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Thermal Effects on Magneto Hydrodynamic Casson Liquid Stream Between Electrically Conducting Plates

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***Abstract:** This paper is intended for studying the thermal effects of transient magneto hydrodynamic Casson liquid stream with presence of heat transfer through inclined parallel plates. The examination reveals various vital parts of flow and heat transfer. The partial differential equation which governs the conditions of the motion of the moving body is changed to standard differential conditions. The flow emphasizes and heat transfer attributes for different inferences of the representing parameters viz. the parameters Casson, heat source, Hartmann and Prandtl's numbers are explored. It was revealed that heat source and magnetic field alters the flow prototype and increments the fluid temperature.*

***Keywords:** Parallel Plate Channel, Boundary Layer, Heat Source, Magnetic Field, Casson liquid.*

I. INTRODUCTION

The investigation of Casson flow and heat transfer in a viscous fluid is of exciting quality as a consequence of their constantly increasing current applications and essential title on a few mechanical events. In the midst of the most recent decades, wide research work has been done on the fluid dynamics of biological fluids in the presence of a magnetic field. For various reasons, the employment of MHD in physiological stream problems is of developing interest. The flow because of extending a flat plane was initially explored by Crane [1]. Kim [2] observed the transient MHD convective temperature past a semi-infinite perpendicular permeable moving surface with variable suction. Andersson [3] studied the MHD flow of sticky liquid on an extending sheet. Under oscillatory suction velocity past a vertical plate, Singh and Singh [4] have investigated the heat and mass transfer in MHD flow of a sticky liquid.

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Adhikary and Misra [5] revealed a precise clarification of the problem of oscillatory flow of a fluid and heat transfer along a porous oscillating channel in the presence of an exterior magnetic field. Some recent studies in this direction may be mentioned in [6-10]. Sanjayanand and Khan [11] considered the sticky–stretchy boundary layer flow and heat transfer owing to an exponentially enlarging sheet [6]. C. K. Kirubhashankar et al [12] have examined Casson fluid flow and heat transfer over an unsteady porous stretching surface. Emmanuel Maurice Arthur et al [13] have investigated the Casson fluid flow over a vertical permeable plane with chemical reaction in the presence of a magnetic field. Pramanik, S [14] considered the Casson fluid flow and heat transfer past an exponentially permeable enlarging surface in the presence of thermal emission. Veeresha et al [15] have scrutinized the Joul heating and thermal dispersion impact on MHD radiative and convective Casson liquid flow past an oscillating semi-infinite vertical porous plate. Heat and mass transfer in transient MHD Casson liquid flow with convective boundary conditions was analyzed by K. Pushpalatha et al [16].

II. MATHEMATICAL ANALYSIS OF THE FLOW

Considering the flow of transient incompressible MHD sticky liquid, making the fluid is electrically transmitted through the plate at an inclined angle φ from the vertical.The magnetic field of strength B0 is uniformly applied in the direction transverse to the liquid flow.

We write the equations of the two-dimensional boundary layer in front of the transverse approach to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial x}=\nu\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho}+g\beta(T-T_0)\cos\varphi \tag{2}$$

$$\frac{\partial T}{\partial t}=\frac{K'}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho C_p}(T-T_0) \tag{3}$$

where ν is the kinematic fluid viscosity, ρ is the fluid density, $\gamma = \mu_B\sqrt{2\pi c}/p_y$ is the Casson parameter, σ is the electrical conductivity of the fluid, and H_0 is the strength of magnetic field applied in the y-direction. ρ and μ is the density and viscosity of the blood while p stands for pressure. K’ is thermal conductivity; C_P is the specific heat at constant pressure.

Q is the quantity of heat, T is the temperature and β is the volumetric expansion parameter while θ is the temperature distribution.

The boundary conditions are taken as:

$\theta = e^{-\lambda^2 t}$, $u = e^{-\lambda^2 t}$ at $y = -1$

$\theta = 0$, $u = 0$ at $y = 1$

Let us introduce the non-dimensional variables,

$$x^* = \frac{x}{h},\ y^* = \frac{y}{h},\ u^* = \frac{u}{m/2\rho h},$$

$$v^* = \frac{v}{m/2\rho h}\ t^* = \frac{t}{\rho h^2/\mu},$$

$$p^*(x,t) = \frac{dp/dx}{\mu m/2\rho^2 h^3},\ \theta^* = \frac{\theta}{\mu m/2\rho^2 h^3} \tag{4}$$

Substituting equation (4) into equations (1) – (3), we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

$$\frac{\partial u}{\partial t} + p = \left(1 + \frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2} - Ha^2 u + g\beta\theta\cos\varphi \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\upsilon P_r}\frac{\partial^2 \theta}{\partial y^2} + \frac{S}{\upsilon P_r}\theta \tag{7}$$

where the heat source parameter, $S = \frac{Qh^2}{K_T}$, Prandtl number, $P_r = \frac{\mu C_p}{K_T}$

III. ANALYTICAL SOLUTION OF THE PROBLEM

In view of the discussions in the previous section, let us choose the solutions of the equations (5) – (7) respectively as

$$u(y,t) = F(y)e^{-\lambda^2 t} \tag{8}$$

$$v(y,t) = G(y)e^{-\lambda^2 t} \tag{9}$$

$$\theta(y,t) = H(y)e^{-\lambda^2 t} \tag{10}$$

The transformed boundary conditions are

$H = 1$, $F = 1$ at $y = -1$

$H = 0$, $F = 0$ at $y = 1$ (11)

With the help of (8) – (10), we obtain the equations (5) – (7) respectively as

$$F''(y) + \frac{\gamma}{1+\gamma}\left(\lambda^2 - Ha^2\right)F(y) = \frac{\gamma}{1+\gamma}\left(p - g\beta H(y)\cos\varphi\right) \tag{12}$$

$$G = C \tag{13}$$

$$H''(y) + \left(S + \lambda^2 P_r \upsilon\right)H(y) = 0 \tag{14}$$

Using the boundary condition (11) the Solution of equation (14) is as follows

$$H(y) = \frac{1}{2\cos\delta}\cos(\delta y) - \frac{1}{2\sin\delta}\sin(\delta y) \tag{15}$$

where $\delta = \sqrt{S + \lambda^2 P_r \upsilon}$

The temperature distribution from equations (10) and (15) is given by

$$\theta(y,t) = \left(\frac{1}{2\cos\delta}\cos(\delta y) - \frac{1}{2\sin\delta}\sin(\delta y)\right)e^{-\lambda^2 t} \tag{16}$$

The equation (16) is used into equation (12), we get

$$F''(y) + \frac{\gamma}{1+\gamma}\left(\lambda^2 - Ha^2\right)F(y) = \frac{\gamma}{1+\gamma}\left(p - g\beta\left(\frac{1}{2\cos\delta}\cos(\delta y) - \frac{1}{2\sin\delta}\sin(\delta y)\cos\varphi\right)\right) \tag{17}$$

The velocity of the flow of the fluid parallel to the direction of the channel is obtained from equation (8) and (17) as,

$$u(y,t) = \left(c_1 + c_2\cos\delta y + c_3\sin\delta y + c_4\cos\alpha y + c_5\sin\alpha y\right)e^{-\lambda^2 t} \tag{18}$$

where $p_1 = \frac{p^*}{e^{-\lambda^2 t}}$, $\alpha = \sqrt{\frac{\gamma}{1+\gamma}\left(\lambda^2 - Ha^2\right)}$, $c_1 = \frac{p_1}{\alpha^2}$,

$c_2 = -\frac{g\beta\cos\varphi}{2\left(\alpha^2 - \delta^2\right)\cos\delta}$, $c_3 = \frac{g\beta\cos\varphi}{2\left(\alpha^2 - \delta^2\right)\sin\delta}$

$c_4 = \frac{1 - 2c_1 + 2c_2\cos\delta}{2\cos\alpha}$, $c_5 = -\frac{1 + 2c_3\sin\delta}{2\sin\alpha}$

From equations (9) and (13), The velocity of the fluid flow perpendicular to the direction of the channel is given by

$$v(y,t) = Ce^{-\lambda^2 t} \tag{19}$$

where C is an arbitrary constant.

Equations (16), (18) and (19) show the temperature distribution, the axial velocity, and normal velocity respectively.

IV. RESULTS AND DISCUSSIONS

Dimensionless systems of linear equations are solved analytically with the boundary conditions, by using the similarity Transform technique. The attained results show the effect of the different dimensionless governing parameters, such as the Casson parameter (β), Magnetic parameter (M), Prandtl number (Pr), inclined angle (φ) and decay parameter (λ) on temperature distribution in the flow of velocity. The attained computational outcomes are illustrated graphically and the difference in velocity and temperature are examined.

A. Effects of different physical parameters on temperature fields

The performance of temperature distributions against y at $\lambda = 0.5$, $P_r = 1$, $\upsilon = 0.5$, and $t = 1$ for a choice of values of heat source parameter (S = 1, 1.75, 2.5, 3.25, 4) is shown in Figure 1. It is studied for Figure 1 that the temperature field reduces with raising the values of S, for $y \le 0.5$, and temperature field raises for $y \ge 0.5$. The utmost effect of heat source is at $y = -1$.

The temperature field distribution for various values of Prandtl number (Pr = 1, 3, 5, 7, 9) at S = 1, λ = 0.5, υ = 0.2, and t = 1 are highlighted in Figure 2. The temperature effect on the Prandtl number steadily reduces with raising the values of Prandtl number.

It is evident from Figure 3 that temperature field distribution reduces with rising the decay parameter up to y ≤ 0.16 and it raises with rising the decay parameter for y ≥ 0.16, at S = 1, Pr = 1, υ = 0.2, and t = 1 for various values of decay parameter (λ = 0.5, 0.75, 1, 1.25, 1.5).

The utmost effect of the decay parameter on the temperature field is between -0.8 ≤ y ≤ -0.4.

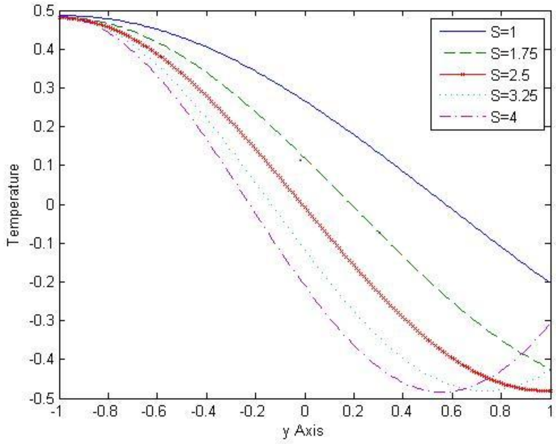

Figure 1. Temperature Field for different values of Heat Source Parameter (*S*)

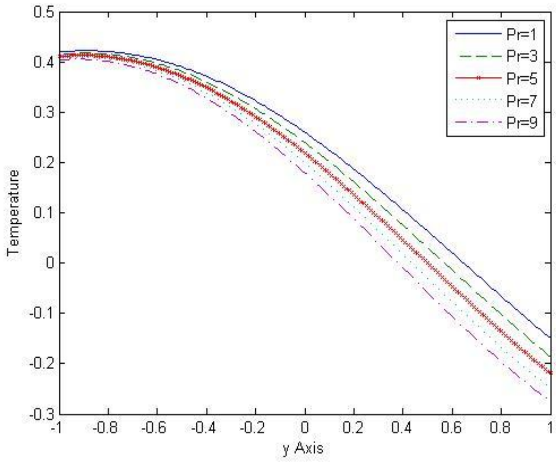

Figure 2. Temperature Field for different values of Prandtl Number (Pr)

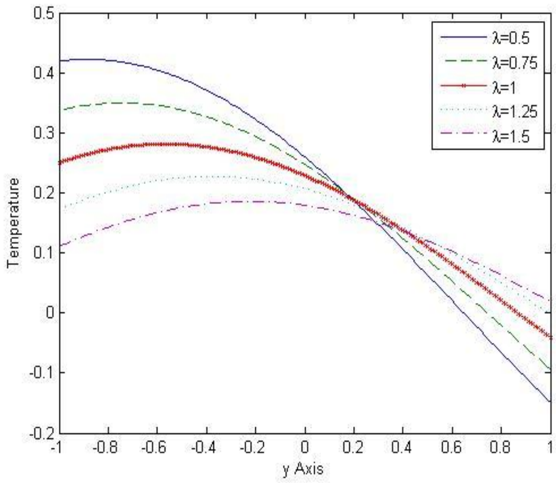

Figure 3. Temperature Field for different values of Prandtl Number (Pr)

B. Effects of different physical parameters on velocity fields

The influence of the axial velocity profiles for several values of heat source parameter (S = 1, 1.75, 2.5, 3.25, 4) at λ = 1.5, Pr = 1, υ = 0.2, t = 1, γ = 0.2, g = 9.8, β = 0.5, p = 0.5, φ = π/4 and Ha = 1 are shown in Figure 4. It is found that axial velocity rises with rising the heart source parameter S for y ≤ -0.3 and the effect reverses for -0.3 ≤ y ≤ 1.

Figure 5 depicts the effect of magnetic field on the axial velocity for various values of Hartmann number (Ha = 1, 1.25, 1.5, 1.75, 2) at S = 1, λ = 0.5, Pr = 1, υ = 0.2, t = 1, γ = 0.2, g = 9.8, β = 0.5, φ = π/4 and p = 0.5. It is revealed that the axial velocity reduces with raising the magnetic field up to y ≤ -0.3 and for y ≥ -0.3 axial velocity raises with raising the magnetic field.

The effect of Prandtl number on the distribution of the axial velocity at S = 1, λ = 0.5, Pr = 1, υ = 0.2, t = 1, γ = 1, g = 9.8, β = 0.5, p = 0.5, φ = π/4 and Ha = 1 is revealed in Figure 6. It is observed that axial velocity reduces with raising the Prandtl number.

Figure 7 exhibits the effect of decay parameter on the axial velocity for various decay parameter values (λ = 0.5, 0.75, 1, 1.25, 1.5) at S = 1, λ = 0.5, Pr = 1, υ = 0.2, t = 1, γ = 1, g = 9.8, β = 0.5, Ha = 2, φ = π/4 and p = 0.5. It is clear that the axial velocity rises with raising the decay parameter up to y ≤ -0.3 and for y ≥ -0.3 axial velocity reduces with raising the decay parameter.

Effects of Casson parameter γ on velocity profiles for transient motion are clearly demonstrated in Figure 8 the performance of velocity with rising γ is noted at S = 1, λ = 0.5, Pr = 1, υ = 0.2, t = 1, γ = 0.2, g = 9.8, β = 0.5, φ = π/4 and p = 0.5. It is clearly seen that the axial velocity reduces with raising the magnetic field up to y ≤ -0.2 and for y ≥ -0.2 axial velocity raises with raising the magnetic field.

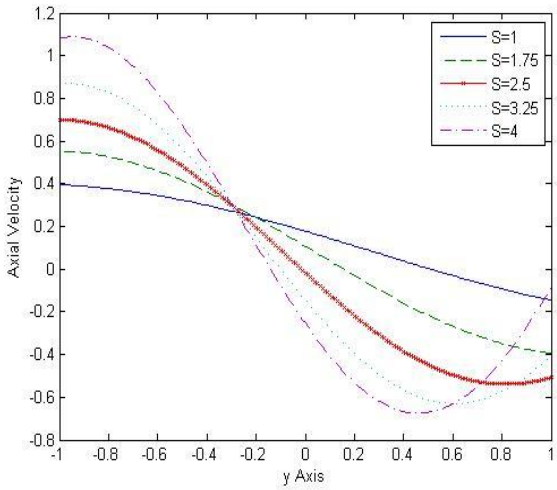

Figure 4. Axial velocity for different values of Heat Source Parameter (S)

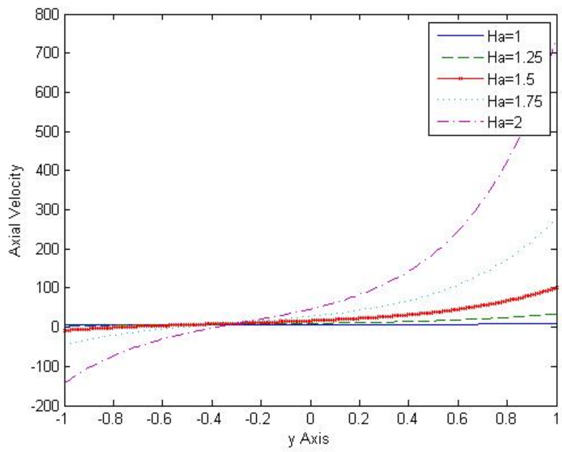

Figure 5. Axial velocity for different values of Magnetic Field Parameter (Ha)

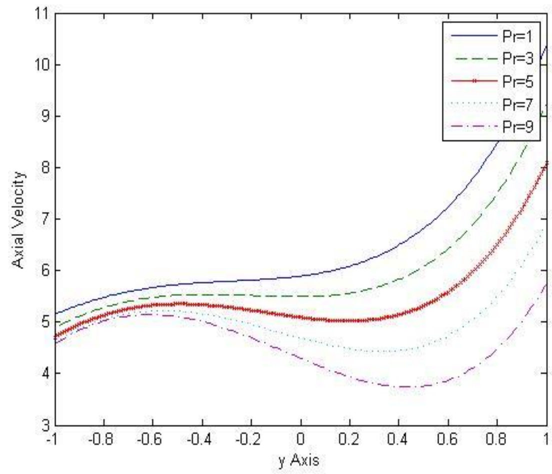

Figure 6. Axial velocity for different values of Prandtl Numer (Pr)

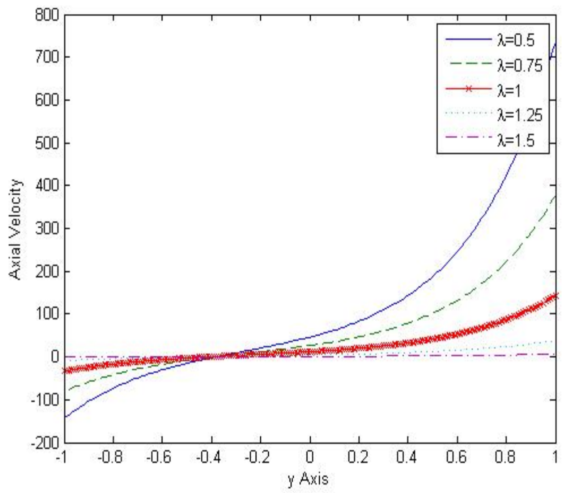

Figure 7. Axial velocity for different values of Decay Parameter (S).

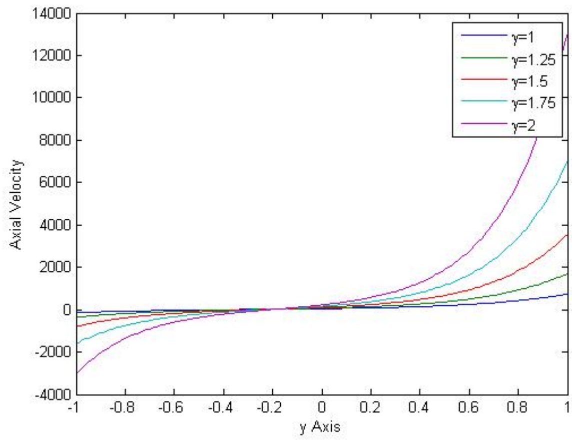

Figure 8. Axial velocity for different values of Casson Parameter (γ).

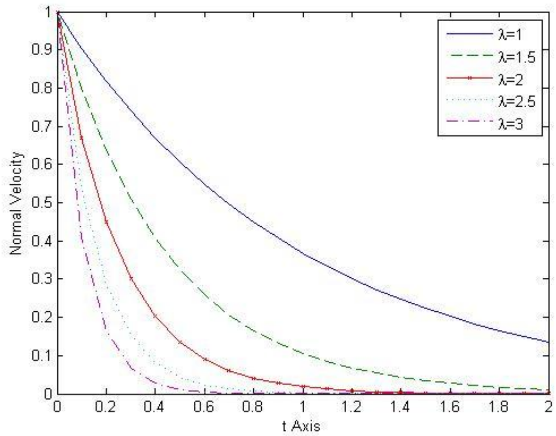

Figure 9. Normal velocity for different values of Decay Parameter (λ).

The normal velocity for various values of the decay parameter is shown in Figure 9. It is represented that normal velocity is reducing with rising values of λ and also for rising values of t. The normal velocity is reducing slowly at low values of the decay parameter ($\lambda = 1$) while it reduces quickly and tends to zero at high values of decay parameter ($\lambda = 3$).

V. CONCLUSION

The present study provides the solution for the Thermal effects on transient Magneto hydrodynamic Casson Fluid flow past moving inclined parallel plates. The model gives a plain form of axial velocity, temperature distribution and normal velocity of the flow. Analytical terminologies are obtained by deciding the axial velocity; temperature distribution and the normal velocity of the flow depend on y and t only. The temperature field reduces with raising the heat source parameter (S), Prandtl number(Pr) and the decay parameter(λ). And temperature field raises with raising the heat source parameter (S) and decay parameter (λ) for $y \geq 0.5$ and $y \geq 0.16$ respectively. For $y \leq -0.3$, the axial velocity raises with raising the heart source parameter (S) and decay parameter (λ) and the effect reverses in $-0.3 \leq y \leq 1$.

The axial velocity reduces with raising the magnetic field (Ha), Prandtl number (Pr) and Casson parameter (γ). And axial velocity raises with raising the magnetic field (Ha) and Casson parameter (γ) for $y \geq -0.5$ and $y \geq -0.2$ respectively. The effect of raising the values of the Casson parameter is to restrain the velocity field. Prandtl number can be used to raise the rate of cooling in conduct flows. The normal velocity reduces with raising the decay parameter and tending to zero very fast for higher values of the decay parameter. The results may be useful for possible technological purposes in liquid-based systems involving stretchable materials.

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