

Combined Impact of Viscous Dissipation, Heat Source and Radiation on Mhd Flow Past a Exponentially Stretching Sheet with Soret and Dufour Effects



B Narsimha Reddy, P Maddileti, B Shashidar Reddy

Abstract: This paper is perpetrated to inspect the impact of viscous dissipation, Heat Source, Radiation and Soret and Dufour on heat mass transfer MHD flow past a exponentially stretched sheet. The equations governing the flow restricted with boundary conditions are altered to a group of non-linear coupled O.D.Es by means of appropriate transformation. Initially the equation of momentum is linearized and then are converted to a group of linear equations by applying implicit finite difference scheme. The group of linear equations is solved numerically by Thomas algorithm. The impact of diverse parameters are discussed through graphical representations and tables. The study reveals that the magnetic field reduces the fluid velocity. Temperature and fluid Concentration drops down with an hike in Dufour number. Raise in Dufour number, rate of heat transfer drops but and mass transfer enhances.

Keywords: Exponentially stretching sheet, Quasi-Linearization, Finite Difference Scheme, Soret and Dufour effect.

I. INTRODUCTION

Due to the various applications in manufacturing procedures in industries such as molten plastic, paper production and soon, the topic of MHD boundary layer flows over a stretching surface has significant importance. In the modern studies, Visco-elastic fluid flows with MHD past stretched surfaces have been evaluated by Liu [1], Cortell [2] and many researchers. Dutta et al.[3] analyzed fluid flow past a stretched surface with heat transfer by considering various physical parameters. Vyas and Ranjan [4] evaluated MHD flow in porous medium past non-linear stretching surface. The closed form solutions for viscous fluid flow over a stretching surface were extended under various physical parameters by Crane[5]. The boundary layer fluid flow over a stretched surface was evaluated by Muhaimina et al. [6]

MHD flows has enormous applications in astrophysics, engineering etc. MHD flow past a stationary plate with the radiation effects was analyzed by Raptis et al [7]. Transfer of heat in MHD flows is an important, as is has various residential and industrial applications. In such process, heat should competently and successfully be released, absorbed or sent from procedure to another to maintain the position quo. Heat transfer occurs due many reasons such as Viscous dissipation, heat source/sink, radiation etc. The radiation effects on MHD fluid flows were investigated by Hakiem [8], Muthukumaraswamy and Senthil Kumar [9]. Shashidar [10] has examined the effects of heat source/sink on MHD flow and heat transfer of a non-Newtonian fluid. Venkata Madhu et al [11] considered the impact of Radiation with heat mass transfer on MHD fluid flow over stretching sheet.

Many authors have allured through the results of heat and mass transfer since they have numerous applications in various industrial and engineering problems as such heat exchangers, rocket boosters etc. Numerical and analytical studies of heat and mass transfer above a stretching plate considering a range of geometries were evaluated by many authors. Sanjayanand and Khan [12] has analyzed boundary layer flow over an exponentially stretching plate.

Under simultaneously occurrence of heat and mass, complex performance will be seen between fluxes and the guiding potentials. Further, the gradients of temperature and concentration will produce energy flux. And hence the energy flux produced by concentration gradient is Dufour effect while the mass flux instigated by temperature gradient will constitute to Soret effect. Shashidar and Saritha [13] has explored the soret and dufour impact on MHD boundary layer flow under slip conditions. Impact of Soret and Dufour on MHD flow over a wavy channel was discussed by Gbadeyan et al [14]

Recently Mohammed [15] has examined the impact of viscous dissipation along with heat generation and radiation on heat and mass transfer MHD flow past a exponentially stretching surface. By considering the above mentioned results, the focus of the current investigation is to evaluate the transfer of heat and mass in MHD flow over a exponentially stretched surface considering the impact of diverse physical parameters on the flow such as Thermal Radiation, Heat absorption, Viscous Dissipation, Soret and Dufour.

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II. FORMULATION OF THE PROBLEM

For mathematical model, In two dimensions consider viscous incompressible fluid over a stretchable sheet is described in current investigation.

A variable magnetic field with strength $B(x)$ is acted in Y-axis direction and the induced magnetic field is negligible. Both the X- axis and Y-axis are considered to the parallel and orthogonal to the flow direction correspondingly. The velocity of the sheet is considered as an exponential function of the distance x from the slit. T_∞ and C_∞ are temperature and concentration far away the from the wall which are considered to be constant values.

With these assumptions, the equations governing flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{----- (1)}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \quad \text{----- (2)}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{v}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B^2}{\rho c_p} u^2 + \frac{Q_0}{\rho c_p} (T - T_\infty) + \frac{Dk_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2} \quad \text{----- (3)}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{Dk_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad \text{----- (4)}$$

Here (u, v) are velocity components along (X, Y) directions correspondingly, T, C are the Temperature and Concentration respectively. v is the kinematic viscosity, σ is the electrical conductivity, B_0 is the transverse magnetic field strength and

is of the form $B(x) = B_0 e^{\frac{x}{2L}}$, ρ is the fluid density, C_p is the specific heat at constant pressure, k is the thermal conductivity, q_r is the radiative heat flux, D is the species diffusivity, μ is the dynamic viscosity, c_s is the concentration susceptibility, k_T is the thermal diffusion ratio, T_m is the mean fluid temperature.

The appropriate boundary conditions are

$$\left. \begin{aligned} u &= U_w = U_0 e^{\frac{x}{2L}}, v = 0 \\ T &= T_w = T_\infty + T_0 e^{\frac{2x}{L}} \\ C &= C_w = C_\infty + C_0 e^{\frac{2x}{L}} \end{aligned} \right\} \text{at } y = 0 \quad \text{----- (5)}$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty$$

Employing the Rosseland's diffusion approximation for radiation [16], q_r is simplified as

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad \text{----- (6)}$$

where σ^* is Stefan - Boltzmann constant and k^* is mean absorption coefficient.

Now by expressing T^4 as a linear function using Taylor's expansion by omitting the higher powers, we obtain,

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2} \quad \text{----- (7)}$$

To reduce equations (2) - (4) into a group of non linear ordinary differential equations, here establish the subsequent similarity variables,

$$\left. \begin{aligned} \eta &= \sqrt{\frac{U_0}{2\nu L}} y e^{\frac{x}{2L}} \\ T &= T_\infty + T_0 e^{\frac{2x}{L}} \theta(\eta) \\ C &= C_\infty + C_0 e^{\frac{2x}{L}} \phi(\eta) \end{aligned} \right\} \quad \text{----- (8)}$$

If the stream function ψ is given as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ then u and v satisfy the equation of continuity and are given by

$$u = U_0 e^{\frac{x}{2L}} f'(\eta), v = -\sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{2L}} \{f(\eta) + f'(\eta)\}, \text{----- (9)}$$

On using similarity transformations, the equations (2) - (5) are transformed to:

$$f''' - 2(f')^2 + ff'' - Mf' = 0 \quad \text{----- (10)}$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + Pr[f\theta' - f'\theta + Ec(f'')^2 + MEc(f')^2 + Q\theta + Du\phi'' = 0 \quad \text{----- (11)}$$

$$\phi'' + Scf\phi' - Scf'\phi + Sr\theta'' = 0 \quad \text{----- (12)}$$

$$\left. \begin{aligned} f(0) &= 0, f'(0) = 1, f'(\infty) \rightarrow 0 \\ \theta(0) &= 1, \theta(\infty) \rightarrow 0 \\ \phi(0) &= 1, \phi(\infty) \rightarrow 0 \end{aligned} \right\} \quad \text{----- (13)}$$

Here the non-dimensional parameters are given by

$M = \frac{2\sigma B_0^2 L}{\rho U_0}$ is Magnetic parameter ,

$Pr = \frac{\rho \nu U_0}{k}$ is Prandtl number ,

$R = \frac{4\sigma^* T_\infty^3}{k^* k}$ is Radiation parameter,

$Sr = \frac{T_0 U_0 e^{\frac{3x}{2L}}}{2\nu L}$ is Soret number ,

$Ec = \frac{U_0^2}{c_p T_0}$ is Eckert number ,

$Q = \frac{2Q_0 L}{U_0 e^{\frac{x}{2L}}}$ is heat source ,

$Sc = \frac{\nu}{D}$ is Schmidt number ,

$Du = -\frac{DC_0 U_0 k_T e^{\frac{3x}{2L}}}{c_s c_p 2\nu L}$ is Dufour number .

In sight of realistic applications in multi-discipline engineering, the terms of Skin friction, Nusselt number and Sherwood number are given by

$$\left. \begin{aligned} C_f &= \frac{2\tau_w}{\rho U_w^2} = \frac{\sqrt{x/L}}{\sqrt{Re/2}} - f''(0) \\ Nu &= \frac{x q_w}{k(T_w - T_\infty)} = -\sqrt{x/L} \sqrt{Re/2} \theta'(0) \\ Sh &= \frac{x J_w}{D(C_w - C_\infty)} = -\sqrt{x/L} \sqrt{Re/2} \phi'(0) \end{aligned} \right\} \quad \text{----- (14)}$$

where $\tau_w = -\mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$, $q_w = -k \left(\frac{\partial T}{\partial y}\right)_{y=0}$,
 $J_w = -D \left(\frac{\partial C}{\partial y}\right)_{y=0}$ and $Re = \frac{x U_w}{\nu}$

III. NUMERICAL PROCEDURE

The collective influence of diverse physical parameters will have great impact on heat and mass transfer characteristics. Since the governing equations are highly non-linear, it is complicated to achieve the closed form solutions. Hence, this has led us to apply numerical method.

In this study, we have applied an efficient implicit finite difference scheme in addition to Quasi-linearization technique [17] has been applied to group of Ordinary differential equations (10), (11), (12) to study the flow behavior subject to the restrictions equation (13). As the differential equation (10) is non-linear, it is first linearized through Quasi-linearization scheme [19].

Then an Finite difference method is applied to convert these coupled ordinary differential equations to a group of linear equations. Now the calculations are done with C Programming to solve this group of linear equations. In this procedure, first momentum equation is solved to find the values of f which are necessary for finding the solution of coupled energy and concentration equations subject to the boundary conditions (13) by employing Thomas Algorithm. In the iteration process, at each stage, the convergence condition for every variable is obtained at the rate of 10^{-5} .

IV. RESULTS AND DISCUSSION

For obtaining the salient features of the flow performance, heat and mass transfer properties, the consequences of velocity, temperature and concentration with associated boundary conditions along with skin friction coefficient,

Nusselt number and Sherwood number have been computed and presented in figures 1 – 10 and in Table 1 under the action of different attractive parameters. It is evident from the table that magnetic parameter enhances the skin friction whereas it decelerates the Nusselt number and Sherwood number. Nusselt number rises with the Prandtl number but Sherwood number decreases.

The influence of Eckert number is to diminish the heat transfer and boost the mass transfer in the fluid flow. Due to heat generation, heat transfer coefficient enhances but the mass transfer shrinks.

The impact of Schmidt is to shrink the Nusselt number and increase the Sherwood number. With the decrease in Soret number or increase in the Dufour number, heat transfer rate falls down whereas the mass transfer raises.

Table- 1:

M	Pr	R	Ec	Q	Sc	Sr	Du	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1.0	0.71	0.5	0.1	0.1	0.6	0.8	0.1	1.711830	0.506201	0.528793
2.0	0.71	0.5	0.1	0.1	0.6	0.8	0.1	2.020512	0.465147	0.518957
3.0	0.71	0.5	0.1	0.1	0.6	0.8	0.1	2.292627	0.434206	0.511413
1.0	1.0	0.5	0.1	0.1	0.6	0.8	0.1	1.711830	0.582408	0.477351
1.0	2.0	0.5	0.1	0.1	0.6	0.8	0.1	1.711830	0.830155	0.312443
1.0	0.71	0.5	0.1	0.1	0.6	0.8	0.1	1.711830	0.506201	0.528793
1.0	0.71	2.0	0.1	0.1	0.6	0.8	0.1	1.711830	0.418724	0.588434
1.0	0.71	0.5	0.5	0.1	0.6	0.8	0.1	1.711830	0.352712	0.608315
1.0	0.71	0.5	1.0	0.1	0.6	0.8	0.1	1.711830	0.160854	0.707721
1.0	0.71	0.5	0.1	1.0	0.6	0.8	0.1	1.711830	0.002764	0.885297
1.0	0.71	0.5	0.1	0.5	0.6	0.8	0.1	1.711830	0.322493	0.655177
1.0	0.71	0.5	0.1	0.1	0.6	0.8	0.1	1.711830	0.506201	0.528793
1.0	0.71	0.5	0.1	0.1	0.6	0.8	0.1	1.711830	0.506201	0.528793
1.0	0.71	0.5	0.1	0.1	0.6	0.8	0.1	1.711830	0.500894	0.623779
1.0	0.71	0.5	0.1	0.1	0.6	0.8	0.1	1.711830	0.494763	0.734771
1.0	0.71	0.5	0.1	0.1	0.6	0.8	0.1	1.711830	0.506201	0.528793
1.0	0.71	0.5	0.1	0.1	0.6	0.4	0.2	1.711830	0.486099	0.599962
1.0	0.71	0.5	0.1	0.1	0.6	0.2	0.4	1.711830	0.445895	0.635547
1.0	0.71	0.5	0.1	0.1	0.6	0.1	0.8	1.711830	0.365489	0.653339

For getting the numerical outcome, the values of $M = 0.0$, $Pr = 1.0$, $R = 1.0$, $Ec = 0.5$, $Q = 1.0$, $Sr = 0.5$ and $Du = 0.1$ are kept as static except the parameter with which the variation is geometrically shown. Figures 1-3 present the influence of magnetic parameter on velocity, temperature and concentration fields. It is evident that velocity drops down under the influence of magnetic field. Whereas the temperature and concentration profiles are accelerated. This is because, magnetic field generates Lorentz force, which lessens the fluid velocity. As a consequence, heat is generated and hence temperature of the fluid is hiked. In general, more energy is produced with the increase in the heat source and hence the temperature is enhanced. This phenomenon can be observed from graph 4.

The influence of altering values of radiation parameter on temperature is revealed in figure 5. It is apparent that the temperature is hiked in the presence of radiation. This is

because, Radiation transfers energy with the particles and hence there will an enhancement of energy in the fluid flow. Figure 6 is drawn to observe the impact of temperature profiles under the action of Viscous Dissipation. Geometrically, It is obvious that with the hike in the Eckert number, fluid temperature enhances. Since temperature enhances with stronger viscous dissipative heat.

Figure 7 reveals the impact of Prandtl number on temperature of the fluid in the flow. As expected, with the hike in Prandtl number, temperature is lessened above the surface. This is the consequence of the phenomenon that with growing values of Prandtl number, thermal boundary layer becomes thinner. The variation in the concentration of the fluid for the various values of Schmidt number is displayed in the figure 8,

which reveals that the growing values of Sc number suppress the concentration boundary layer thickness.

Figures 9 and 10 are graphical representations of the temperature and concentration fields for altering values of Soret and Dufour number. The values of Soret and Dufour are varied such that the product of Soret and Dufour number is fixed. With the rise in Dufour number or drop in Soret number, both the temperature and concentration of the fluid in the flow in increased. This is because of the phenomenon that mass flux caused by temperature produces Soret effect and the energy flux caused by concentration gradient gives Dufour effect.

FIGURES

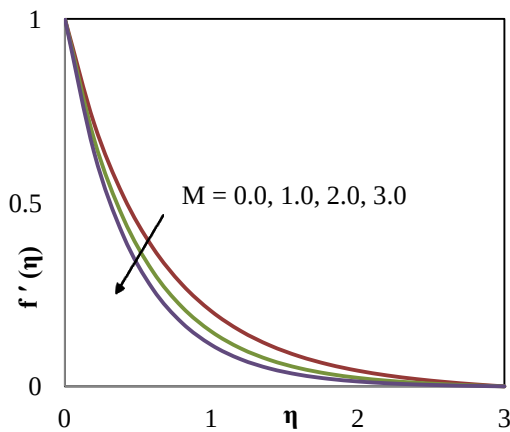


Fig.1 Velocity with M

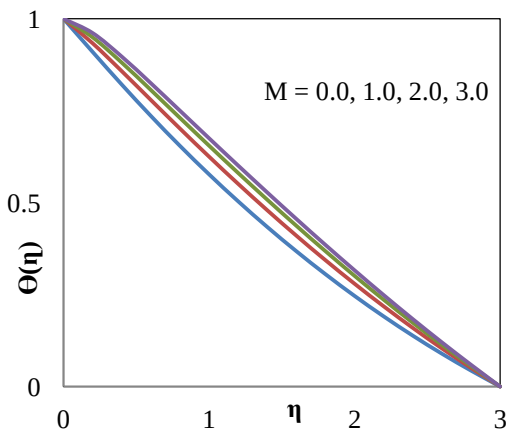


Fig.2 Temperature with M

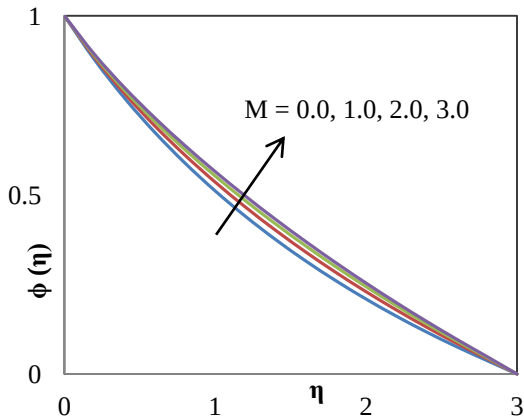


Fig.3 Concentration with M

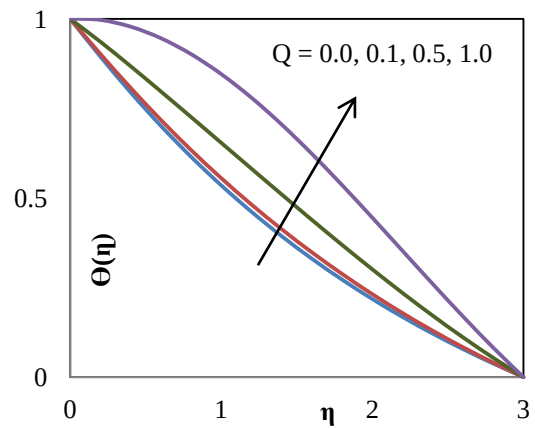


Fig.4 Temperature with Q

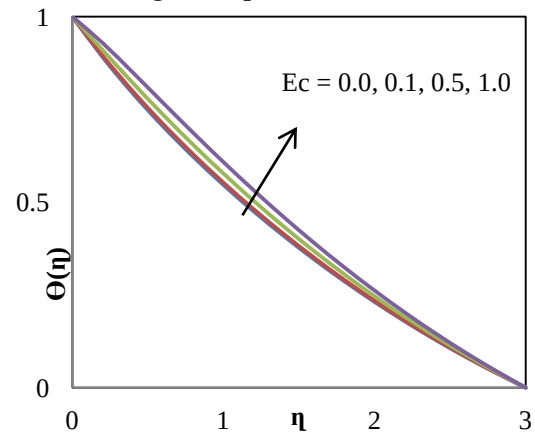


Fig. 5 Temperature with Ec

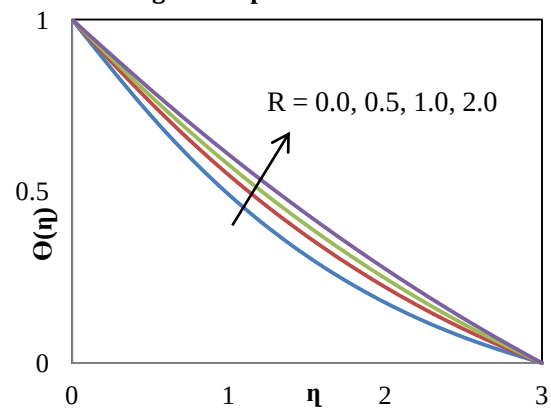


Fig.6 Temperature with R

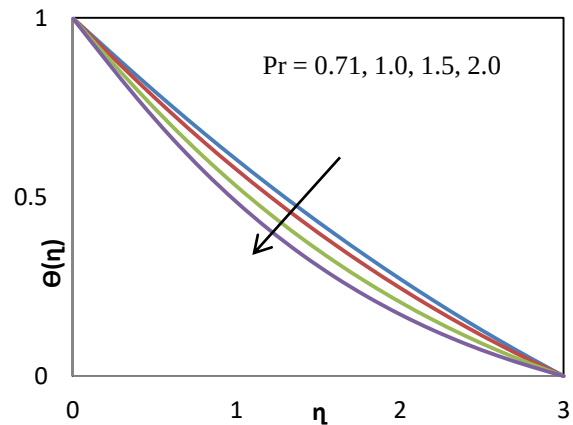


Fig.7 Temperature with Pr

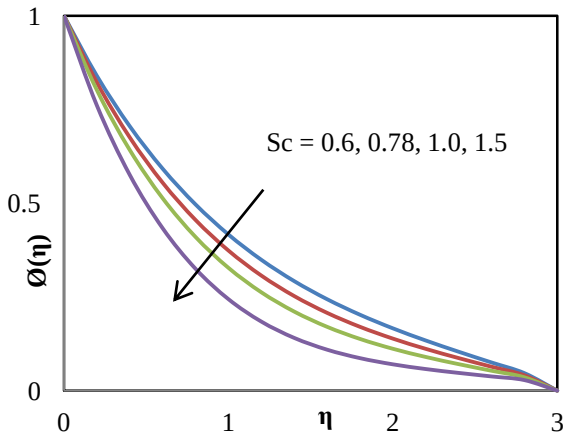


Fig.8 Concentration with Sc

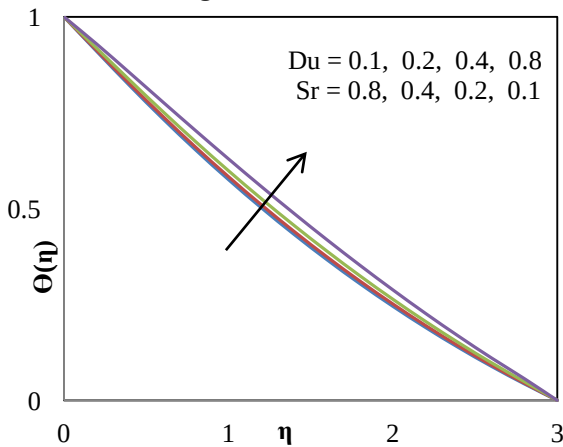


Fig.9 Temperature with Sr and Du

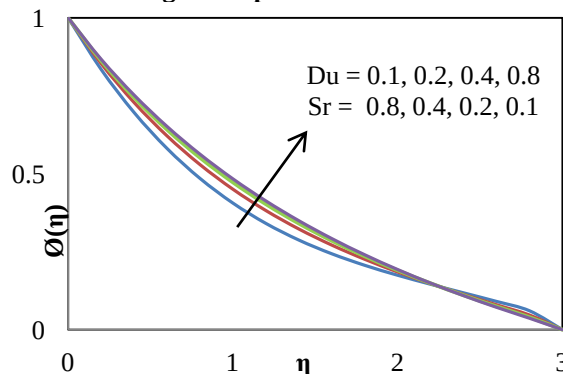


Fig.10 Concentration with Sr and Du

V. CONCLUSIONS

Numerical investigation is done on impact of Soret and Dufour, Viscous dissipation, Heat generation and Radiative MHD flow past a exponentially stretching sheet. The equations governing the flow are treated with finite difference scheme and the numerical computations are carried out for altering values of governing parameters of the problem. The significant findings of the investigation are as follows:

1. Fluid velocity retards with magnetic parameter.
2. Eckert number, Heat generation and Radiation parameters has the tendency to boost the temperature of fluid.
3. Rise in Prandtl number decline fluid temperature.
4. Growing values of Schmidt number suppresses the concentration profiles of the flow.
5. Rise in Dufour number have the tendency to reduce

the temperature as well as concentration fields.

6. With Dufour effect, Nusselt number shrinks and Sherwood number enhances.

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numerical techniques.

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