

Mathematical Analysis and Simulation of Ahrens Model in Pulse Combustor

Suresh Sorathiya, M.B. Prajapati

Abstract: Ahrens model accepted large data of Kilicarslan but with some real differences. In Kilicarslan model, released heat in combustion having more P^2R that is released under Ahrens. As per Ahrens if any specific object divides reaction and cool zone then heat release / cycle is very less compare with Kilicarslan. But with one condition of one object that is, it should be pressure independent otherwise density of reactants remains constant (or not enough increased); as a result, more heat released with less satisfied output. Mathematical model of Ahrens brief about object zones and its separation. This paper also brief about dynamic work function of tailpipe with adjustment of frequency which will proceed for system stability and its accuracy.

Keywords: Burning Mass Equation, Frequency Analysis, Roll of Oscillation, Stability, Modified Ahrens Equation

I. INTRODUCTION

Ahrens model is Kilicarslan's modified model. Assuming the separation of two zones is possible by introducing thin flame sheet between them. Now process of heat spanning combustion chamber cross section with two zones, where pressure is independent and criteria of Rayleigh satisfied [1].

Above mathematical process indicates 2nd order differential equation, similar given by Kilicarslan but not constant damping. So as per Ahrens released heat is mathematically damped oscillation graph. Because at t_0 time reactants having low pressure and at 2nd piece of cycle it increases (at 3rd it stabilized again and last cycle again pressure is low). Ahrens used pulse combustor for stabilization with some parameters having fixed flame speed.

II. MATHEMATICS OF MODEL

A. Stabilized Energy

In Kilicarslan model equations gases are in perfect and ideal condition and ratio of heats are constant at the end of 4th cycle in reactants. So total Constant combustion energy E_{cc} is uniform in two separated zones. For separating two zones five major calculations need to be focused: 1. Energy Stability, 2. Rate of change of burning Mass, 3. Reaction of combustion chamber, 4. Reaction of change in mass and 5.

Area covered before and after burning process. That is why, Defined equation which is same as Kilicarslan [3].

$$\frac{d}{dt}(E_{cc}) = \dot{Q} + h_r \dot{m}_r - h_e \dot{m}_e$$

Where, E_{cc} =Total Energy output, Q = Rate of change of released heat, m_r and m_e are mass flux in out in combustion chamber respectively. H_r and h_e are amount of enthalpy of reactance and products, while heat being released.

Total energy equation while combustion chamber is in process of releasing heat.

$$E_{cc} = \frac{PV_{cc}}{\gamma - 1}$$

Where, $\Gamma = C_p/C_v$ is heat constant. V_{cc} =Total Volume while heat release. And P is constant pressure.

Now rate of heat release in combustion chamber is:

$$Q = m_b * (\Delta H_f / (1 + r)) = m_b (h_e - h_r)$$

Where, m_b =Burning amount of mass of reactant, ΔH_f heat of combustion per mass of fuel, r is amount of air used in fuel burning process.

Note: Kilicarslan assumes that the quantity of rate of change of burning and releasing mass are equal. Where as Ahrens says they are different [1].

B. Rate of Change of Burning Mass

Burning rate indicates how much amount of heat released. Also specify the reaction of mixture and content, amount for cycle. Burning amount is cause of separating two zones where reactants having unburned material and products has burnt gases. If burning speed (product material) per cycle in cross section is depends on density of reactant, cross section area and its relative speed then total rate of change of mass burning is:

$$m_b = \rho_r * A_b * U_f$$

Where, m_b = Burning amount of mass of reactant, ρ_r (=Density, A_b =Cross Section Area for burning process and U_f =Flame speed. If T_0 = is initial time where pressure is initiated which is constant in entire process then burning equation is defined by [5]:

$$m_b = (P/RT_0) * A_b * U_f$$

R =Specific Gas Constant

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C. Process of Combustion Chamber

If related reactive mass is present (Unburn material after burning process) then it is difference of rate of change of reactant mass and burning mass denoted by:

$$\frac{dM_r}{dt} = \dot{m}_r - \dot{m}_b + I^2 R \dots \text{(Author Equation)}$$

Ideal situation R.H.S. is zero in every cycle.

$$\int_{Cycle}^x (\dot{m}_r - \dot{m}_b) \sim 0$$

- It near to zero cause of heat loss.

D. Rate of Change of Mass Flow

Above derivation and previous paper of Kilicarslan [3] describe how and what amount of change is possible while burning process.

Based on that mass change in reactant is:

$$\dot{m}_r(t) = k_g * (1 + r) * \sqrt{|P|} 1_{(-x,0)}(p)$$

K_g = Constant flow parameter, $1_x(x)$ =indicator function, x is element for negative reactant in $X \subset R$ and Zero otherwise.

The value of K_g is:

$$k_g = \sqrt{2 * p_g} * C_{D,g} * A_{v,g}$$

- Where, P_g is density of gas, C_{dg} = Discharge Coefficient of gas valve, A_{vg} = Effective flow area of gas pipe and r =gas to fuel Ratio.

E. Total Density of Release in Cross Section Area

Let suppose, L_{tp} =Tail Pipe Length, A_{tp} =Tail Pipe Cross Section Area, m_e =Leaving Mass after burning process per cycle.

$$L_{tp} * \left(\frac{d * (m_e)}{dt} \right) = \rho A_{tp}$$

III. FREQUENCY EQUATION AND ANALYSIS

After covering above process, equation leads to linear (In ideal condition it is linear: Zero R.H.S.) per cycle but damping reactance is not linear.

The equation can be written as:

$$\ddot{x} + h(x, \dot{x}) + (w_0)^2 x \sim 0$$

Where, $x=p(t)$ and dot represent time derivation. The nonlinear damping term is $h(x, \dot{x})$

$$h(x, \dot{x}) = -(A * F'(x) + B) \dot{x}$$

Where $F'(x)$ is:

$$F'(x) = -\frac{1}{2 * (\text{Root}|P|)} * (1_{(-x,0)}(X))$$

In $F'(x)$ having negative sign (Due to realising heat) ≤ 0 , that is why damping is non-linear. And final equation of non-linear reactant frequency is:

$$f_0 = \frac{(\gamma - 1) * h_e}{2 * P_i} * \left(\sqrt{\frac{A_{tp}}{V_{cc} * L_{tp}}} \right)$$

IV. OSCILLATION STABILITY

Finding stable oscillation, need to defined the value of maximum amplitude value of the oscillation. For satisfying stability criteria what is the value of realising heat in stable oscillation and unstable maximum value needed.

$$\int_{Cycle}^x (\dot{m}_r - \dot{m}_b) \sim 0$$

Based on above equation,

The maximum value of unstable oscillation is:

$$P_{max} = \left(\frac{2I}{\pi} \right)^{-1} * ((P_0 * (1 + \gamma) * h_r) / (\Delta H_f))$$

And the value of stable oscillation is:

$$U_f = \left(\frac{I}{2 * \pi} \right)^{0.5} * \left(\frac{RT_0 * (1 + \gamma)^{3/2}}{A_b * (P_0)^{0.5}} \right) * \sqrt{(h_r) / (\Delta H_f)}$$

Where I is integral Factor that describe the Shape of curve.

$$I = \left(\frac{1}{2 * T_p} \right) * \int_{Cycle}^x \sqrt{|p(\hat{t})|} * 1_{(-x, 0)} * (p(\hat{t})) dt$$

V. RESULT AND SIMULATION

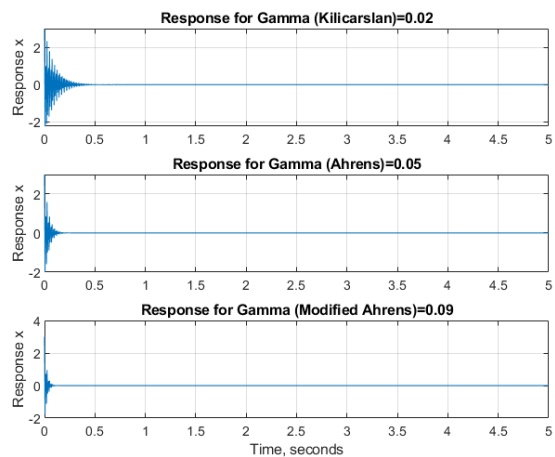


Figure 1. Comparison of previous v/s modified models

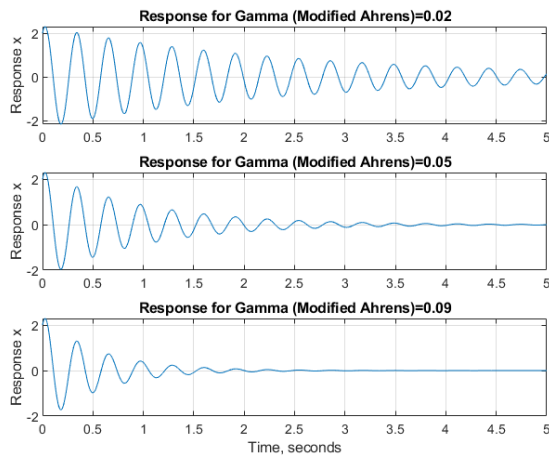


Figure 2. Adjustment of less difference heat constant (Gamma tens to zero)

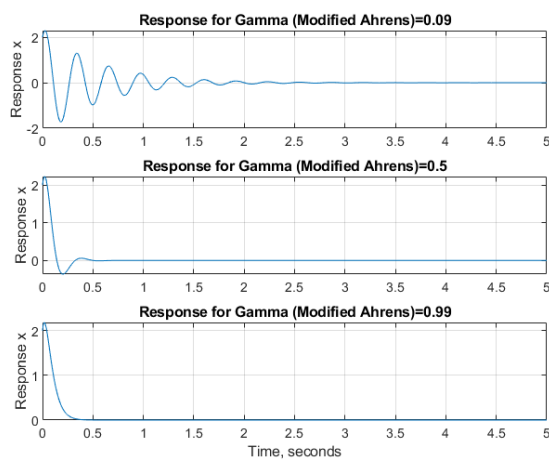
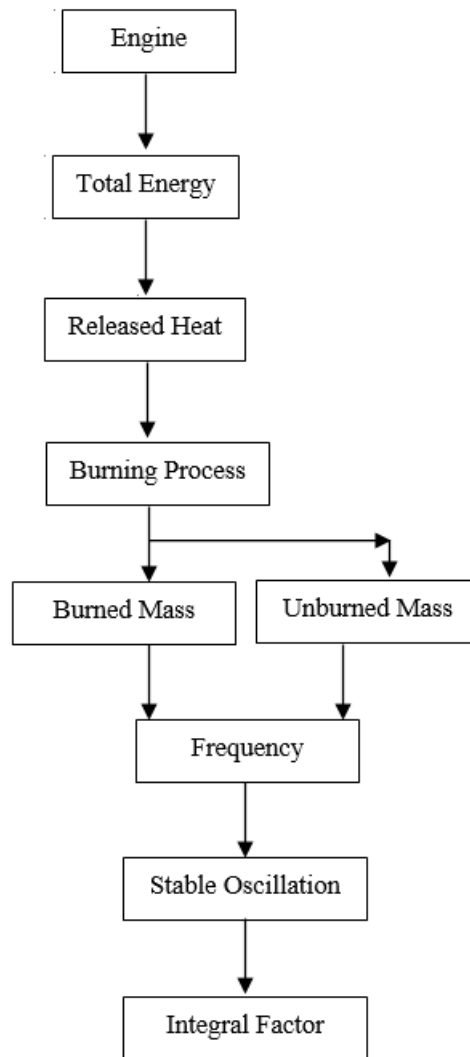


Figure 3. Adjustment of large difference heat constant (Gamma zero to One)

TABLE 1: COMPARISON

Parameter	Kilicarslan	Ahrens	Modified Ahrens
Total Energy	Low	Moderate	High
Heat	High	Moderate	Low
Burning Mass	Minimum	Moderate	Maximum
Unburned Material	More	Moderate	Less
Oscillation	Unstable	Marginal Stable	Stable
Curve (I)	Sharp	Conditional Smooth	Smooth

VI. FLOW CHART



VII. CONCLUSION

Based on above mathematical equation and stabilized oscillation condition, We conclude that little modification in Ahrens model in analysis will give smoothest output compare to Kilicarslan and Ahrens while releasing heat in pulse combustor [Reference figure 3]

Pulse combustor age and average is increased by this process and it can chase 10 times higher frequency compare with initial conditions. Flame speed is also potential as per user requirement.

Figure 2 is simulated result of High constant pressure in Low volume. In this condition if constant pressure is very high for even a second that will create large oscillation which will damage the system and accordingly.

Figure 3 is simulated result of variation of pressure vs volume. When both are same it always in moderate condition due to its inverse proportionality. Means when this ratio tends to one, it creates nearly zero oscillation. Which will be required for any system.

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