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MHD Flow of a Stratified Dusty Fluid in Porous Medium Past An Infinite Porous Vertical Plate

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Abstract: In this article, MHD flow of a stratified dusty viscous fluid through a porous medium past an infinite porous vertical plate with time dependent suction has been investigated assuming that the free stream oscillates about a constant mean and suction velocity as exponentially decreasing function of time. The liquid velocity, particle velocity and skin friction for liquid and dust particles have been obtained. The effect of magnetic field, Grashoff number, permeability and stratification factor on velocity profiles and skin friction have been discussed with the help of graphs and tables.

Keywords and phases: permeability, Grashoff number, skin fraction, Temperature.

I. INTRODUCTION

Nature has provided mechanism for bringing about changes in temperature that gives rise to variation in density. Quite a good meaning of meteorological phenomena are manifestations of stratified flow. In particular the hydromagnetic wave like instabilities in a rotating stratified fluid may be helpful to study how far such waves within the earth's liquid core may be responsible for the slow west ward drift with time of the geomagnetic field. In the presence of magnetic field, the stratified dusty flows are of great importance in fluidization (flow through packet beds), soil solvation by natural winds (Tso and Hung, 2009), dust entertainment in clouds during explosion, pulp and paper technology etc. The study of the effects of various physical variables on the free stream of stratified viscous flow has been by so many researchers. Lighthill (1954) studied the two dimensional time dependent flow problems dealing with the response of the boundary layer to external unsteady fluctuations of the free stream about a mean value. After that a large number of papers dealing with this subject have been brought out by many authors. Yih(1954) and Messiha(1966) have studied effect of density variation on fluid flow and laminar boundary layer in oscillating flow along an infinite flat plate with variable suction respectively.

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Channabasappa and Raganna(1976), Gupta and Sharma (1978), Govindraj and Kandaswami(1982),Gupta and Gupta (1986) have studied problems on stratified fluid flow under different geometrical situations and varied boundary conditions. Following Saffman (1962) and using Laplace transform technique following Sneddon(1979), Singh and Singh(1989) have studied unsteady flow of dusty viscous stratified fluid through porous medium in an open rectangular channel under variable electromagnetic field. Recently Kumar and Mohan (1991) have studied stratified fluctuating MHD flow through a porous medium past an infinite porous vertical plate with time dependent suction. Nandeppanavar and Siddalingappa (2013) studied the effect of viscous dissipation and thermal radiation on heat transfer over a non-linearly stretching sheet through porous medium.

In the present investigation flow of a dusty viscous, incompressible, electrically conducting stratified flow in a porous medium past an infinite vertical plate with time dependent suction in the presence of transversely applied magnetic field of very small magnetic Reynolds number has been studied. Exact solution for the liquid velocity, particle velocity and expressions for skin friction of liquid and dust particles have been obtained. To find the solution, method of permutation has been used. The effects of various parameters on velocity field and skin friction are discussed with the help of graphs and tables.

II. FORMULATION OF THE PROBLEM

Consider the flow of an incompressible, electrically conducting viscous liquid embedded with uniform, non-conducting, solid, spherical dust particles through a porous medium past an infinite porous vertical plate. Taking x-axis in the plane of the plate along the direction of the flow and y-axis perpendicular to the plate and passes through the x-axis. Let ρ_0 be the density and μ_0 be the viscosity of the liquid at the plate y = 0 and β is the stratification factor. A magnetic field B_0 (assumed to be an exponentially decreasing function of the distance from the plate) is applied perpendicular to the flow region and the suction velocity V is considered to the exponentially decreasing function of time. In the analysis all fluid properties are assumed to be constant except the influence of the density variation with temperature only in body force term. In the influence of density variation other terms of the momentum and variation of expansion coefficient with temperature are considered negligible. The free convection currents are in existence due to temperature difference ($T - T_\infty$), when T is the wall temperature and T_∞ is the free stream temperature.

It is further assumed that U_o is the mean free stream velocity which oscillates about a constant mean and n be the frequency of oscillation of the free stream and the magnetic Reynolds number is very small (<<1) so that the influence of induced magnetic field is not considered. The governing equations of the motion for the present configuration following Saffman(1962) and Gupta and Gupta(1982) are-

$$\rho\left[\frac{\partial u_1}{\partial t}+V^*\frac{\partial u_1}{\partial y}\right]= -\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u_1}{\partial y}\right)-\frac{\mu}{k}u_1-\sigma B_0^2 u_1 - \rho g_x - KN(v_1-u_1) \quad \text{....(2.1)}$$

$$\frac{\partial}{\partial y}(\rho V)=0 \quad \text{....(2.2)}$$

And $m\left(\frac{\partial v_1}{\partial t}\right)=(u_1-v_1)k$ ....(2.3)

Where $\rho = \rho_0 e^{-\beta y}$, $\mu = \mu_0 e^{-By}$, $B = B_0 e^{-\beta y}$, and $N = N_0 e^{-\beta y}$, For flow of free stream

$$\rho\frac{du_1}{dt}=-\frac{\partial p}{\partial x}-\frac{\mu}{k}u_1-\sigma B^2 U_1-\rho_\infty g_x + KN(V_1-U_1) \quad \text{...(2.4)}$$

And $m\left(\frac{\partial V}{\partial t}\right)=k(u_1-v_1)$ ...(2.5)

The equation (2.2) reveals that suction velocity is a function of time t only. Following Messiha (1966) we assume

$$V^* = -v(1+\epsilon\, e^{-nt}) \text{...(2.6)}$$

where v is the non-zero suction velocity.
From equation of state, we have

$$g_x(\rho_\infty-\rho) = g_x\beta^*\rho(T-T_\infty) \text{...(2.7)}$$

where T is wall temperature and T_∞ is the free stream temperature and k the permeability of medium, σ the electrical conductivity, B_0 be the transverse magnetic field(constant) and N_0 be the density of dust particle and m be the mass of dust particle, k be the stocks resistance.

Eliminating $\frac{\partial p}{\partial x}$ from equation (2.4) and (2.1) and using (2.6) and 2.7), we get –

$$\frac{\partial u_1}{\partial t}-v(1+\epsilon\, e^{-nt})\frac{\partial u_1}{\partial y}=\frac{du_1}{dt}-\nu_0\frac{\partial^2 u_1}{\partial y^2}+ g_x\beta^*\rho(T-T_\infty)+\frac{\nu_0}{k}(U_1-u_1) - \frac{\sigma B_0^2}{\rho}(U_1-u_1)-\frac{KN_0}{\rho}[(V_1-v_1)-(U_1-u_1)] \quad \text{...(2.8)}$$

We introduce the following non-dimensional variables-

$$u_1^* = u_1/U_0,\ U_1^* = U_1/U_0,\ v_1^* = v_1/U_0,\ V_1^* = V_1/U_0,\ y^* = y\,U_0/v_0,$$
$$t^* = t\,U_0^2/v_0,\ n^* = n\,v_0/U_0 \text{ and}$$
$$k^* = k\,U_0^2/v_0^2.$$

On substitute the above non-dimensional quantities in equations (2.8),(2.5) and (2.3) and dropping the asterisks over them, we obtain-

$$\frac{\partial u_1}{\partial t}-v(1+\epsilon\, e^{-nt})\frac{\partial u_1}{\partial y}=\frac{du_1}{dt}+\frac{\partial^2 u_1}{\partial y^2}-a\frac{\partial u_1}{\partial y}+G +\left(M^2+\frac{1}{K}\right)(U_1-u_1) - \frac{l}{b}[(V_1-v_1)-(U_1-u_1)] \quad \text{.....(2.9)}$$

$$b\frac{\partial V_1}{\partial t}=U_1-V_1 \quad \text{...(2.10)}$$

and

$$b\frac{\partial v_1}{\partial t}=u_1-v_1 \quad \text{...(2.11)}$$

where $a=\frac{v_0\beta}{\mu_0}$ (Stratification factor)

$G=\frac{v_0 g_x\beta^*(T-T_\infty)}{U_0 u_0^2}$ (Grashoff number)

$M=\frac{B_0}{v_0}\sqrt{\frac{\sigma v_0}{\rho_0}}$ (Hartmann number)

$l=\frac{N_0 m}{\rho}$ (Mass Concentration of dust particles)

and b = $\frac{m\,u_0^2}{v_0 k}$ (relaxation time parameter of dust particles

The non-dimensional boundary condition are-

$$\left.\begin{array}{l} t>0;\ u_1=v_1=0\ \ at\ y=0 \\ u_1=v_1=1+\epsilon\, e^{-nt}\ \ as\ \ y\to\infty \end{array}\right] \quad \text{... (2.12)}$$

III. SOLUTION OF THE PROBLEM

In the neighbourhood of the plate following Kumar and Mohan (1991) we assume:-

$$\left.\begin{array}{l} u_1(y,t) = f_1(y)+\epsilon\, e^{-nt}f_2(y) \\ v_1(y,t) = g_1(y)+\epsilon\, e^{-nt}g_2(y) \end{array}\right] \quad \text{... (3.1)}$$

and

$$\left.\begin{array}{l} u_1(t) = 1+\epsilon\, e^{-nt}C_1 \\ v_1(t) = 1+\epsilon\, e^{-nt}C_2 \end{array}\right] \quad \text{... (3.2)}$$

Where C_1 and C_2 are real constants which are to be determined later on.

Substituting (3.1) and (3.2) in the equation (2.10) and (2.11) and comparing the similar terms, we get-

$$f_1(y)=g_1(y) \quad \text{... (3.3)}$$

$$f_2(y)=(1-nb)g_2(y) \quad \text{... (3.4)}$$

and

$$C_2 = \frac{C_1}{(1-nb)} \quad \ldots (3.5)$$

Using (3.1) to (3.5) in the equation (2.9), we have

$$f_1'' + (v-a)f_1' - \left(M^2 + \frac{1}{k}\right) f_1 = -G - \left(M^2 + \frac{1}{k}\right) \quad \ldots (3.6)$$

$$f_2'' + (v-a)f_2' - \left(M^2 + \frac{1}{k}\right) - n - \frac{ln}{(1-bn)} f_1 = -V_1 f_1'(y) - \left(M^2 + \frac{1}{k} - n - \frac{l\,n}{(1-bn)}\right) C_1 \quad \ldots (3.7)$$

The boundary conditions are transformed to –

$$\left.\begin{array}{l} f_1 = f_2 = g_1 = g_2 = 0 \quad at\ y = 0 \\ f_1 = f_2 = g_1 = g_2 = 1 \quad as\ y \to \infty \end{array}\right\} \ldots (3.8)$$

On solving the equations (3.6) and (3.7) under the boundary conditions (3.8) we obtain $f_1(y)$
And $f_2(y)$ using these in (3.3) and (3.4) we get-

$$u_1(y,t) = a_0 - a_0 e^{-H_2 y} + \in [1 - a_1 e^{-H_2 y} + (a_1 - 1)e^{-H_4 y}]e^{-nt} \quad \ldots (3.9)$$

$$v_1(y,t) = a_0 - a_0 e^{-H_2 y} + \in [1 - a_2 e^{-H_2 y} + (a_2 - 1)e^{-H_4 y}]e^{-nt} \quad \ldots (3.10)$$

where $C_1 = 1$, $C_2 = \frac{1}{(1-bn)}$, $a_0 = \frac{G}{M^2 + \frac{1}{k}}$

$$a_1 = \frac{a_0 H_2}{H_2^2 - (v-a)H_2 - \left(M^2 + \frac{1}{k}\right) - n - \frac{lm}{(1-bn)}}$$

$$a_2 = \frac{a_1}{(1-bn)}$$

$$H_2 = \frac{1}{2}\left[(V-a) + \left[(V-a)^2 + 4\left(M^2 + \frac{1}{k}\right)\right]^{1/2}\right]$$

$$H_4 = \frac{1}{2}\left[(V-a) + \left[(V-a)^2 + 4\left(M^2 + \frac{1}{k} - n - \frac{l\,n}{(1-b\,n)}\right)\right]^{1/2}\right]$$

Table I

Effect of magnetic field on skin friction

(l = 0.3, b = 0.2, n = 1.0, ∈ = 0.05, G = 1.0, v = 0.5, k = 0.25 and a = 0.05)

T	M = 1.0		M=2.0	
	τ_1	τ_2	τ_1	τ_2
1.0	2.5139	2.5145	2.7272	2.7268
2.0	2.4876	2.4878	2.7004	2.7003
3.0	2.4779	2.4781	2.6905	2.6905
4.0	2.4744	2.4745	2.6869	2.6868
5.0	2.4731	2.4732	2.6856	2.6856

Table II

Effect of permeability on skin friction

(l = 0.3, b = 0.2, n = 1.0, ∈ = 0.05, M = 1.0, G = 1.0, v = 0.5, t= 1.0)

A	K = .20		K=0.25	
	τ_1	τ_2	τ_1	τ_2
.05	2.7305	2.7309	2.5139	2.5145
.10	2.7023	2.7033	2.4859	2.4864
.15	2.6754	2.6758	2.4592	2.4599
.20	2.6482	2.6486	2.4311	2.4316
.25	2.6213	2.6217	2.4040	2.4046

Table III

Effect of Grashoff number on skin friction

(l = 0.3, b = 0.2, n=1.0, ∈=0.05, M = 1.0, v = 0.5, a = .05, k=.25)

A	G = 0.0		G = 1.0	
	τ_1	τ_2	τ_1	τ_2
1.0	2.6957	2.6957	2.5139	2.5144
2.0	2.4868	2.4868	2.4876	2.4878
3.0	2.4777	2.4777	2.4779	2.4780
4.0	2.4743	2.4743	2.4744	2.4744
5.0	2.4724	2.4724	2.4731	2.4731

IV. NON-DIMENSIONAL SKIN-FRICTION

Skin- friction for liquid

$$\tau_1 = \left(\frac{\partial u_1}{\partial y}\right) y = 0$$

$$= H_2 + \in [a_1 H_2 - (a_1 - 1)H_4]e^{-nt} \ldots (4.1)$$

Skin- friction for dust particles $\tau_2 = \left(\frac{\partial v_1}{\partial y}\right) y = 0$

$$= H_2 + \in [a_2 H_2 - (a_2 - 1)H_4]e^{-nt} \quad \ldots (4.2)$$

V. PARTICULAR CASES

Case I:

When Grashoff number i.e. G = 0, the fluid velocity is equal to the particle velocity and skin friction for liquid is equal to skin friction for dust particle.

Case II:

When the fluid is non-dusty i.e. K = 0 then the results are exactly same as obtained by Kumar and Mohan (1991).

Case III:

When the fluid is non-dusty i.e. k = 0 and suction is independent of time, the results are similar to Gupta and Gupta (1982).

VI. RESULT AND DISCUSION

Table I indicates the effect of magnetic field on skin-friction for liquid and dust particles at (l=0.3, b = 0.2, n = 1.0, ∈ = 0.05, G = 1.0, v = 0.5, k = 0.25 and a = 0.05). It is clear from the table that when the intensity of magnetic field increase as the skin- friction increase for liquid and dust particles both for all values of time. Besides, when the time increase the skin-friction decreases (in both cases) for given magnitude of magnetic intensity.

Table II shows the effect of permeability on skin-friction for liquid and dust particles at (l=0.3, b = 0.2, n = 1.0, ∈ = 0.05, G = 1.0, v = 0.5, M = 1.0 and t = 1.0). It is obvious that for all values of stratification factor á' the skin-friction for liquid and dust particles decreases. It is also obvious that for all values of á' the skin-friction decrease for liquid and particles velocity for each given value of permeability parameter.

Table III reveals the effect of Grashoff number on the skin-friction for the liquid and dust particles at (l=0.3, b = 0.2, n = 1.0, ∈ = 0.05, M = 1.0, v = 0.5, k = 0.25 and a = 0.05).It is clear from the table that skin-friction for liquid and dust particles decreases for all values of time. Besides, skin-friction for given Grashoff number decreases (in both cases) when 't' increases. It is interesting to note that when Grashoff number is zero, the Skin-friction for liquid and dust particles is equal for all values of time.

Velocity profile for liquid and dust particles have been calculated for the values l = 0.3, b = 0.2, t = 1.0, ∈ = 0.05, v = 0.5 and different values of G, M, k and a.

	G	M	k	a
I	1.0	1.0	0.25	0.05
II	1.0	3.0	0.25	0.05
III	0.0	1.0	0.25	0.05
IV	1.0	1.0	0.20	0.05
V	1.0	1.0	0.25	0.15

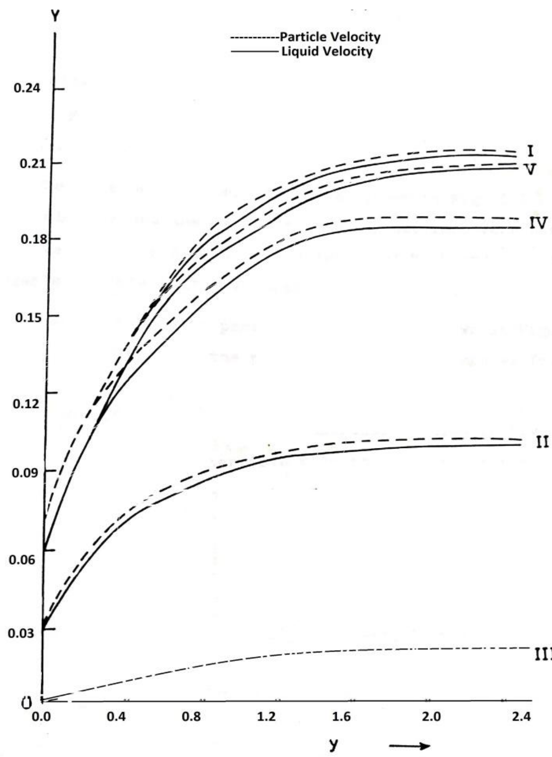

Fig.1:Velocity profile for liquid and dust particles

From figure1, It is obvious that:

(i) When the magnetic field increases (shown in fig I &II) the velocity of liquid and dust particles decreases for all values of y while on increasing 'y' the velocity of both increases.

(ii) When Grashoff number decreases (shown in fig. I& III) the liquid velocity and the particle velocity declines very sharply for all values of y and for given values of M, k and a. It is remarkable to note that when Grashoff number is zero.

(iii) When permeability parameter decreases (shown in fig. I &IV) the liquid velocity and the particle velocity decreases for all values of 'y' and for given values of M, G and a.

(iv) When stratification factor increases (shown in fig. I & V) the liquid velocity and particle velocity decline very slowly for all values of y and values of M, G and k.

(v) When y increases (shown in fig. I & V) the velocity of liquid and dust particles increases for given values of the parameters.

(vi) The particle velocity is always more than liquid velocity for all values of y and for all given values of the parameters except the case when Grashoff number is zero.

VII. CONCLUSION

From Table I, it is observed that when the intensity of magnetic field increases, the skin- friction increases for both liquid and dust particles for all values of time. Additionally, when the time increases, the skin-friction decreases for both liquid and dust for given magnitude of magnetic intensity. From Table II, it is observed that for all values of stratification factor á', the skin-friction for liquid and dust particles decreases. From Table III, it is observed that skin-friction for liquid and dust particles decreases for all values of time. In addition, skin-friction for given Grashoff number decreases for both liquid and dust when 't' increases.

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