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Slip Velocity Effect on Blood Flow through a Multi-Irregular Constricted Artery

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Abstract: ***In this research paper, fluid flow is considered in uni direction through the multi – irregular constricted artery. Blood has been considered Hershcel –Bulkley i.e. non- Newtonion. Analytical techniques are carried out to solve the problem. Mathematical expressions for several variables of fluid flow has been estabilished. The effect of slip velocity, flow behaviour index and yield stress on axial velocity volumetric flow rate and wall shear stress have been depicted through graphs. It has noticed that velocity in axial direction and fluid flow rate increases as increase in slip velocity at arterial wall. It has also observed that shear stress increases with increasing of yield stress. Flow rate reaches lowest value at some points in the portion of stenosis for clinical investigation.***

Keywords **:** ***Slip velocity, non-Newtonian fluid, blood flow, multi-irregular stenosis.***

I. INTRODUCTION

It is vital field to study blood flow characteristics through stenosed and normal artery since it has been proved by experimentally that many cardiovascular diseases like (brain stroke, heart attack, headache hypertension etc) evolved according to the blood flow nature and mechanical properties of blood tubes walls. The experimentally investigations and theoretically studies about flow of blood are very helpful in the treatment of cardiovascular diseases. The exact reason of construction of atherosclerosis in an artery is not well known. It is considered that the deposition of fatty substances and cholesterol etc. at interior wall of artery are main factors of stenosis in an artery. A number of studies have been tackled experimentally and theoretically in order to manifest the influence of atherosclerosis on blood flow through stenotic and non stenotic portion of an artery, by considering blood as a Newtonian fluid by Biswas and Chakraborty [2]. A non-Newtonian blood is one in which shear stress and shear strain rate does not express in linear form. Blood shows its non-Newtonian nature in the smaller arteries at low shear rate as suggested by experimental studies Huckabe CE. [6]. Mandal [8] prepared power-law model in which blood taken as non-Newtonian in nature blood flow from a stenosed arterial segment solved numerically and the blood flow characteristics generalized.

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The model studied potentially to calculate flow characteristics and ,the wall shear stress and the resistive impedance. A mathematical model was developed to investigate properties of blood as viscosity ,viscoelasticity, and thixotropy taking blood as non- Newtonian by Thurston. Arun Kumar Maiti [7] considered blood as Casson fluid and effect of slip velocity on blood flow studied by taking stenosis as cosine shaped and multiple stenosed artery. Amit Bhatnagar [1] prepared a model to understand the influence of Slip Velocity on flow charecteristics of blood through composite stenosed portion of artery. Model observed variation in flow characteristics corresponding to slip velocity. Many researcher considered cosine shaped, multiple shaped stenosed, composite shaped stenosis to understand slip effect on characteristics of blood flow. Siddiqui [9], Biswas and Laskar [3], Kumar Harjeet [5] have been prepared mathematical models to study the blood flow characteristics through a stenosed portion of artery due to slip condition. But they have considered the effect of single stenosis. Recently Gupta Amit *et. al*. [4] studied the effect of slip on blood flow through bell shaped stenosed arterial segments. Present model analyzed influence of slip velocity on flow characteristic of blood flow considering blood as non - Newtonian.

II. MATHEMATICAL FORMULATION

Geometry of multi-irregular stenosis mathematically represented as:

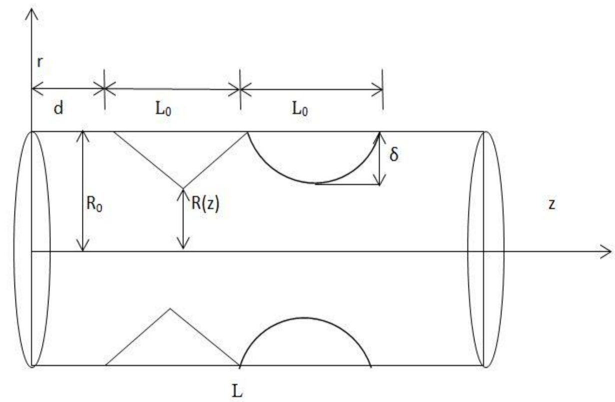

Figure 1. Multi irregular stenosed .

$$R(z)=\begin{cases} R_0-\frac{2\delta}{L_0}(z-d) & d\le z\le d+\frac{L_0}{2} \\ R_0-\frac{2\delta}{L_0}(z-d-L_0) & d+\frac{L_0}{2}<z\le d+L_0 \\ R_0-\delta+\frac{4\delta}{L_0^{\,2}}\left(z-d-\frac{3L_0}{2}\right)^2 & d+L_0<z\le d+2L_0 \end{cases} \quad (1)$$

Where
$R(z)$ = Radius of stenosed portion of artery.
R_0 = Radius of artery without stenosis.
$2L_0$ = Total length of stenosed arterial segment.
δ = highest height of the stenosis
d = d indicates stenosis location.
The equations describing motion of blood flow are given as

$$-\frac{dp}{dz}-\frac{1}{r}\frac{d(r\tau)}{dr}=0$$

or $-\frac{dp}{dz}=\frac{1}{r}\frac{d(r\tau)}{dr}$ (2)

$$0=\frac{dp}{dr} \quad (3)$$

Where r and z represent the radial and axial coordinates respectively, p is the pressure and τ shows the shear stress. Consider the blood as a Hershel - Bulkley type fluid, for which the constitutive equations for shear stress and shear rate, is given by

$$\tau = k^{\frac{1}{n}}\left(-\frac{du}{dr}\right)^{\frac{1}{n}}+\tau_0 \qquad \tau \geq \tau_0. \quad (4)$$

$$\left(-\frac{du}{dr}\right)=0, \qquad \tau < \tau_0. \quad (5)$$

Above equations may be written as

$$-\frac{du}{dr}=\frac{1}{k}(\tau-\tau_0)^n \qquad \tau \geq \tau_0. \quad (6)$$

$$\left(-\frac{du}{dr}\right)=0, \qquad \tau < \tau_0. \quad (7)$$

And boundary conditions are

(1) $u = u_s$ at $r = R(z)$, velocity slip condition. (8)
(2) τ is finite at r = 0 , regularity condition. (9)
where u is axial velocity and τ_0 is yield stress and k is the fluid viscosity n is fluid behavior index.

III. SOLUTION OF THE PROBLEM

Solving equation (2) applying the boundary condition (9) expression for shearing stress as

$$\tau=-\frac{1}{2}r\frac{dp}{dz} \quad (10)$$

The wall shear stress written as

$$\tau_R=-\frac{1}{2}R\frac{dp}{dz} \quad (11)$$

Corresponding yield stress is obtained as

$$\tau_0=-\frac{1}{2}r_0\frac{dp}{dz} \quad (12)$$

Where r_0 is core region radius.
From equation (2) using (10) and (12) the axial velocity may be expressed as

$$\frac{du}{dr}=-\frac{1}{k}\left(-\frac{1}{2}\frac{dp}{dz}\right)^n(r-r_0)^n \quad (13)$$

On integrating equation (13) between limits r and R we obtained

$$\int_r^R du=\int_r^R -\frac{1}{k}\left(-\frac{1}{2}\frac{dp}{dz}\right)^n(r-r_0)^n dr$$

Consider $-\frac{1}{2}\frac{dp}{dz}=P$ above equation becomes

$$\int_r^R du=-\frac{1}{k}\int_r^R\left(\frac{P}{2}\right)^n(r-r_0)^n dr$$

$$u(R)-u(r)=-\frac{1}{k}\left(\frac{P}{2}\right)^n\left[\frac{(R-r_0)^{(n+1)}}{(n+1)}-\frac{(r-r_0)^{(n+1)}}{(n+1)}\right] \quad (14)$$

Applying slip boundary condition (8)

$$u=u_s+\frac{1}{k(n+1)}\left(\frac{P}{2}\right)^n\left[(R-r_0)^{(n+1)}-(r-r_0)^{(n+1)}\right] \quad (15)$$

For core region $r = r_0$

$$u=u_s+\frac{1}{k(n+1)}\left(\frac{P}{2}\right)^n\left[(R-r_0)^{(n+1)}\right] \quad (16)$$

Volumetric flow rate is

$$Q=\int_0^R 2\pi r u dr$$

$$Q=2\pi\left[\int_0^{r_0} r u dr+\int_{r_0}^R r u dr\right] \quad (17)$$

Using equations (15) and (16) in equation (17)

$$Q=\pi R^2 u_s+\frac{\pi R^3\tau_R^n}{k(n+3)}\left[1+\frac{2\alpha}{(n+2)}+\frac{2\alpha^2}{(n+1)(n+2)}\right](1-\alpha^2) \quad (18)$$

Where $\alpha=\frac{\tau_0}{\tau_R}=\frac{r_0}{R}$

When $\frac{\tau_0}{\tau_R}\leq 1$ equation (18) becomes

$$Q=\pi R^2 u_s+\frac{\pi R^3\tau_R^n}{k(n+3)}\left[1-\frac{n+3}{n+2}\alpha\right] \quad (19)$$

Above equation may be written as

$$\tau_R=\left[\frac{k(n+3)(Q-\pi R^2 u_s)}{\pi R^3}\right]^{\frac{1}{n}}+\frac{(n+3)}{(n+2)}\tau_0 \quad (20)$$

$$\tau_N=\left[\frac{k(n+3)(Q-\pi R_0^2 u_s)}{\pi R_0^3}\right]^{\frac{1}{n}}+\frac{(n+3)}{(n+2)}\tau_0 \text{ for absence of stenosis.} \quad (21)$$

Wall shear stress may be given as $\tau=\frac{\tau_R}{\tau_N}$

$$\tau=\frac{\left[\frac{k(n+3)(Q-\pi R^2 u_s)}{\pi R^3}\right]^{\frac{1}{n}}+\frac{(n+3)}{(n+2)}\tau_0}{\left[\frac{k(n+3)(Q-\pi R_0^2 u_s)}{\pi R_0^3}\right]^{\frac{1}{n}}+\frac{(n+3)}{(n+2)}\tau_0}$$

$$\tau=\frac{\left[k(n+3)Q-\pi R_0^2\left(\frac{R}{R_0}\right)^2 u_s)\right]^{\frac{1}{n}}+\left(\frac{n+3}{n+2}\right)(\pi R_0^3)^{\frac{1}{n}}\left(\frac{R}{R_0}^3\right)^{\frac{1}{n}}\tau_0}{\left(\frac{R}{R_0}\right)^{\frac{3}{n}}\left[k(n+3)(Q-\pi R_0^2 u_s)\right]^{\frac{1}{n}}+\left(\frac{n+3}{n+2}\right)(\pi R_0^3)^{\frac{1}{n}}\tau_0} \quad (22)$$

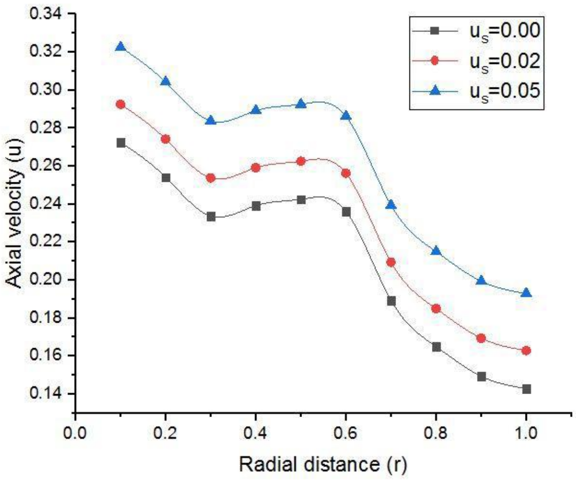

Fig.2 : Axial velocity (u) profile for u_s = 0,0.02,0.05

IV. RESULT AND DISCUSSION

The flow characteristics of blood flow are explained by various analytical expressions and depicted graphically as shown below.

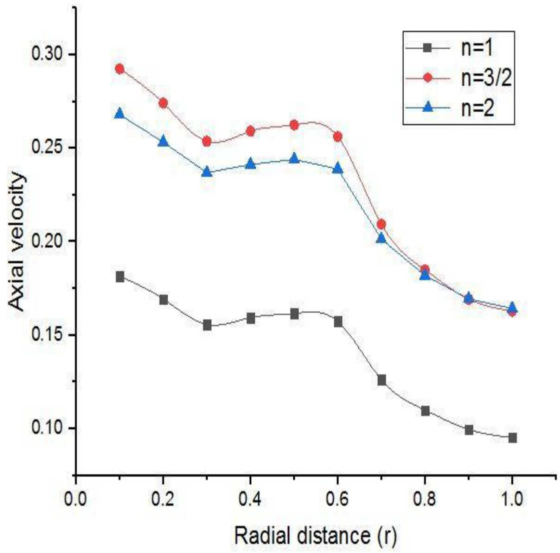

Fig. 3 Axial velocity (u) profile for n = 1, 3/2, 2.

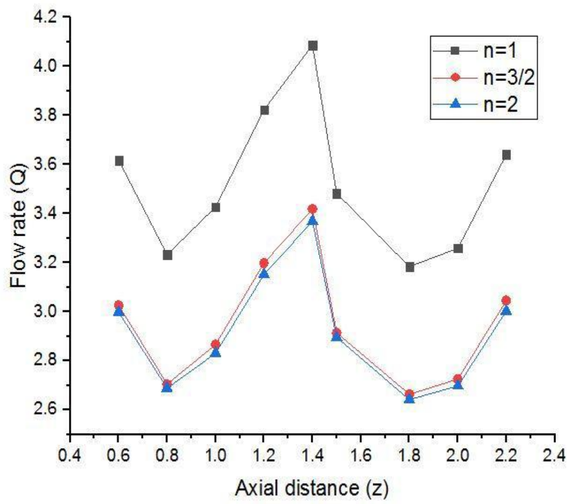

Fig.4 : Distribution of Volumetric flow rate (Q) for u_s = 0.0. 02, 0.05.

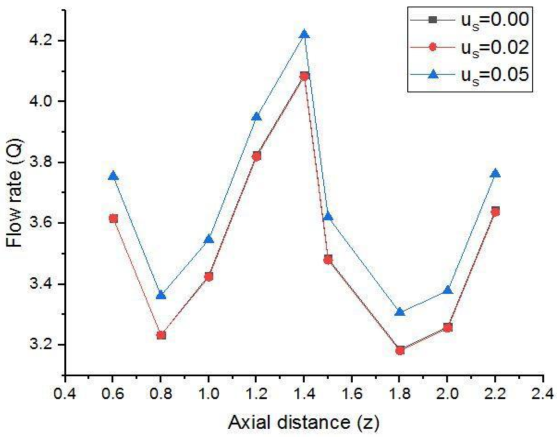

Fig.5 : Distribution of volumetric flow rate(Q) n = 1, 3/2, 2.

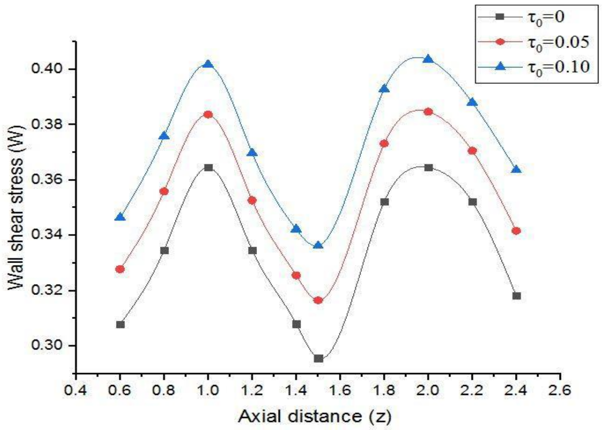

Fig.6. Wall shear rate for distinct τ_0

In the present model , slip velocity (u_s) effect on characteristics of fluid flow through multi- irregular has been discussed. Axial velocity profile , wall shear stress distribution , flow rate have been depicted graphically. The values of parameters are taken as, slip velocity (u_s = 0, 0.02 and 0.05), behavior index of blood flow (n = 1, 3/2 and 2), yield stress (τ_0= 0, 0.05,0.10) .The axial velocity (u) profile for u_s = 0,0.02,0.05 and for n = 1, 3/2, 2 shown in figure 2 and figure 3 respectively. It is noticed that increase in slip velocity, increases in axial velocity in stenosed artery. Figure 3 reveals that the axial velocity increases, with increasing behavior index n. The distribution of the volumetric flow rate for u_s = 0,0.02,0.05 and for n = 1, 3/2, 2.shown in figures 4 and 5. It is seen that flow rate increases, with increasing slip velocity while it decreases with increasing of behaviour index n. Wall shear rate for distinct yield stress (τ_0) is shown in figure 6. The change in wall shear stress against axial distance can be seen at any point in stenosed arterial segment. It is maximum at two points z =1 and z =2 which are stenotics throats and minimum at z =1.5 which is critical height of stenosis.

V. CONCLUSION

In this model the characteristics of fluid flow through a multi irregular stenosed arterial segment with axial slip velocity has been investigated theoretically.

Blood is considered as Hershel Bulkley fluid. Various analytical expressions for characteristics of blood flow are obtained. It is noticed that axial velocity (u) and volumetric flow rate (Q) increases due to increases in slip velocity. Axial velocity increases while flow rate (Q) decreases with increasing of fluid behaviour index. It may also be conclude that wall shear increases with increasing of yield stress. This model reveals that yield stress and slip velocity both are sensitive parameters for flow characteristics. Physicians can control various cardiovascular diseases by adjusting these parameters with the help of various diagnostic tools.

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