International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-4, February 2020

Natural Convection MHD Effect on Heat Transfer in Vertical Cylinder

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***Abstract:** In this paper, natural convection MHD effect on Heat transfer in vertical cylinder is analyzed. Firstly heat transfer equation and convert it in dimensionless form by introducing dimensionless variables has considered. By use of finite Hankel and inverse Hankel transforms in dimensionless heat transfer equation, the values of temperature and Nusselt number is obtained. Finally, graphical representation is used to study the effect of Prandtl number pr on temperature and Nusselt number Nu. It is observed that the fluid at the center region of the cylinder is not heated for value of Prandtl number Pr greater than 1.7 for small values of t.*

***Keywords:** Free convection, Heat transfer, Liquid metal, Mixed convection, MHD*

I. INTRODUCTION

The phenomenon of heat transfer due to natural convection and mixed convection has been analyzed by many researchers. R.U.Haq, S. Nadeem, Z.H.Khan and N.F.M.Noor [6] examined the thermal conductivity within the base fluids in presence of Carbon Nanotubes. Tao Fan, Hang Xu and I. Pop [10] computed the influence of parameters such as Brownian motion, the thermophoresis and Lewis number on the temperature and nanoparticle concentration distributions. They analyzed that the nanoparticles enhance the heat transfer attributes of the flow in the horizontal channel. E.V.Murphree [2] proposed a first phenomenology between fluid friction and heat transfer. In an alternate phenomenology, H.T.Lin [4] obtained the analogies of Reynolds, Prandtl, Karman and Colburn for Prandtl number with uniform wall temperature. S. Cuevas and B.F.Picologlou [7], [8] emphasized to specify turbulent velocity profiles to solve the heat transfer equation. Dawid Taler [1] proposed turbulent heat transfer in the tubes and calculated Nusselt numbers to find a relevant heat transfer correlation. He also compared this heat transfer correlation with experimental data. llyas Khan, Nehad Ali Shah, Asifa Tassaddiq, Norzieha Mustapha and Seripah Awang Kechil [5] computed the heat transfer from the surface of the cylinder to the fluid.

Hulin Huang and Ying Fang [3] investigated heat transfer by applying the MHD “k- ε” model and the induced magnetic field equation. Srinathuni Lavanya and D. Chenna Kesavaiah [9] focused on effects of velocity for dusty gas and dust particle, magnetic field, temperature and concentration. Ya. Listratov, D. Ognerubov, E. Sviridov [11] determined the threshold value of the Grashof number at which buoyancy begins to affect heat transfer.

Revised Manuscript Received on February 28, 2020.
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In present paper natural convection MHD effect on heat transfer in vertical cylinder has been obtained by using Hankel transformation.

II. MATHEMATICAL FORMULATION OF A PROBLEM

Consider a vertical cylinder of radius r_0 and axis of the cylinder is in upward direction. At time t, cylinder starts to oscillate along its axis then cylinder temperature raised to T_c as shown in Fig. 1.

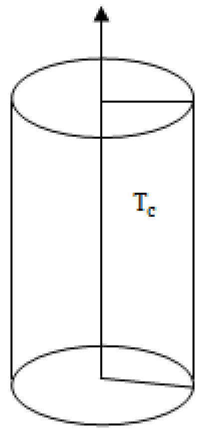

Fig.1.shows cylinder of radius r_0 oscillates on its axis.

Assuming that the temperature T which is a function of r and t. First consider the PDE for heat transfer turbulent flow is governed by

$$\frac{\partial^2 T(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T(r,t)}{\partial r} = \frac{1}{\alpha}\frac{\partial T(r,t)}{\partial t} \quad ; 0 < r < r_0 , t > 0, \qquad (1)$$

With initial condition and mixed boundary condition

$$T(r, 0) = T_c \quad ; 0 \le r \le r_0 , t > 0, \qquad (2)$$

$$\frac{\partial T(r_0,t)}{\partial r} = -hT \quad , \qquad (3)$$

Where h is a positive constant.

Let us assume the following dimensionless variables as

$$\Theta = \frac{T+h}{T_c+h} \ , \ r^* = \frac{r}{r_0} \ , t^* = \frac{t\,\vartheta}{{r_0}^2} .$$

Dropping the star notation, the equations (1) to (3) reduced to

$$\Pr.\frac{\partial \Theta(r,t)}{\partial t} = \frac{\partial^2 \Theta(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial \Theta(r,t)}{\partial r} , \qquad (4)$$

$$\Theta(r, 0) = 1 ; 0 \le r \le 1, t > 0, \qquad (5)$$

$$\frac{\partial \Theta(1,t)}{\partial r} = -h\,\Theta(1,t) + g \quad , \qquad (6)$$

$$\text{Where, } g = \frac{h^2}{T_c+h} \quad , \Pr = \frac{\vartheta}{\alpha} ,$$

Using the finite Hankel transform of order zero, the equation (4) reduced to

$$\frac{d\Theta_H(s_n,t)}{dt} = -\frac{{s_n}^2}{Pr}\,\Theta_H(s_n,t),$$

$$\frac{d\Theta_H}{dx} + \frac{{s_n}^2 t}{Pr}\,\Theta_H = 0. \qquad (7)$$

Where, $\Theta_H(s_n,t) = \int_0^1 r\,\Theta(r,t)J_0(s_n r)dr$, is finite Hankel transform of $\Theta(r,t)$ and s_n, n = 0, 1, 2 , 3……are the roots of equation $J_0(x) = 0$ where $J_0(x)$ represents the Bessel functions of first kind of order zero.

So that $\qquad \Theta_H(s_n,0) = \int_0^1 rJ_0(s_n r)dr = \frac{J_1(s_n)}{s_n} \qquad (8)$

Using initial condition (8), the solution of equation (7) is given by:

$$\Theta_H(s_n,t) = \frac{J_1(s_n)}{s_n}.e^{-\frac{{s_n}^2 t}{Pr}} \qquad (9)$$

Taking inverse Hankel transform, expression for temperature is given by

$$\Theta(r,t) \quad = 2\sum_{n=1}^{\infty}\frac{s_n J_1(s_n)}{h^2+{s_n}^2}.\frac{J_0(s_n r)}{{J_0}^2(s_n)}.e^{-\frac{{s_n}^2 t}{Pr}}. \qquad (10)$$

The Nusselt number is defined as:

$$Nu = \frac{\partial\Theta(r,t)}{\partial r} = 2\sum_{n=1}^{\infty} e^{-\frac{{s_n}^2 t}{Pr}}.(11) \qquad (11)$$

III. RESULTS AND DISCUSSIONS

Representative results for temperature as a function of radial coordinate for distinct values of Prandtl number at time t are obtained. Additionally we have computed the Nusselt number Nu. To observe the effect of the Prandtl number Pr on fluid temperature $\Theta(r,t)$ and Nusselt number Nu, results are shown graphically. In Table I, Table II and Table III, as the value of r increases the temperature decreases. Figures 2(a), 2(b) and 2(c) show the temperature, for distinct values of the Prandtl number Pr at time t. Temperature was obtained for three different values of time 0.1, 0.2 and 0.3 corresponding to the distinct values of Prandtl number. In Table IV, it is shown that Nusselt number decreases with increase in value of t or decrease in value of Pr and vice-versa. The equation (10) shows that the temperature tends to zero for small values of the Prandtl number or large values of time t. Finally, it is observed that for small values of the Prandtl number, the heat transfer from the cylinder surface to the fluid is meaningful because for the values of the Prandtl number Pr larger than 1.7, the fluid at center region of the cylinder is not heated.

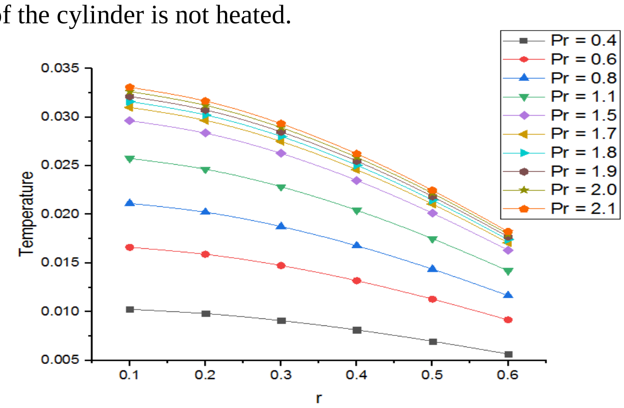

Fig. 2(a). Temperature for distinct values of the Prandtl number at time t = 0.1.

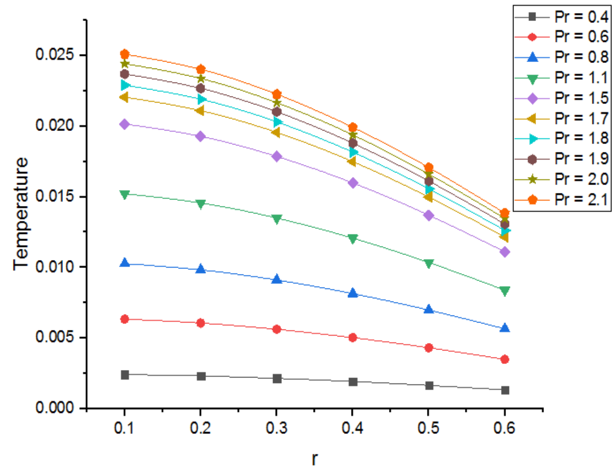

Fig. 2(b). Temperature for distinct values of the Prandtl number at time t = 0.2.

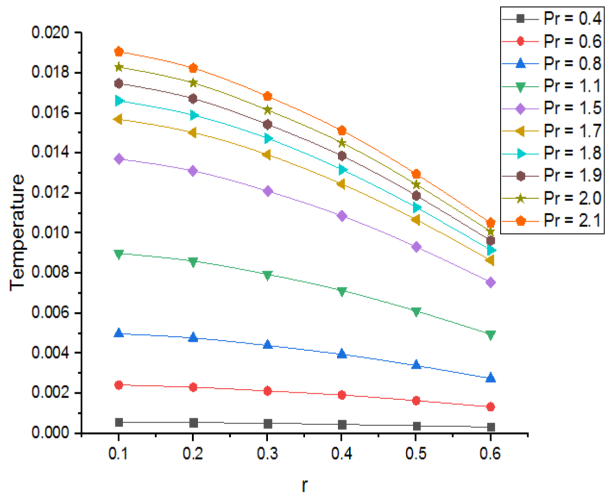

Fig. 2(c). Temperature for distinct values of the Prandtl number at time t = 0.3.

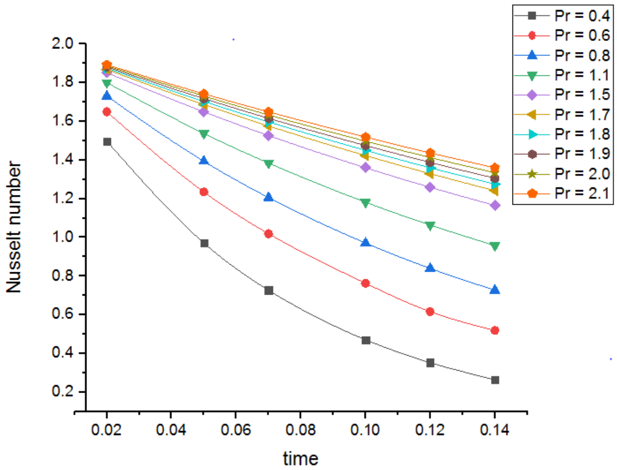

Fig. 3. The Nusselt number for distinct values of the Prandtl number at time t.

Table I: Temperature for distinct values of Prandtl number at time t=0.1

		Θ								
R	Pr=0.4	Pr =0.6	Pr =0.8	Pr =1.1	Pr =1.5	Pr =1.7	Pr = 1.8	Pr =1.9	Pr =2.0	Pr=2.1
0.1	0.01025	0.01662	0.02115	0.02576	0.02964	0.03101	0.03161	0.03214	0.03264	0.03308
0.2	0.00981	0.01591	0.02024	0.02465	0.02836	0.02968	0.03024	0.03075	0.03122	0.03166
0.3	0.00909	0.01475	0.01876	0.02285	0.02628	0.02751	0.02803	0.0285	0.02894	0.02934
0.4	0.00813	0.01319	0.01678	0.02043	0.02351	0.0246	0.02507	0.02549	0.02588	0.02624
0.5	0.00697	0.0113	0.01438	0.01751	0.02014	0.02108	0.02148	0.02185	0.02218	0.02248
0.6	0.00565	0.00917	0.01166	0.01421	0.01634	0.0171	0.01743	0.01772	0.01799	0.01824

Table II: Temperature for distinct values of Prandtl number at time t=0.2

		Θ								
R	Pr=0.4	Pr =0.6	Pr =0.8	Pr =1.1	Pr =1.5	Pr =1.7	Pr = 1.8	Pr =1.9	Pr =2.0	Pr=2.1
0.1	0.00241	0.00634	0.01027	0.01523	0.02016	0.02207	0.02292	0.02371	0.02444	0.02512
0.2	0.00231	0.00606	0.00983	0.01457	0.01929	0.02111	0.02192	0.02269	0.02339	0.02404
0.3	0.00214	0.00562	0.00911	0.0135	0.01787	0.01957	0.02032	0.02103	0.02168	0.02228
0.4	0.00192	0.00503	0.00815	0.01208	0.01599	0.0175	0.01817	0.01881	0.01939	0.01993
0.5	0.00164	0.00431	0.00698	0.01035	0.0137	0.01499	0.01557	0.01612	0.01661	0.01708
0.6	0.00133	0.00349	0.00566	0.00839	0.01111	0.01217	0.01263	0.01308	0.01348	0.01385

Table III: Temperature for distinct values of Prandtl number at time t=0.3

		Θ								
R	Pr=0.4	Pr =0.6	Pr =0.8	Pr =1.1	Pr =1.5	Pr =1.7	Pr=1.8	Pr =1.9	Pr =2.0	Pr=2.1
0.1	0.00057	0.00242	0.00498	0.009	0.01371	0.0157	0.01662	0.01749	0.0183	0.01908
0.2	0.00055	0.00231	0.00477	0.00861	0.01312	0.01503	0.0159	0.01673	0.01751	0.01825
0.3	0.00051	0.00213	0.0044	0.00795	0.01211	0.01393	0.01474	0.01544	0.01616	0.01684
0.4	0.00045	0.00192	0.00395	0.00714	0.01087	0.01246	0.01318	0.01387	0.01452	0.01513
0.5	0.00039	0.00164	0.00339	0.00612	0.00932	0.01068	0.01129	0.01188	0.01244	0.01296
0.6	0.00032	0.00133	0.00275	0.00496	0.00756	0.00866	0.00916	0.00964	0.01009	0.01052

Table IV: The Nusselt number Nu for distinct values of Prandtl number Pr at time t

		Nu								
T	Pr=0.4	Pr =0.6	Pr =0.8	Pr =1.1	Pr =1.5	Pr =1.7	Pr =1.8	Pr =1.9	Pr =2.0	Pr=2.1
0.02	1.49778	1.64932	1.73078	1.80038	1.85158	1.86844	1.87552	1.88188	1.88762	1.89282
0.05	0.9707	1.23518	1.39334	1.53768	1.64934	1.68716	1.7032	1.71766	1.73078	1.74274
0.07	0.72694	1.01862	1.20578	1.3842	1.52694	1.5762	1.5972	1.6162	1.63352	1.64934
0.1	0.47112	0.76284	0.9707	1.1822	1.36016	1.42328	1.45044	1.47516	1.49778	1.51856
0.12	0.35282	0.6161	0.84002	1.06424	1.25922	1.32966	1.36016	1.38804	1.41362	1.43718
0.14	0.26422	0.51878	0.72694	0.95802	1.16576	1.2422	1.2755	1.30606	1.33418	1.36016

IV. CONCLUSION

Natural convection MHD effect on heat transfer in vertical cylinder has investigated. The fluid temperature and Nusselt number has obtained for distinct values of the Prandtl number Pr by using finite Hankel and inverse Hankel represents that, when the value of time t increases, the temperature decreases i.e., for large values of time t, heat transfer is most influential. Fig.3. reveals the effect of the Prandtl number Pr on the Nusselt number Nu. Equation (11) represents that when the values of the Prandtl number Pr increases, the Nusselt number Nu increases, as shown in Fig.3.

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