International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-4, February 2020

Nano M Locally Closed Sets and Maps in Nano Topological Spaces

A. Padma, M. Saraswathi, A. Vadivel, M. Seenivasan

Abstract: *The concepts of $\mathfrak{N}\mathcal{M}lc$ sets, $\mathfrak{N}\mathcal{M}lc$ continuous and $\mathfrak{N}\mathcal{M}lc$ irresolute functions are introduced & some of its characteristics are discussed. Also nano $\mathcal{M}$ submaximal spaces are defined & its properties are discussed.*

Keywords and phrases: $\mathfrak{N}\mathcal{M}lc$ sets, $\mathfrak{N}\mathcal{M}$ submaximal, $\mathfrak{N}\mathcal{M}lc$ continuous and $\mathfrak{N}\mathcal{M}lc$ irresolute functions. AMS (2000) subject classification: 54B05.

I. INTRODUCTION AND PRELIMINARIES

Nano topology (briefly, $\mathfrak{N}\mathfrak{T}$) was introduced by Thivagar [7] in the year 2013. Also he introduced nano closed ($\mathfrak{N}c$) sets & nano-interior (resp. closure ($\mathfrak{N}cl$)) ($\mathfrak{N}int$) in a nano topological spaces (briefly, $\mathfrak{N}ts$). Various forms of $\mathfrak{N}s$ were discussed in [1] -[15]. Nano regular open (briefly, $\mathfrak{N}ro$) sets, nano θ (resp. nano δ interior (resp. closure)) (briefly, $\mathfrak{N}int_\theta(A)$ (resp. $\mathfrak{N}cl_\theta(A)$, $\mathfrak{N}int_\delta(A)$, $\mathfrak{N}cl_\delta(A)$)) and also nano θ (resp. δ open (resp. closed)) (briefly, $\mathfrak{N}\theta o$ (resp. $\mathfrak{N}\theta c$, $\mathfrak{N}\delta o$, $\mathfrak{N}\delta c$)) sets were introduced in [4, 7, 11, 12]. Nano δ-pre (resp. δ-semi, e, $\mathcal{M}$, θ-pre &θ-semi) open (briefly $\mathfrak{N}\delta\mathcal{P}o$ (resp. $\mathfrak{N}\delta\mathcal{S}o$, $\mathfrak{N}eo$, $\mathfrak{N}\mathcal{M}o$, $\mathfrak{N}\theta\mathcal{P}o\&\mathfrak{N}\theta\mathcal{S}o$)), nano δ-pre (resp. δ-semi, e, $\mathcal{M}$&θ-semi) interior (briefly, $\mathfrak{N}\mathcal{P}int_\delta(K)$ (resp. $\mathfrak{N}\mathcal{S}int_\delta(K)$, $\mathfrak{N}eint(K)$, $\mathfrak{N}\mathcal{M}int(K)\&\mathfrak{N}\mathcal{S}int_\theta(K)$)), nano δ-pre (resp. δ-semi, e, $\mathcal{M}$&θ-semi) closure (briefly, $\mathfrak{N}\mathcal{P}cl_\delta(K)$ (resp. $\mathfrak{N}\mathcal{S}cl_\delta(K)$, $\mathfrak{N}ecl(K)$, $\mathfrak{N}\mathcal{M}cl(K)\&\mathfrak{N}\mathcal{S}cl_\theta(K)$)) were introduced in [10, 11, 12]. The collection of all $\mathfrak{N}\delta\mathcal{P}o$ (resp. $\mathfrak{N}\delta\mathcal{S}o$, $\mathfrak{N}eo$, $\mathfrak{N}\mathcal{M}o$ and $\mathfrak{N}\theta\mathcal{S}o$) sets is denoted by $\mathfrak{N}\delta\mathcal{P}O(U,\tau_R(P))$, (resp. $\mathfrak{N}\delta\mathcal{S}O(U,\tau_R(P))$, $\mathfrak{N}eO(U,\tau_R(P))$, $\mathfrak{N}\mathcal{M}O(U,\tau_R(P))$ and $\mathfrak{N}\theta\mathcal{S}O(U,\tau_R(P))$) and the collection of all nano δ-pre (resp. δ-semi, e, nano $\mathcal{M}$&θ-semi) closed (briefly, $\mathfrak{N}\delta\mathcal{P}c$ (resp. $\mathfrak{N}\delta\mathcal{S}c$, $\mathfrak{N}ec$, $\mathfrak{N}\mathcal{M}c\&\mathfrak{N}\theta\mathcal{S}c$)) sets is denoted by $\mathfrak{N}\delta\mathcal{P}C(U,\tau_R(P))$, (resp. $\mathfrak{N}\delta\mathcal{S}C(U,\tau_R(P))$, $\mathfrak{N}eC(U,\tau_R(P))$, $\mathfrak{N}\mathcal{M}C(U,\tau_R(P))\&\mathfrak{N}\theta\mathcal{S}C(U,\tau_R(P))$).

Nano generalized (resp. θ, θ semi, δ, δ semi, δ pre, e&$\mathcal{M}$) closed (briefly, $\mathfrak{N}gc$, $\mathfrak{N}g\theta c$, $\mathfrak{N}g\theta\mathcal{S}c$, $\mathfrak{N}g\delta c$, $\mathfrak{N}g\delta\mathcal{S}c$, $\mathfrak{N}g\delta\mathcal{P}c$, $\mathfrak{N}gec\&\mathfrak{N}g\mathcal{M}c$) were introduced in [1, 2, 4, 5, 9]. A subset K of a nano generalized (resp. θ, θ semi, δ, δ semi &δ pre) open [1, 2, 4, 5] (briefly, $\mathfrak{N}go$(resp. $\mathfrak{N}g\theta o$, $\mathfrak{N}g\theta\mathcal{S}o$, $\mathfrak{N}g\delta o$, $\mathfrak{N}g\delta\mathcal{S}o\&\mathfrak{N}g\delta\mathcal{P}o$)) if its complement K^C is nano generalized (resp. θ, θ semi, δ, δ semi and δ pre) closed (briefly, $\mathfrak{N}gc$(resp. $\mathfrak{N}g\theta c$, $\mathfrak{N}g\theta\mathcal{S}c$, $\mathfrak{N}g\delta c$, $\mathfrak{N}g\delta\mathcal{S}c\&\mathfrak{N}g\delta\mathcal{P}c$)). The system of all nano generalized (resp. θ, θ semi, δ, δ semi, δ pre, $\mathfrak{N}ge$ and $\mathfrak{N}g\mathcal{M}$) open sets of $(U,\tau_R(P))$ is denoted by $\mathfrak{N}GO(U,P)$ (resp. $\mathfrak{N}G\theta O(U,P)$, $\mathfrak{N}G\theta\mathcal{S}O(U,P)$, $\mathfrak{N}G\delta O(U,P)$, $\mathfrak{N}G\delta\mathcal{S}O(U,P)$, $\mathfrak{N}G\delta\mathcal{P}O(U,P)$, $\mathfrak{N}GeC(U,P)\&\mathfrak{N}G\mathcal{M}C(U,P)$). Nano continuous and irresolute (briefly, $\mathfrak{N}Cts$ and $\mathfrak{N}Irr$) were introduced by [3, 8]. Nano dense, nano locally closed, (resp. continuous and irresolute) (briefly, $\mathfrak{N}lc$ (resp. $\mathfrak{N}lCts$ and $\mathfrak{N}lIrr$)), nano submaximal (briefly, $\mathfrak{N}submax$) and nano door space were introduced [2, 13, 14].

II. NANO M LOCALLY CLOSED SETS

In this section the three forms of $\mathfrak{N}lc$ sets denoted by $\mathfrak{N}\mathcal{M}LC(U,P)$, $\mathfrak{N}\mathcal{M}LC^*(U,P)\&\mathfrak{N}\mathcal{M}LC^{**}(U,P)$ are introduced and obtained its properties.

Definition 2.1 A subset K of a $\mathfrak{N}ts(U,\tau_R(P))$ is called Nano $\mathcal{M}$ locally closed (resp. closed *& closed **) (briefly, $\mathfrak{N}\mathcal{M}lc$ (resp. $\mathfrak{N}\mathcal{M}lc^*\&\mathfrak{N}\mathcal{M}lc^{**}$)) set if $K = G \cap F$, G is $\mathfrak{N}\mathcal{M}o$ (resp. $\mathfrak{N}\mathcal{M}o\&\mathfrak{N}o$) and F is $\mathfrak{N}\mathcal{M}c$ (resp. $\mathfrak{N}c\&\mathfrak{N}\mathcal{M}c$) in $(U,\tau_R(P))$.

The class of all $\mathfrak{N}\mathcal{M}lc$ (resp. $\mathfrak{N}\mathcal{M}lc^*\&\mathfrak{N}\mathcal{M}lc^{**}$) sets is denoted by $\mathfrak{N}\mathcal{M}LC(U,P)$ (resp. $\mathfrak{N}\mathcal{M}LC^*(U,P)\&\mathfrak{N}\mathcal{M}LC^{**}(U,P)$).

Theorem 2.1 Let $(U,\tau_R(P))$ and $(V,\sigma_{R'}(Q))$ be $\mathfrak{N}ts$'s. Then every $\mathfrak{N}lc$ (resp. $\mathfrak{N}\mathcal{M}lc^*\&\mathfrak{N}\mathcal{M}lc^{**}$) set is $\mathfrak{N}\mathcal{M}lc$ set, but converse is not.

Proof. Let $K = G \cap F$ be $\mathfrak{N}lc$ set where G is $\mathfrak{N}o\&F$ is $\mathfrak{N}c$ in U. Because each $\mathfrak{N}o$ set is $\mathfrak{N}\mathcal{M}o$ set &$\mathfrak{N}c$ is $\mathfrak{N}\mathcal{M}c$ set. Hence K is $\mathfrak{N}\mathcal{M}lc$ in U. The other results can be proved in the similar manner

Example 2.1 Let $U = \{e,d,c,b,a\}$ with $U/R = \{\{b,a\},\{c\},\{e,d\}\}\&P = \{c,a\}$. The $\mathfrak{N}t\tau_R(P) = \{U,\phi,\{c\},\{b,a\},\{c,b,a\}\}$.

Revised Manuscript Received on February 28, 2020.
* Correspondence Author
A. Padma, Department of Mathematics, Annai Women's College, Karur, Tamil Nadu, India. E-mail: cbepadma22@gmail.com
M. Saraswathi, Department of Mathematics, Kandaswamy Kandar's College, P-velur, Tamil Nadu, India. E-mail: vimesh.sarash75@gmail.com
A. Vadivel*, PG and Research, Department of Mathematics, Government Arts College (Autonomous), Karur - 639 005; Department of Mathematics, Annamalai University, Annamalai Nagar, India. E-mail: avmaths@gmail.com
M. Seenivasan, Department of Mathematics, Annamalai University, Annamalai Nagar, India. E-mail: emseeni@rediffmail.com

The $\mathfrak{N}s\{e,d,b,a\}$ (resp. $\{e,d,c,a\}$) is $\mathfrak{N}\mathcal{M}lc$ set but not $\mathfrak{N}lc$ and $\mathfrak{N}\mathcal{M}lc^{**}$ (resp. $\mathfrak{N}\mathcal{M}lc^{*}$) set.

Theorem 2.2 For $K \subset U$, the conditions

1. $K \in \mathfrak{N}\mathcal{M}LC^{*}(U,P)$.
2. $K = G \cap \mathfrak{N}cl(K)$ for some $\mathfrak{N}\mathcal{M}o$ set G.
3. $\mathfrak{N}cl(K) - K$ is $\mathfrak{N}\mathcal{M}c$.
4. $K \cup (U - \mathfrak{N}cl(K))$ is $\mathfrak{N}\mathcal{M}o$ are equivalent.

Proof. (i) $\Rightarrow$ (ii): Let $K \in \mathfrak{N}\mathcal{M}LC^{*}(U,P)$. Then $\exists \mathfrak{N}\mathcal{M}o$ set $G \& \mathfrak{N}c$ set $F \ni K = G \cap F$. Since $K \subseteq G \& K \subseteq \mathfrak{N}cl(K)$ we have $K \subseteq G \cap \mathfrak{N}cl(K)$. Conversely, since $K \subseteq F$, $\mathfrak{N}cl(K) \subseteq \mathfrak{N}cl(F)$, $F \supseteq \mathfrak{N}cl(K)$, $\mathfrak{N}cl(K) \cap G \subseteq F \cap G = K$, $G \cap \mathfrak{N}cl(K) \subseteq K$. Thus $K = G \cap \mathfrak{N}cl(K)$.

(ii) $\Rightarrow$ (i): Let $K = G \cap \mathfrak{N}cl(K)$ for some $\mathfrak{N}\mathcal{M}o$ set G. Clearly $\mathfrak{N}cl(K)$ is $\mathfrak{N}c \&$ hence $K = G \cap \mathfrak{N}cl(K) \in \mathfrak{N}\mathcal{M}LC^{*}(U,P)$, $K \in \mathfrak{N}\mathcal{M}LC^{*}(U,P)$.

(iii) $\Rightarrow$ (iv): Let $P = \mathfrak{N}cl(K) - K$. Then P is $\mathfrak{N}\mathcal{M}c$ by assumption $\& U - P = U \cap (U - (\mathfrak{N}cl(K) - K)) = K \cup (U - \mathfrak{N}cl(K))$. But $U - P$ is $\mathfrak{N}\mathcal{M}o$. Therefore $K \cup (U - \mathfrak{N}cl(K))$ is $\mathfrak{N}\mathcal{M}o$.

(iv) $\Rightarrow$ (iii): Let $Q = K \cup (U - \mathfrak{N}cl(K))$. Then Q is $\mathfrak{N}\mathcal{M}o$. So $U - Q$ is $\mathfrak{N}\mathcal{M}c \& U - Q = U - (U \cup K - (\mathfrak{N}cl(K))) = \mathfrak{N}cl(K) - K$. Thus $\mathfrak{N}cl(K) - K$ is $\mathfrak{N}\mathcal{M}c$.

(iv) $\Rightarrow$ (ii): Let $G = K \cup (U - \mathfrak{N}cl(K))$. Then G is $\mathfrak{N}\mathcal{M}o$. Now $G \cap \mathfrak{N}cl(K) = [K \cup (U - \mathfrak{N}cl(K))] \cap \mathfrak{N}cl(K) = [\mathfrak{N}cl(K) \cap K] \cup [\mathfrak{N}cl(K) \cap (U - \mathfrak{N}cl(K))] = K \cup \phi = K$. Therefore, $K = G \cap \mathfrak{N}cl(K)$.

(ii) $\Rightarrow$ (iv): Let $K = G \cap \mathfrak{N}cl(K)$ for some $\mathfrak{N}\mathcal{M}o$ set G. Then $K \cup (U - \mathfrak{N}cl(K)) = (G \cap \mathfrak{N}cl(K)) \cup (U - \mathfrak{N}cl(K)) = G \cap U = G$, which is $\mathfrak{N}\mathcal{M}o$. Thus $K \cup (U - \mathfrak{N}cl(K))$ is $\mathfrak{N}\mathcal{M}o$.

Theorem 2.3 Let $K \subset U \& K \in \mathfrak{N}\mathcal{M}LC^{**}(U,P)$, then $K = G \cap \mathfrak{N}\mathcal{M}cl(K)$ for some $\mathfrak{N}o$ set G.

Proof. Let $K \in \mathfrak{N}\mathcal{M}LC^{**}(U,P)$. Then $\exists \mathfrak{N}o$ set $G \& \mathfrak{N}\mathcal{M}c$ set $F \ni K = G \cap F$. Since $K \subseteq G \& K \subseteq \mathfrak{N}\mathcal{M}cl(K) \&$ hence $K \subseteq G \cap \mathfrak{N}\mathcal{M}cl(K)$. Conversely, if $x \in G \cap \mathfrak{N}\mathcal{M}cl(K)$ then $x \in G \& x \in \mathfrak{N}\mathcal{M}cl(K) \subseteq F$. So $x \in G \cap F = K$. Hence $G \cap \mathfrak{N}\mathcal{M}cl(K) \subseteq K$. Therefore $K = G \cap \mathfrak{N}\mathcal{M}cl(K)$.

Theorem 2.4 Let $K \subset U \&$ if K is $\mathfrak{N}lc$ set then K is $\mathfrak{N}\mathcal{M}lc^{*}$ set or $\mathfrak{N}\mathcal{M}lc^{**}$ set.

Theorem 2.5 Let $K \& L$ be subsets of $U \& \mathfrak{N}\mathcal{M}O(U,P)$ is closed under finite intersection (briefly, f.i.). If $K, L \in \mathfrak{N}\mathcal{M}LC^{*}(U,P)$ then $K \cap L \in \mathfrak{N}\mathcal{M}LC^{*}(U,P)$.

Proof. Let $K, L \in \mathfrak{N}\mathcal{M}LC^{*}(U,P)$.

Then by Theorem 2.2, $\exists \mathfrak{N}\mathcal{M}o$ sets $P \& Q \ni K = P \cap \mathfrak{N}cl(K) \& L = Q \cap \mathfrak{N}cl(L)$. Therefore $K \cap L = P \cap \mathfrak{N}cl(K) \cap Q \cap \mathfrak{N}cl(L) = P \cap Q \cap \mathfrak{N}cl(K) \cap \mathfrak{N}cl(L)$ where $P \cap Q$ is $\mathfrak{N}\mathcal{M}o \& \mathfrak{N}cl(K) \cap \mathfrak{N}cl(L)$ is $\mathfrak{N}c$. Thus $K \cap L \in \mathfrak{N}\mathcal{M}LC^{*}(U,P)$.

Theorem 2.6 Let $K \& L$ be subsets of $(U, \tau_R(P)) \& \mathfrak{N}\mathcal{M}O(U,P)$ and $\mathfrak{N}\mathcal{M}C(U,P)$ is closed under f.i. If $K, L \in \mathfrak{N}\mathcal{M}LC(U,P)$ then $K \cap L \in \mathfrak{N}\mathcal{M}LC(U,P)$.

Proof. Let $K, L \in \mathfrak{N}\mathcal{M}LC(U,P)$. Then $\exists \mathfrak{N}\mathcal{M}o$ sets $P \& Q \ni K = P \cap \mathfrak{N}cl(K) \& L = Q \cap \mathfrak{N}cl(L)$. Therefore $K \cap L = P \cap \mathfrak{N}cl(K) \cap Q \cap \mathfrak{N}cl(L) = P \cap Q \cap \mathfrak{N}cl(K) \cap \mathfrak{N}cl(L)$ where $P \cap Q$ is $\mathfrak{N}\mathcal{M}o \& \mathfrak{N}cl(K) \cap \mathfrak{N}cl(L)$ is $\mathfrak{N}c$. Since every $\mathfrak{N}c$ is $\mathfrak{N}\mathcal{M}c$, $\mathfrak{N}cl(K) \cap \mathfrak{N}cl(L)$ is $\mathfrak{N}\mathcal{M}c$. So $K \cap L \in \mathfrak{N}\mathcal{M}LC(U,P)$.

Theorem 2.7 Let $K \& L$ be subsets of a $\mathfrak{N}ts(U, \tau_R(P)) \& \mathfrak{N}\mathcal{M}C(U,P)$ is closed under finite intersection. If $K \in \mathfrak{N}\mathcal{M}LC^{**}(U,P) \& L$ is $\mathfrak{N}o$ or $\mathfrak{N}c$ then $K \cap L \in \mathfrak{N}\mathcal{M}LC^{**}(U,P)$.

Proof. Let $K \in \mathfrak{N}\mathcal{M}LC^{**}(U,P)$. Then $\exists \mathfrak{N}o$ sets $G \& \mathfrak{N}\mathcal{M}c$ set F such that $K = G \cap F$. So $K \cap L = G \cap F \cap L$. Consider L is $\mathfrak{N}o$, then $K \cap L = (G \cap L) \cap F$ where $G \cap L$ is $\mathfrak{N}o$. This shows that $K \cap L \in \mathfrak{N}\mathcal{M}LC^{**}(U,P)$. Consider L is $\mathfrak{N}c$, then $F \cap L$ is $\mathfrak{N}\mathcal{M}c$. Clearly $K \cap L \in \mathfrak{N}\mathcal{M}LC^{**}(U,P)$.

Theorem 2.8 Let $K \& L$ be subsets of a $\mathfrak{N}ts(U, \tau_R(P)) \& \mathfrak{N}\mathcal{M}O(U,P) \& \mathfrak{N}\mathcal{M}C(U,P)$ is closed under arbitrary intersection. If $K \in \mathfrak{N}\mathcal{M}LC(U,P) \& L$ is $\mathfrak{N}\mathcal{M}o$ or $\mathfrak{N}c$ then $K \cap L \in \mathfrak{N}\mathcal{M}LC(U,P)$.

Proof. Suppose $K \in \mathfrak{N}\mathcal{M}LC(U,P)$. Then $\exists \mathfrak{N}\mathcal{M}o$ sets $G \& \mathfrak{N}\mathcal{M}c$ set $F \ni K = G \cap F$. So $K \cap L = G \cap F \cap L$. If L is $\mathfrak{N}\mathcal{M}o$, then $K \cap L = (G \cap L) \cap F$ where $G \cap L$ is $\mathfrak{N}\mathcal{M}o$. This implies $K \cap L = \mathfrak{N}\mathcal{M}LC(U,P)$. If L is $\mathfrak{N}c$, $(F \cap L)$ is $\mathfrak{N}\mathcal{M}c$, Since every $\mathfrak{N}c$ is $\mathfrak{N}\mathcal{M}c$. Therefore $K \cap L \in \mathfrak{N}\mathcal{M}LC(U,P)$.

Theorem 2.9 Let $K \& L$ be subsets of a $\mathfrak{N}ts(U, \tau_R(P)) \& \mathfrak{N}\mathcal{M}O(U,P) \& \mathfrak{N}\mathcal{M}C(U,P)$ is closed under arbitrary intersection. If $K \in \mathfrak{N}\mathcal{M}LC^{*}(U,P) \& L$ is $\mathfrak{N}\mathcal{M}o$ or $\mathfrak{N}c$ then $K \cap L \in \mathfrak{N}\mathcal{M}LC^{*}(U,P)$.

Proof. Let $K \in \mathfrak{N}\mathcal{M}LC^{*}(U,P)$. Then $\exists \mathfrak{N}\mathcal{M}o$ sets $G \& \mathfrak{N}\mathcal{M}c$ set $F \ni K = G \cap F$. So $K \cap L = G \cap F \cap L$. If L is $\mathfrak{N}\mathcal{M}o$, then $K \cap L = (G \cap L) \cap F$ where $G \cap L$ is $\mathfrak{N}\mathcal{M}o$. This implies $K \cap L \in \mathfrak{N}\mathcal{M}LC^{*}(U,P)$. If L is $\mathfrak{N}c$, $(F \cap L)$ is $\mathfrak{N}c$. Therefore $K \cap L \in \mathfrak{N}\mathcal{M}LC^{*}(U,P)$.

Definition 2.2 A $\mathfrak{N}s$ of a $\mathfrak{N}tsU$ is called nano $\mathcal{M}$ dense if $\mathfrak{N}\mathcal{M}cl(K) = U$.

Definition 2.3 A $\mathfrak{N}ts(U, \tau_R(P))$ is said to be $\mathfrak{N}\mathcal{M}$ submaximal (briefly, $\mathfrak{N}\mathcal{M}submax$) if every Nano $\mathcal{M}$ dense subset of $(U, \tau_R(P))$ is $\mathfrak{N}\mathcal{M}o$ in $(U, \tau_R(P))$.

Theorem 2.10 Every $\mathfrak{N}submax$ space is $\mathfrak{N}\mathcal{M}submax$ but not conversely.

Proof. Let U be a $\mathfrak{N}submax\&K$ be Nano dense subset of U. Then by assumption K is $\mathfrak{N}o$ in U. But every $\mathfrak{N}o$ set is $\mathfrak{N}\mathcal{M}o$ and so K is $\mathfrak{N}\mathcal{M}o$ in U. Therefore U is $\mathfrak{N}\mathcal{M}submax$.

Example 2.2 In Example 2.1, the $\mathfrak{N}s\{e,c,b,a\}$ is $\mathfrak{N}\mathcal{M}submax$ but not $\mathfrak{N}submax$.

Theorem 2.11 A $\mathfrak{N}ts(U,\tau_R(P))$ is $\mathfrak{N}\mathcal{M}submax$ iff $\mathfrak{N}\mathcal{M}LC^*(U,P)=\mathcal{P}(U)$.

Proof. Suppose $K\in\mathcal{P}(U)\&$ let $G=K\cup(U-\mathfrak{N}cl(K))$. Then $\mathfrak{N}cl(G)=\mathfrak{N}cl(K\cup(U-\mathfrak{N}cl(K)))=U$, $\mathfrak{N}cl(G)=U$, G is Nano dense subset of U. Since U is $\mathfrak{N}\mathcal{M}submax$, G is $\mathfrak{N}\mathcal{M}o$ in U. i.e., $K\cup(U-\mathfrak{N}cl(K))$ is $\mathfrak{N}\mathcal{M}o$. By Theorem 2.2, $K\in\mathfrak{N}\mathcal{M}LC^*(U,P)\&$ hence $\mathfrak{N}\mathcal{M}LC^*(U,P)=\mathcal{P}(U)$.

Conversely, consider K be Nano dense subset of $U\&$ let $\mathfrak{N}\mathcal{M}LC^*(U,P)=\mathcal{P}(U)$. Then by hypothesis, $K\cup(U-\mathfrak{N}cl(K))=K\cup\emptyset=K$. By Theorem 2.2, K is $\mathfrak{N}\mathcal{M}o$ in $U\&K\in\mathfrak{N}\mathcal{M}LC^*(U,P)$. Hence U is $\mathfrak{N}\mathcal{M}submax$.

III. NANO M CONTINUOUS MAPS

Definition 3.1 A map $f:(U,\tau_R(P))\to(V,\sigma_{R'}(Q))$ is called $\mathfrak{N}\mathcal{M}lc$ (resp. $\mathfrak{N}\mathcal{M}LC^*\&\mathfrak{N}\mathcal{M}LC^{**}$)-continuous (briefly, $\mathfrak{N}\mathcal{M}lc$-$Cts$ (resp. $\mathfrak{N}\mathcal{M}lc^*$-$Cts\&\mathfrak{N}\mathcal{M}lc^{**}$-$Cts$)) function if the inverse image of every $\mathfrak{N}o$ set in V is $\mathfrak{N}\mathcal{M}lc$ (resp. $\mathfrak{N}\mathcal{M}lc^*\&\mathfrak{N}\mathcal{M}lc^{**}$) set.

Definition 3.2 A map $f:(U,\tau_R(P))\to(V,\sigma_{R'}(Q))$ is called $\mathfrak{N}\mathcal{M}lc$ (resp. $\mathfrak{N}\mathcal{M}lc^*\&\mathfrak{N}\mathcal{M}lc^{**}$)-irresolute (briefly, $\mathfrak{N}\mathcal{M}lc$-$irr$ (resp. $\mathfrak{N}\mathcal{M}lc^*$-$irr\&\mathfrak{N}\mathcal{M}lc^{**}$-$irr$)) function if the inverse image of every $\mathfrak{N}\mathcal{M}lc$ (resp. $\mathfrak{N}\mathcal{M}lc^*\&\mathfrak{N}\mathcal{M}lc^{**}$) set in V is $\mathfrak{N}\mathcal{M}lc$ (resp. $\mathfrak{N}\mathcal{M}lc^*\&\mathfrak{N}\mathcal{M}lc^{**}$) set.

Theorem 3.1 Let $f:(U,\tau_R(P))$ and $(V,\sigma_{R'}(Q))$ be $\mathfrak{N}ts$'s. Then every $\mathfrak{N}lc$ - Cts (resp. $\mathfrak{N}\mathcal{M}lc^*$-$Cts\&\mathfrak{N}\mathcal{M}lc^{**}$-$Cts$) is $\mathfrak{N}\mathcal{M}lc$-$Cts$. But not conversely.

Proof. Assume that f is $\mathfrak{N}lc$-Cts. Let K be $\mathfrak{N}o$ in V. Then $f^{-1}(K)$ is $\mathfrak{N}lc$ in U. But each $\mathfrak{N}lc$ set is $\mathfrak{N}\mathcal{M}lc$ set. Therefore $f^{-1}(K)$ is $\mathfrak{N}\mathcal{M}lc$ set in U. Hence f is $\mathfrak{N}\mathcal{M}lc$-$Cts$. The others are in a similar manner

Example 3.1 Let $U=V=\{v,w,x,y,z\}$ with $U/R=\{\{x\},\{v,w\},\{y,z\}\}$, $P=\{v,x\},\tau_R(P)=\{U,\phi,\{x\},\{v,w\},\{v,w,x\}\}$, $V/R'=\{\{y\},\{v,w\},\{x,z\}\}$ and $Q=\{y,z\}$, $\tau'_R(Q)=\{U,\phi,\{v,w\},\{x,z\},\{v,w,x,z\}\}$ Then, the mapping $f:(U,\tau_R(P))\to(V,\tau_{R'}(Q))$ defined by $f(v)=w,f(w)=z,f(x)=y,f(y)=v$ and $f(z)=x$ is $\mathfrak{N}\mathcal{M}lcCts$ but not $\mathfrak{N}lcCts$, $\mathfrak{N}\mathcal{M}lc^*Cts\&\mathfrak{N}\mathcal{M}lc^{**}Cts$ the set $\{w,x,z\}$ is $\mathfrak{N}o$ in V but $f^{-1}(\{w,x,z\})=\{v,w,x\}$ is not $\mathfrak{N}lc$, $\mathfrak{N}lc^*\&\mathfrak{N}lc^{**}$ in U.

Theorem 3.2 Let f be a function $\&$ if f is $\mathfrak{N}lc$-Cts then f is $\mathfrak{N}\mathcal{M}lc^*$-$Cts\&\mathfrak{N}\mathcal{M}lc^{**}$-$Cts$.

Proof. Let K be a $\mathfrak{N}o$ in V. Then by definition, $f^{-1}(K)$ is $\mathfrak{N}lc$ in V. By Theorem 2.4, $f^{-1}(K)$ is $\mathfrak{N}\mathcal{M}lc^*\&f^{-1}(K)$ is $\mathfrak{N}\mathcal{M}lc^{**}$ set. Hence f is $\mathfrak{N}\mathcal{M}lc^*$-$Cts\&\mathfrak{N}\mathcal{M}lc^{**}$-$Cts$.

Theorem 3.3 Let f be a function $\&$ if f is $\mathfrak{N}\mathcal{M}lc$ - irr ($\mathfrak{N}\mathcal{M}lc^*$ - irr or $\mathfrak{N}\mathcal{M}lc^{**}$ - irr) then f is $\mathfrak{N}\mathcal{M}lc$-$Cts$ ($\mathfrak{N}\mathcal{M}lc^*$-$Cts\&\mathfrak{N}\mathcal{M}lc^{**}$-$Cts$).

Proof. Let G be $\mathfrak{N}o$ set in W. Because every $\mathfrak{N}o$ set is $\mathfrak{N}lc$ set [2] $\&$ by Theorem 2.1(i), every $\mathfrak{N}lc$ set is $\mathfrak{N}\mathcal{M}lc$ set, G is $\mathfrak{N}\mathcal{M}lc$. Because f is $\mathfrak{N}\mathcal{M}lc$-$irr$, $f^{-1}(G)$ is $\mathfrak{N}\mathcal{M}lc$ in U ($\mathfrak{N}\mathcal{M}lc^*$-$irr$ or $\mathfrak{N}\mathcal{M}lc^{**}$-$irr$). Hence f is $\mathfrak{N}\mathcal{M}lc$-$Cts$ ($\mathfrak{N}\mathcal{M}lc^*$-$Cts\&\mathfrak{N}\mathcal{M}lc^{**}$-$Cts$).

Definition 3.3 A $\mathfrak{N}ts(U,\tau_R(P))$ is called $\mathfrak{N}\mathcal{M}$ door space if each subset of U is either $\mathfrak{N}\mathcal{M}o$ or $\mathfrak{N}\mathcal{M}c$ in U.

Theorem 3.4 Any function defined from a Nano $\mathcal{M}$ door space into a $\mathfrak{N}ts$ is $\mathfrak{N}\mathcal{M}lc$-$irr$.

Proof. Let U be Nano $\mathcal{M}$ door space $\&V$ be $\mathfrak{N}ts$. Let $f:(U,\tau_R(P))\to(V,\sigma_{R'}(Q))$ be a function. Let K be $\mathfrak{N}\mathcal{M}lc$ in V. Then $f^{-1}(K)$ is either $\mathfrak{N}o$ or a $\mathfrak{N}c$. Since every $\mathfrak{N}o$ set or $\mathfrak{N}c$ set is $\mathfrak{N}lc$ set [2] $\&$ by Theorem 2.1(i), every $\mathfrak{N}lc$ set is $\mathfrak{N}\mathcal{M}lc$ set. In both cases, $f^{-1}(K)$ is $\mathfrak{N}\mathcal{M}lc$ set. Hence f is $\mathfrak{N}\mathcal{M}lc$-$irr$.

Theorem 3.5 If $f:(U,\tau_R(P))\to(V,\sigma_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}lc$ - irr (resp. $\mathfrak{N}\mathcal{M}lc^*$ - irr ($\mathfrak{N}\mathcal{M}lc^{**}$ - irr)) $\&g:(V,\sigma_{R'}(Q))\to(W,\mu_{R''}(R))$ is $\mathfrak{N}lc$ - Cts, then $g\circ f:(U,\tau_R(P))\to(W,\mu_{R''}(R))$ is $\mathfrak{N}\mathcal{M}lc$-$Cts$ (resp. $\mathfrak{N}\mathcal{M}lc^*$-$Cts$ ($\mathfrak{N}\mathcal{M}lc^{**}$-$Cts$)).

IV. CONCLUSION

In our paper, the concepts of $\mathfrak{N}\mathcal{M}lc$ sets, $\mathfrak{N}\mathcal{M}lc$ continuous $\&\mathfrak{N}\mathcal{M}lc$ irresolute functions are introduced and some of its characteristics are discussed.

REFERENCES

1. S. Bamini, M. Saraswathi, A. Vadivel and G. Saravanakumar, *Generalizations of nano δ-closed sets in nano topological spaces*, Adalya J., **8** (11) (2019), 488-493.
2. K. Bhuvaneshwari and K. Mythili Gnanapriya, *Nano Generalized closed sets*, Int. J. Sci. & Res. Pub., **4**(5) 2014, 1-3.
3. K. Bhuvaneswari and A. Ezhilarasi, *Nano semi generalized irresolute maps in Nano topological spaces*, IJMA, **7** (3) (2016), 68-75.
4. Carmel Richard, *Studies on nano topological spaces*, Ph.D Thesis, MKU, India, (2013).
5. P. Dhanasekaran, A. Vadivel, G. Saravanakumar and M. Angayarkanni, *Generalizations of Nano θ- closed sets in Nano Topological Spaces*, J. of Information and Computational Science, **9** (11) (2019), 669-675.
6. Erdal Ekici, On e-open sets, DP^*-sets and DPE^*-sets and decomposition of continuity, Ara. Jou. Scie., **33** (2) (2008), 269-282.
7. M. L. Thivagar, and Carmel Richard, *On Nano forms of weakly open sets*, Int. J. of Mathematics and Statistics Inv., **1** 2013, 31-37.
8. M. L. Thivagar and Carmel Richard, *On nano continuity*, Math. Theory and Modelling, **3** (7) (2013), 32-37.

9. P. Manivannan, A. Vadivel and V. Chandrasekar, *Nano generalized e-closed sets in nano topological spaces*, Submitted.
10. A. Padma, M. Saraswathi, A. Vadivel and G. Saravanakumar, *New Notions of Nano $\mathcal{M}$-open Sets*, Malaya Journal of Matematik, **S** (1) (2019), 656-660.
11. V. Pankajam and K. Kavitha, *δopen sets and δ nano continuity in δ nano topological space*, Int. J. Ino. Sci. and Res. Tec., **2** (12) (2017), 110-118.
12. A. Revathy and I. Gnanambal, *Int. Jr. of Engineering, Contemporary Math. & Sci.*, 1 **(2)** (2015), 1-6.
13. P. Sathishmohan, On θg^* closed sets in topological spaces, J. of Math. and Infornmatics, **13** (2018), 65-80.
14. S. B. Shalini, *A study on nano generalized β closed sets and its maps in nano topological spaces*, Ph.D Thesis, Nirmala College for Women(A), Coimbatore, (2018).
15. M. Sujatha, M. Angayarkanni, *New Notions via Nano θ open Sets With an Application in Diagnosis of Type - II Diabetics*, Adalya Journal, **8** (10) (2019), 643-651.

AUTHORS PROFILE

A. Padma, I have 12 years of experience in teaching. At present I am pursuing my Ph.D from 2016 onwards in Kandasamy Kandar's College, P.Velur, Namakkal. I have completed M.Phil Mathematics, Avinashilingam University, Coimbatore; M.Sc Mathematics in P.S.G College of Arts and Science, Coimbatore; B.Sc Mathematics in Nirmala College for Women, Coimbatore. My area of specialization is topology. I am interested in doing further research in applications of topology in different fields.

M. Saraswathi, has published 25 research publications both in national and international journals to his credit. Under her guidance she has guiding 4 Ph.D scholars. At present she is Assistant Professor, Department of Mathematics, Kandaswami Kandar's College, P-Velur, Namakkal, Tamil Nadu, India.

A Vadivel, obtained his early and collegiate education respectively from Kandaswami Kandar's Boys School and Kandaswami Kandar's College, P-Velur, Namakkal. He obtained doctoral degree from Annamalai University, Annamalai Nagar. Under his able guidance 8 scholars obtained their doctoral degrees. Dr. A. Vadivel has published 180 research publications both in national and international journals to his credit. He serves as referee for 5 peer reviewed international journals. He is Assistant Professor in the Department of Mathematics, Annamalai University, Annamalai Nagar, at present he is deputed to Assistant Professor in the Department of Mathematics, Government Arts College (A), Karur-5, Tamil Nadu, India

Dr. M. Seenivasan is working as Associate Professor in Department of Mathematics, Annamalai University. He was awarded Ph.D degree by Annamalai University in the year 2012. He was 17 years of teaching and 15 years of research experience. His area of research is Stochastic Processes, Graph theory and Computer Networks. He has published more than 20 research articles in reputed indexed international journals. He has guided 20 M.Phil scholars. At present, he is guiding three Ph.D scholars. He acted as editorial members of more than 13 international journals. He organized four international conferences in abroad. He visited Thailand , Malaysia, Singapore and USA.