniform. There are more number of coefficients present at the center and less near the periphery. This enables non uniform undersampling of MR images. All these makes the natural fit of MRI onto CS. Compressive sensing is the best choice to reduce the scanning time. The reconstruction of undersampled data is treated as optimization problem. The norm function of linear algebra can be used to solve this optimization problem.

In this manuscript how compressive sensing technique can be applied for the rapid MRI process is demonstrated. The rest of the manuscript discusses the following section II deals with fundamentals of compressive sensing, section III briefs about the principle of MRI, section IV describes the methodology, results and discussions are presented in section V and conclusion is given in section VI.

II. COMPRESSIVE SENSING

Nyquists Sampling theory states that reconstruction of the signal/image is possible if and only if samples are taken at a rate more than the Nyquists rate. Any violation in this theorem will lead to aliasing. Generally sampling rate is too high results in so many samples, creates storage difficulty. The transmission of data and storage is possible if the redundancy is removed. To do so compression algorithms are used. In this process a lot of samples which were captured will be discarded. These samples were acquired by knowing that they will be discarded later just to satisfy the sampling theorem. The idea behind compressive sensing is to acquire only the important samples so there is no question of redundancy. This reduces the sampling rate drastically enables sampling of the signal at information rate. This merges sampling and compression into one. But main concern here is how the signal/image can be sampled at an information rate? This can be achieved by gathering information about sparsity domain of the signal. The philosophy of compressive sensing [2] can be explained by the mathematical theory behind this. Let the 'S' be the signal to be sampled and this can be made sparse using the basis Ø is shown in equation (1)

$$S = ØX \qquad (1)$$

where Ø is n x n representation matrix, X is a sparse vector of length n. if X contains only k number of nonzero coefficients then signal S is called sparse.

The signal S can be sampled using sampling or measurement matrix Θ is as shown in equation (2)

$$Y = \Theta S \quad (2)$$

where Y is the measured or sampled data, the sampling matrix Θ is having dimension m x n, m<n indicating compressive sampling. The measured data Y can be represented as shown in equation (3)

$$Y = \Theta \text{Ø} X = AX \quad (3)$$

where A=ΘØ is called reconstruction matrix with dimension m x n. The reconstruction of the signal S from the coefficient X is similar to the solution of the problem AX = Y of linear algebra. Since the matrix A has less number of rows than the columns, number of solutions of this equation is infinite. The best solution will reconstruct the signal exactly and equally. This is as shown in Fig.1. The task of obtaining the exact solution is like an optimization problem. The problem shall be solved by the use of norm function of linear algebra. The Lp [3] norm of a signal 's' is defined by the equation (4).

$$\|s\|_p = \left(\sum_{i=0}^{n} |s_i|^p\right)^{\frac{1}{p}} \quad (4)$$

If the value of p=1 then $\|s\|_1$ is known as L_1 norm of the signal s, which describes sparsity, the number nonzero components present in the signal's'. With p=2 $\|s\|_2$ called as L_2 norm of the signal's'.

The Total Variation (TV) [4] is the norm of gradient of an image. The anisotropic TV is the L_1 norm of gradient whereas isotropic norm is the L_2 norm of the gradient of an image as given in equation (5) and (6) respectively.

$$TV(s) = \sum_{j,k} \left\| s_{j+1,k} - s_{j,k}, s_{j,k+1} - s_{j,k} \right\|_1 \quad (5)$$

$$TV(s) = \sum_{j,k} \left\| s_{j+1,k} - s_{j,k}, s_{j,k+1} - s_{j,k} \right\|_2 \quad (6)$$

The TV norm will be useful when an image/signal is having sparse gradient. It is also able preserve edges of an image.

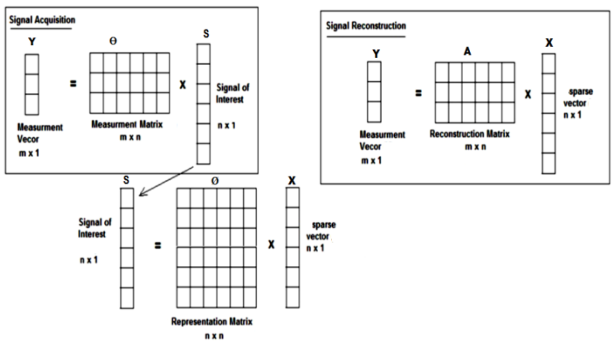


Fig.1 Compressive Sensing Model

The compressive sensing is works on three key factors sparsity, undersampling and nonlinear reconstruction. Applications of compressive sensing range widely. This novel sampling theory is prominently useful in reducing the acquisition time of MRI scanners.

III. PRINCIPLE OF MRI

MRI technique is prominently used in the field of medical to understand the disorders of the human body organs by capturing quality images. In MRI scanner strong static filed of 0.2 to 0.3 Tesla is applied. The scanner also generates the time varying magnetic field by Radio Frequency (RF) current. The human body is mostly made of water. These hydrogen protons [5] possess spin and it will be aligned under strong static field. Due to time varying magnetic field produced by gradient magnets, they absorb energy and show spin in different direction which is termed as resonance. The frequency of resonance is called Larmor frequency given in equation (7).

$$\omega_0 = \frac{\gamma \beta_0}{2\pi} \quad (7)$$

where γ is gyromagnetic ratio and β_0 is the magnetic field strength. When the magnetic field is off they come back to original alignment by releasing RF signal. The MRI scanner contains receiver coils to capture these signals. These signals are corresponds to the position of gradient magnets or in other words the body part being scanned. This data is the Fourier samples of the body part. By the use of Inverse Fourier Transform (IFT) clear image of the organ can be obtained. MRI scanner produces good resolution image but the major problem is that the patients should be still during the entire process of 30 to 90 minutes. If there is any movement which effects the quality of an image leads to the repetition of the process. By the application of CS scanning duration can be reduced drastically.

IV. METHODOLOGY

The methodology of the proposed implementation is as shown in Fig. 2. The representation matrix Ø is the Fourier matrix. The test image is sparse in Fourier domain. The measurement matrix Θ is the VDS sampling mask. The reconstruction algorithm used is the Reconstruction from Partial Fourier algorithm (RecPF) which solves the TVL1-L2 model optimization problem.

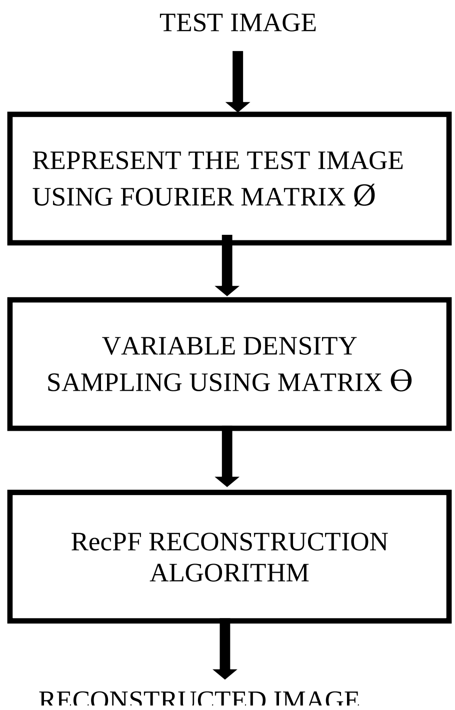


Fig. 2 Block Diagram

A. Sampling Mask

One of the key challenges in the CS-MRI is mask designing problem. This is the active research area. Mask is the sampling trajectory or measurement matrix used in the compressive sensing. Traditional sampling theory uses Cartesian sampling grid to satisfy the sampling theorem. In CS non Cartesian sampling is used. The design of undersampling mask [6] is the key requirement in Compressive sampling. The K space data is not uniform in nature. It has non uniform energy distribution, dense at the center than periphery. This enables the use of non uniform sampling. The random sampling trajectories are preferred in CS for undersampling. The use of Variable Density Sampling (VDS) [7] makes the process easy. Since the K space energy is concentrated at the center, capturing more samples from the center will avoid artifacts in the CS MRI reconstruction. The VDS allows us reconstruct the image using very few measurements. In this work VDS with elliptical, square and triangular sampling density masks are considered as shown in Fig: 3. The K space data is sampled using the VDS mask. The mask is generated using radius and angle function K_r and K_Θ as given in equation (8-11) [8].

Elliptical:

$$K_r\,(k_y,\ k_z) = \sqrt{(\alpha k_y)^2 + {k_z}^2} \qquad (8)$$

Elliptical with spiral twist:

$$K_r\,(k_y,\ k_z) = \sqrt{(\alpha k_y)^2 + k_z}$$

$$K_\Theta(k_y,\ k_z) = \arg\left\{(\alpha k_y + i k_z \times e^{i2\pi\beta Kr\,(ky,kz)}\right\} \qquad (9)$$

Square:

$$K_r\,(k_y,\ k_z) = \max\left\{\left|\alpha k_y\right|, \left|k_z\right|\right\} \qquad (10)$$

Triangular:

$$K_r\,(k_y,\ k_z) = \left|\alpha k_y\right| + \left|k_z\right| \qquad (11)$$

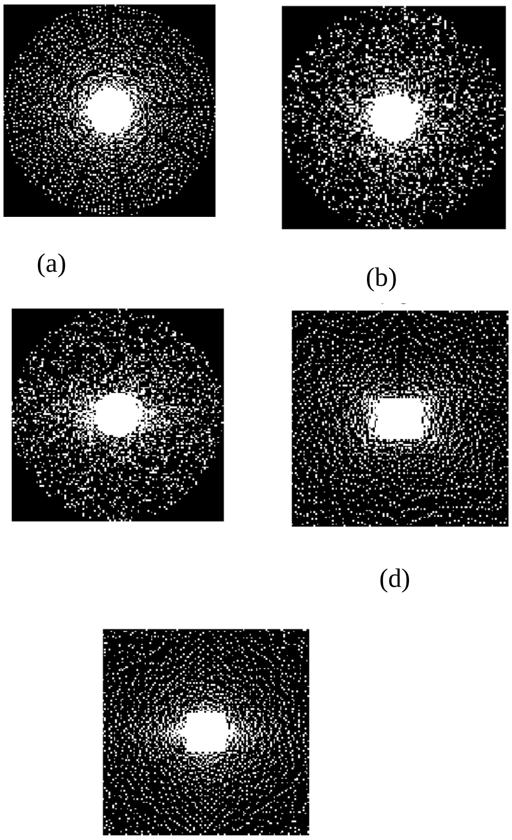
where k_y, k_z denotes K space dimensions, α controls isotropic behavior of the mask and β is used to add spiral twist to each radial spoke.

B. Reconstruction Algorithm

The reconstruction algorithm [9] used is Reconstruction from Partial Fourier algorithm (RecPF) which solves the TVL1-L2 model optimization problem given in equation (12) [10]. This algorithm picks the data from Fourier domain.

$$\min a_{TV} * TV(u) + a_{L1} * |ST * u|_1 + 0.5|Fp * u - B|_2{}^2 \qquad (12)$$

where a_{TV}, a_{L1} are regularization parameters, B is the measurement vector, u is the signal to be reconstructed, ST is the sparsifying transform and Fp is the partial Fourier matrix.



(a) (b) (c) (d) (e)

Fig: 3 Sampling Density Masks (a) Elliptical (b) Elliptical with spiral twist (c) Elliptical with spiral twist (anisotropic) (d) Square (e) Triangular

V. RESULTS AND DISCUSSION

Simulation was carried out using MATLAB phantom test image of size 128x128. The K space data is the Fourier coefficients of the image. This data is undersampled using the proposed variable density masks listed in Fig: 3. Analysis is done for different undersampling patterns and Signal to Noise Ratio (SNR) and execution time are measured. The reconstruction is performed using regularization parameters a_{TV} =10^{-7}, a_{L1} =0. The simulation is also done by varying percentage of samples. A comparative analysis is as shown in Table: 1. The VDS allows reconstruction using few measurements. The simulation shown in Table: 1 uses only 18% of the Fourier coefficients The experiment is also conducted to observe the effect of regularization parameter a_{TV} on the reconstruction algorithm. It is observed that as the regularization parameter is increased the performance parameters SNR and Execution time are decreased as depicted in Fig: 4. To have an optimum performance the selection of a_{TV} is as crucial. It is observed that the value of a_{TV} is found around 10^{-7} for optimum performance, above this limit performance degrades.

Table- I Simulation Parameters

Sampling mask	SNR (dB)	Execution Time(sec)
Elliptical (isotropic)	33.87	4.31
Elliptical with spiral twist (isotropic)	33.88	4.54
Elliptical with spiral twist (anisotropic)	34.37	4.67
Square (anisotropic)	40.07	3.81
Triangle (anisotropic)	41.99	3.89

The comparison of proposed VDS sampling masks show that the proposed triangular and square sampling masks measure more samples from the center. Therefore they perform better than the elliptical mask. The anisotropic spacing increases randomness of the mask. This will be useful in remove motion artifacts of the image. As depicted from the Table: I performance of anisotropic mask is enhanced compared to isotropic mask.

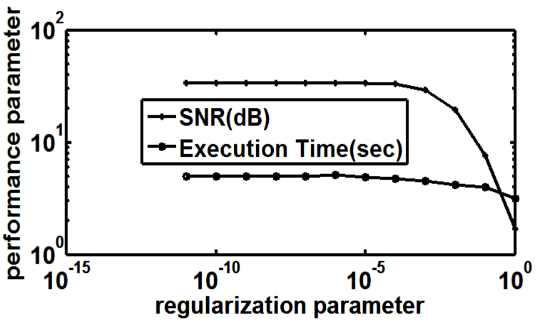


Fig: 4 Performance parameter variations with regularization parameter in RecPF

VI. CONCLUSION

Compressive sensing is a novel theory which can be used for rapid signal acquisition. Application of CS in MRI will mitigate major shortcoming of the technology. By exploiting the energy distribution of the MRI K space, the VDS mask can be designed to mitigate undersampling artifacts. VDS enables the application of CS into MRI. In this work some of the VDS masks used to demonstrate compressive sensed MRI and a comparative analysis is done.

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AUTHORS PROFILE

Kavitha S, received her B.E. degree in Electronics and Communication Engineering in 2008, she completed her M.Tech degree in Digital Electronics and Communication and pursuing her Ph.d from VTU Belagavi. Her area of interest are signal processing, image processing and communication.

Dr. B. Aziz Musthafa, received his B. E. degree in Electronics and Tele-communication Engineering from Gulbarga University in 1988. He received M.Tech degree in information technology Karnataka state university. He received his Ph.D. from the Mangalore University. He has 10 years of industrial experience and 15 years of teaching experience. He has published 10 research papers in international journals and conferences. His area of research include wireless networks, image processing, medical image processing. Presently he is working as a Professor in BIT Mangalore.