International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-4, February 2020

Direct Synthesis Based Fractional Order PID Controller Design for Non - Integer Order Systems with Time Delay

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Abstract: ***Present study focuses on design and implementation of PI$^{\lambda}$D$^{\mu}$ controller to obtain the closed loop response of for non-integer order systems (NIOS). Controller is designed using direct synthesis (DS) method. Performance analysis in terms of IAE, ISE and ITAE is made and compared with that of literature reported methods. Robustness in terms of Maximum Sensitivity (Ms) is also analyzed. Tuning parameters q and λ, are selected with an arbitrary value for set-point trajectory and disturbance rejection. The closed loop response is studied for various non-integer order systems. Tuning parameters with respect to q (adjustable tuning parameter in assumed closed loop transfer function) and λ are arrived at for different case studies, q varying from 0.05 to 0.5 and λ varying from 0.5 to 6. FOMCON tool box of Simulink in MATLAB is employed for the simulation study.***

Keywords : Direct Synthesis, Fractional Order, FOPID Controller, Non-Integer Order Systems, Performance Analysis, and Robustness.

I. INTRODUCTION

Fractional calculus or non-integer calculus has the same age that of integer calculus. It is said that, the foundation for Fractional Calculus was first laid by Leibniz asking about the concept of differential operator to L'Hopital in 1965. Later on, several famous mathematicians made efforts for the development of the theory of fractional calculus. But due to great complexity in solving and lack of physical and geometrical understanding, non-integer calculus was not considered suitable for any applications. In recent past, fractional calculus has been applied in the modeling and control of different kinds of real systems, as is well known and documented in the theory of control or in the literature of application.

Attention is given to Fractional differential equations due to their applications in engineering and science via process control, electro chemistry, visco-elasticity, porous media and electromagnetism theory.

Revised Manuscript Received on February 28, 2020.

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Fractional calculus extends the integer calculus by out spreading the ordinary differential equations to arbitrary order differential equations, i.e. those having fractional orders of derivatives and integrals. The fractional calculus is a name for the theory of derivatives and integrals of arbitrary order which generalize and unify the notions of integer-order differentiation and n-fold integration.

A. Non – Integer Order Systems (NIOS)

A linear Non – Integer order continuous time dynamic system can be expressed by a non-integer differential equation as in (1) of the form:

$$a_m D^{\gamma_m} y(t) + a_{m-1} D^{\gamma_{m-1}} y(t) + a_{m-2} D^{\gamma_{m-2}} y(t) + \ldots + a_0 D^{\gamma_0} y(t) = b_n D^{\delta_n} x(t) + b_{n-1} D^{\delta_{n-1}} x(t) + b_{n-2} D^{\delta_{n-2}} x(t) + \ldots + b_0 D^{\delta_0} x(t) \quad (1)$$

On applying Laplace Transform to Eq(1) with zero initial conditions, Non – Integer transfer function can be obtained as *G(s)*

$$\frac{b_n s^{\delta_n} + b_{n-1} s^{\delta_{n-1}} + b_{n-2} s^{\delta_{n-2}} + \ldots + b_0 s^{\delta_0}}{a_m s^{\gamma_m} + a_{m-1} s^{\gamma_{m-1}} + a_{m-2} s^{\gamma_{m-2}} + \ldots + a_0 s^{\gamma_0}} \quad (2)$$

If the system with proportionate order "r" then transfer function model is

$$G(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{i=n}^{i=0} b_i \left(s^r\right)^i}{\sum_{i=n}^{i=n} a_i \left(s^r\right)^i} \quad (3)$$

In the recent decade, considerable courtesy has been paid towards control systems whose processes and/or controllers are of fractional order. This is mainly due to the fact that many real-time systems are well characterized by fractional-order differential equations, i.e., equations involving arbitrary order derivatives. Well known definitions for the general fractional differential-integro operations are the Riemann- Liouvilleand Caputo definition and the Grünwal – d – Letnikov (GdL) definition [1]. They are represented in Eq (4), Eq (5) and Eq (6) below.

Riemann- Liouville definition for f(t) is

$$D^{\mu} f(t) = \frac{d^m}{dt^m}\left[\frac{1}{\Gamma(m-\mu)}\int_0^t (t-\tau)^{m-\mu-1} f(\tau)\,d\tau, \qquad \ldots\ldots (4)$$

Where $m-1<\mu\leq m \in \mathrm{N}$

Caputo definition for f(t) is

$$D^{\mu} f(t) = \frac{1}{\Gamma(m-\mu)}\int_0^t (t-\tau)^{m-\mu-1} f^{(m)}(\tau)\,d\tau, \qquad ..(5)$$

Where $m-1<\mu\leq m \in \mathrm{N}$

Grünwal – d – Letnikov definition for f (t) is

$$D^{\mu} f(t) = \lim_{h\to\infty}\frac{1}{h^{\mu}}\sum_{i=0}^{\infty}\left[(-1)^i\binom{\mu}{i} f(t-ih)\right] \qquad ..(6)$$

Where $\binom{\mu}{i} = \frac{\Gamma(\mu+1)}{\Gamma(i+1)\Gamma(\mu-i+1)}$

D^{μ}: Fractional differentiation or integration depending on the sign of μ,

$\Gamma(.)$: Euler's Gamma function, h: Finite Sampling Interval

Dynamic systems of fractional order were considered in Podulbny [1] and proposed a design of fractional order controllers for the systems. The method for designing fractional order controllers is based on Laplace Transform formula for a new function of the Mittag-Leffler type. IMC based Fractional Order – 2 DOF controller is proposed by Vinopraba et al [2] to meet the required bandwidth specification by overcoming the disadvantages with IMC based Fractional Order - One Degree of Freedom controller. Fractional order controller cascaded with filter based on Internal Model Control (IMC) method is designed by Dazi Li et al [3] for different types of non-integer order systems with dead time. NIOS considered are of the form as in

$$G_p(s) = \frac{b_1 s^{\beta_1} + b_0}{a_2 s^{\alpha_2} + a_1 s^{\alpha_1} + a_0} e^{-\tau s}, \alpha_2 > \alpha_1, \alpha_2 > \beta_1$$

Logical fractional order controllers are designed Bettayeb and Mansouri [4] for NIOS by using IMC method. They considered that the controller is disintegrated into two transfer functions: a $PI^{v}D^{\mu}$-controller and a simple fractional which is named $PI^{v}D^{\mu}$-FOF-controller. Simulation studies were carried out for NIOS with fractional order < 1 and > 1.

Fractional-Order PI controllers based on fractional calculus and Bode's ideal transfer function is proposed for first-order-plus-dead time process model [5]. Tuning rules were derived for both servo and regulatory problems. The proposed controller has reliably improved performance over other similar controllers and conventional integer PI controllers. Two fractional order proportional integral controller designs are proposed for a class of fractional order systems [6], fractional order proportional integral (FOPI) and fractional order [proportional integral] (FO[PI]) with integer order PID (IOPID) controller. Fractional Order – [Proportional Derivative] (FO-[PD]) controller design for simple NIOS is proposed [7]. Padula and Visioli [8] analyzed that the tuning of fractional order PID controller need more attention when compared to the conventional PID controller. Robustness and performance of the fractional order PID controller thus designed is carried out on first order plus dead time systems. Five design specifications needs to be defined in the fractional order $PI^{\lambda}D^{\mu}$ controller design [9].

FOPID controller is designed by various optimization techniques viz., Tabu Search Algorithm (TSA) [10], Cuckoo Search Algorithm (CSA) based two-degree of freedom fractional order proportional-integral-derivative (2-DOF FOPID) controller [11], Genetic Algorithm (GA) based optimal $^{\alpha}$ PFC (Fractional-Order Predictive Functional Controller) is designed for $^{\alpha}$NMSS (Fractional Order Non-Minimal input- output State Space) model [12], Differential Evolution (DE) algorithm [13] with design specifications in time and frequency domains.

Fuzzy based FOPID controller design Pan and Das [14], gives better performance when compared to conventional PID and Integer Order Fuzzy PID controllers in linear and nonlinear operating regions; Enhanced robust fractional order proportional-plus-integral (ERFOPI) controller based on neural network is proposed by Zhang and Pi [15] for permanent magnet synchronous motor (PMSM). This analysis is based on open-loop gain variation of controlled plant. A novel graphical tuning technique of FOPID controllers for interval fractional order plant family is proposed by [16] with wide-range sets of FOPID controller parameters assuring detailed H_{∞}-norm constraint. Innovative model reduction method and a clear PID tuning rule for the purpose of PID auto-tuning is proposed by Sung et al [17] on the base of a fractional order plus dead time model.

Direct Synthesis based PID controller with a filter is designed for various integrating systems with dead time [18, 19, 20]. In Anil and Padma Sree [18] Maximum Sensitivity (Ms) value (Robustness Parameter) is the tuning parameter and set-point weighting is introduced to minimize overshoot [19]. Application of fractional order systems viz., classical heat transfer wall problem, a continuous stirred tank heater with transportation delays, an industrial scaled primary separation cell, and froth heaters [21]. FOMCON [22] is a new fractional-order modeling and control toolbox for MATLAB. It offers a set of tools for researchers in the field of fractional-order control.

B. Motivation of the Work

As against the integer order PID controller, Fractional order controllers for fractional order systems would give a precise response of a system. Out of several methods available for designing controllers Direct Synthesis (DS) method [18, 19, 20] is found to be better algorithm for designing Integer Order PID controllers. In literature very little focus is found on design of FOPID controllers by DS method. In the present study fractional order controller is designed based on DS method One Non-Integer and Two Non-Integer Order Systems with Time Delay.

C. Conventional PID Controller Vs FOPID Controller

The properly designed and tuned Proportional – Integral – Derivative (PID) controllers have wide applications in process control industry due to its simple structure and can be easily understood and implemented in practice. The maintenance and operation of PID controllers are not so difficult.

The design methods for PID controller are mainly based on frequency domain [23] or time domain performance criteria, The parallel form of conventional PID structure is represented by (7)

$$G_c(s)=K_p+\frac{K_I}{s}+K_D s=K_p\left(+\frac{1}{\tau_I s}+\tau_D s\right) \quad (7)$$

Although conventional PID controller shows good control performance, now-a-days, to improve the system control performance, FOPID controller has drawn attention. The common form of FOPID controller is $PI^{\mu}D^{\upsilon}$ and it provides extra degree of freedom as it has 5 design specifications but in the case of conventional PID controller the design specifications are limited to 3 [9]. The design of Proportional, Integral and Derivative gains are common for FOPID and conventional PID controllers but there is a need to design orders of Integral and Derivative in FOPID whereas it is not necessary in the design of conventional PID controller. Complexity in the design procedure of FOPID increases when compared to conventional PID controller but FOPID is precise when compared to conventional PID controller. The parallel form of FOPID structure is represented by (8):

$$G_c(s)=K_p+\frac{K_I}{s^{\mu}}+K_D s^{\nu}=K_p\left(+\frac{1}{\tau_I s^{\mu}}+\tau_D s^{\nu}\right) \quad (8)$$

Where μ and ν are non-integers

II. PROPOSED METHODOLOGY FOR DESIGN OF DS - FOPID CONTROLLER

A. One Non – Integer Order System with Time Delay (Type – A)

Process transfer function can be considered as

$$G_p(s)=\frac{k}{Ts^{\alpha}+1}e^{-\tau s} \quad (9)$$

The desired closed loop transfer function is considered as Eq (10)

$$\left(\frac{y}{y_r}\right)_d=\frac{\eta s^q+1}{(\lambda s^q+1)^2}e^{-\tau s}\Rightarrow\left(\frac{y}{y_r}\right)_d=\frac{\eta s^{\alpha}+1}{(\lambda s^{\alpha}+1)^2}e^{-\tau s} \quad (10)$$

Using DS method, controller transfer function can be obtained as in Eq (11)

$$G_c(s)=\frac{1}{G_p}\left[\frac{\left(\frac{y}{y_r}\right)_d}{1-\left(\frac{y}{y_r}\right)_d}\right] \quad (11)$$

By using Eq (9) and Eq(10) in Eq (11) then controller transfer function is rewritten as

$$G_c(s)=\frac{Ts^{\alpha}+1}{k}\left[\frac{\frac{\eta s^{\alpha}+1}{(\lambda s^{\alpha}+1)^2}}{1-\left(\frac{\eta s^{\alpha}+1}{(\lambda s^{\alpha}+1)^2}\right)}\right] \quad (12)$$

Equation (13) represents the time delay approximation by first order Pade' approximation.

$$e-\tau s=\frac{1-0.5\tau s}{1+0.5\tau s} \quad (13)$$

Then the controller transfer function can be deduced to (14)

$$G_c(s)=\frac{Ts^{\alpha}+1}{k}\left[\frac{(\eta s^{\alpha}+1)*(1+0.5\tau s)}{s^{2\alpha+1}(0.5\tau\lambda^2)+s^{\alpha+1}(\tau\lambda+0.5\eta\tau)+s^{2\alpha}(\lambda^2)+s^{\alpha}(2\lambda-\eta)+\tau s}\right] \quad (14)$$

For obtaining FO-PID controller, coefficient of $s^{\alpha}$is equated to zero, so

$$(2\lambda-\eta)=0 \text{ i.e., } 2\lambda=\eta \quad (15)$$

Then (15) is re-written in the form of (16)

$$G_c(s)=\frac{1}{k\tau}\frac{(\eta s^{\alpha}+1)*(1+0.5\tau s)}{s}\left[\frac{Ts^{\alpha}+1}{s^{2\alpha}(0.5\lambda^2)+s^{\alpha}(\lambda+0.5\eta)+s^{2\alpha-1}\left(\frac{\lambda^2}{\tau}\right)+1}\right] \quad (16)$$

The resultant DS-FOPID Controller for One Non-Integer Order Time Delay System is in the form

$$G_c(s)=\frac{T+\eta}{k\tau}\left[1+\frac{1}{(T+\eta)s^{\alpha}}+\frac{\eta T}{T+\eta}s^{2\alpha-1}\right]*\left[\frac{(1+0.5\tau s)}{s^{1-\alpha}\left[s^{2\alpha}(0.5\lambda^2)+s^{\alpha}(\lambda+0.5\eta)+s^{2\alpha-1}\left(\frac{\lambda^2}{\tau}\right)+1\right]}\right] \quad (17)$$

B. Two Non – Integer Dissimilar Order Systems With Time Delay (Type – B)

Process transfer function can be considered as

$$G_p(s)=\frac{b_1 s^{\beta_1}+b_0}{a_2 s^{\alpha_2}+a_1 s^{\alpha_1}+a_0}e^{-\tau s} \quad (18)$$

The required closed loop trajectory is considered as

$$\left(\frac{y}{y_r}\right)_d=\frac{\eta s^q+1}{(\lambda s^q+1)^2}e^{-\tau s} \quad (19)$$

Using DS method, transfer function of the controller is written as in (12). By using (18) and (19) in (12) the controller transfer function is rewritten as

$$G_c(s)=\frac{1}{\frac{b_1 s^{\beta_1}+b_0}{a_2 s^{\alpha_2}+a_1 s^{\alpha_1}+a_0}e^{-\tau s}}*\left[\frac{\frac{\eta s^q+1}{(\lambda s^q+1)^2}e^{-\tau s}}{1-\left[\frac{\eta s^q+1}{(\lambda s^q+1)^2}e^{-\tau s}\right]}\right] \quad (20)$$

Equation (20) with Pade' first order approximation for time delay, becomes

$$G_c(s)=\left[\frac{[\eta s^q+1][a_2 s^{\alpha_2}+a_1 s^{\alpha_1}+a_0]}{[b_1 s^{\beta_1}+b_0][\lambda^2 s^{2q}+(2\lambda-\eta)s^q+\eta\tau s^{q+1}+\tau s]}\right] \quad (21)$$

For obtaining FO-PID controller, coefficient of $s^q$is equated to zero, so $(2\lambda-\eta)=0$ i.e.,$2\lambda=\eta$ then the required DS – FOPID with filter is obtained as

$$G_c(s)=\left[\frac{[\eta s^q+1][1+0.5\tau s]}{s^{1-\alpha_1}\left[s^{2q}0.5\lambda^2+s^q(\lambda+0.5\eta)+s^{2q-1}\left(\frac{\lambda^2}{\tau}\right)+1\right]}\right]*\frac{a_1}{\tau[b_1 s^{\beta_1}+b_0]}\left[1+\frac{1}{\frac{a_1}{a_0}s^{\alpha_1}}+\frac{a_2}{a_1}s^{\alpha_2-\alpha_1}\right] \quad (22)$$

Where $K_c=\frac{a_1}{\tau},\tau_I=\frac{a_1}{a_0},\tau_D=\frac{a_2}{a_1},2\lambda=\eta$ (23)

C. Algorithm

In concise the proposed method involves the following steps in determining the controller settings.

- Select the non integer process transfer function
- Desired fractional closed loop trajectory is assumed
- Coefficient of s^{α} is equated to zero, to obtain the tuning parameter
- First order Pade's Approximation is chosen for time delay
- The resultant DS-FOPID controller transfer function with filter is obtained
- The controller is compared with that of conventional PID controller for controller settings.

III. SIMULATION STUDY

Simulation results for one non-integer order transfer function models and two non-integer order transfer function models are studied for servo and regulatory problems. Simulink block diagram of MATLAB is employed for obtaining the closed loop behavior (ref Figure 1).

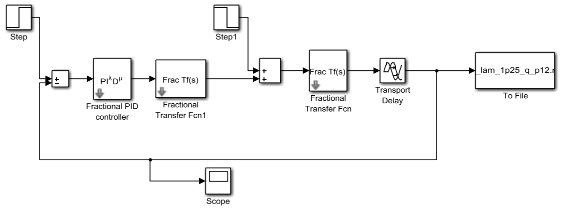

Fig 1: Simulink Block diagram for closed loop behavior

The PID tuning rules for the proposed method are obtained. Maximum magnitude of sensitivity function, Ms Value is evaluated which indicates the robustness of the controller. The performance comparison of the proposed DS-FOPID controller is made with the fractional order - PID controllers IMC- FOPID [3], IMC-FOPID-FOF controller [4] in terms of ISE, IAE and ITAE. For Case Study C i.e., for two non – integer order time delay systems with one non – integer twice as other [22], performance comparison of the proposed DS-FOPID controller for Servo problem is made with the Fractional Order-PID controllers designed by IMC-FOPID method in terms of ISE, IAE and ITAE.

A. One Non-Integer Order Systems with Time Delay (Case Study 1)

One Non-Integer Order transfer function $G_p(s)=\frac{66.16}{12.72s^{0.5}+1}e^{-1.93s}$ is considered [3]. DS-FOPID, IMC-FOPID and IMC-FOPID-FOF controller parameters are given in Table 1. Closed loop response for servo and regulatory problems is shown in Figure 2. At time t=0 sec, servo problem is studied by incorporating changes in set point and at time t = 300 seconds, disturbance rejection is analyzed with D = -0.1 as final value. Figure 2 depicts comparison of proposed FOPID controller (DS-FOPID), IMC-FOPID controller and IMC-FOPID-FOF.

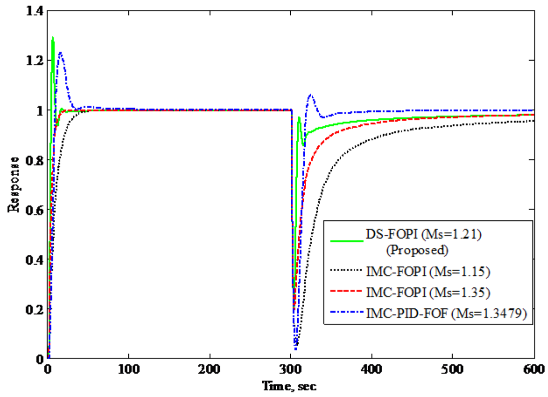

Fig. 2: Servo and regulatory response for case study 1.

From Figure 2 it can be concluded that servo response with proposed method is slightly oscillatory when compared with literature reported methods. But the rise time and settling time are faster than the reported methods. With respect to regulatory response, proposed DS-FOPID controller is better when compared with literature reported methods. Also proposed controller is robust when compared to literature reported methods. Robustness analysis in terms of Ms Value is given in Table 1 and performance analysis is given in Table 2.

Table 1: FOPID controller settings and Ms value for case study 1($0 < \alpha < 1$)

Case Study	Method	Controller Settings						Tuning Parameter	Ms
		k_c	τ_I	τ_D	τ_f	μ	ν		
1	DS-FOPID	0.0274	122.45	-	$\frac{1+0.965*s}{s^{0.5}(8s+8s^{0.5}+1)}$	0.5	--	λ=4, q=0.5	1.21
	IMC-FOPID	0.19	12.658	-	$\frac{1+0.96s}{s^{0.5}(4.53s+10.99)}$	0.5	--	Ms=1.15,k = 1	1.15
	IMC-FOPID	0.19	12.658	-	$\frac{1+0.96s}{s^{0.5}(1.75s+5.44)}$	0.5	--	Ms=1.35, k = 1	1.35
	IMC-PID-FOF	0.199	12.72	1.035	$\frac{1}{s^{0.5}[1+22.14s^{0.33}]}$	0.5	0.5	$\Phi_m = 60^0$ ω_c = 0.1 rad/sec	1.3479

Table 2: Performance analysis for all case studies 1 to 4

Case Study	Method	Servo Problem			Regulatory Problem		
		ISE	IAE	ITAE	ISE	IAE	ITAE
1	DS FO-PID	19.72	32.53	289.69	18.64	65.79	2.16*10^4
	IMC-FOPID(Ms=1.15)	40.78	72.59	711.15	115.71	172.37	5.50*10^4
	IMC-FOPID(Ms=1.35)	23.20	36.043	178.25	40.157	91.02	2.88*10^4
	IMC-PID-FOF	34.94	61.302	639.81	42.73	74.98	2.35*10^04
2	DS-FOPID	16.68	25.13	75.23	1.253	7.22	1.48*e+03
	IMC-FOPID	17.71	25.39	78.65	1.35	7.17	1.47e+03
	IMC-FOPID-FOF	9.087	18.72	189.38	1.045	7.31	1.53e+03
3	DS- FOPID	47.94	56.11	3.53	0.0028	0.31	9.295
	IMC-FOPID	0.72	0.993	0.23	0.015	0.37	11.74
	IMC-FOPID-FOF	2.32	3.945	6.35	0.24	1.85	60.68
4	DS-FOPID	17.31	18.54	2.82	8.52*e-07	0.0014	0.021
	IMC-FOPID	2.314	3.163	1.283	0.381	2.50	41.28
	IMC-FOPID-FOF	2.81	4.481	3.70	0.0029	0.081	1.25

B. One Non-Integer Order Systems With Time Delay (Case Study – 2)

One Non-Integer Order transfer function $G_p(s)=\frac{5}{1.5s^{1.5}+1}e^{-s}$ is considered [4] and Fractional Order PID controller parameters for various methods are given in Table 3. Closed loop response for both servo and regulatory problems is represented in Figure 3. At time t=0 sec servo problem is studied by incorporating changes in set point and at time t = 200 seconds, disturbance rejection is analyzed with D = -0.05 as final value. Figure 3 shows a comparison between proposed FOPID controller (DS FOPID) and IMC-FOPID controller [3] and IMC-FOPID-FOF [4].

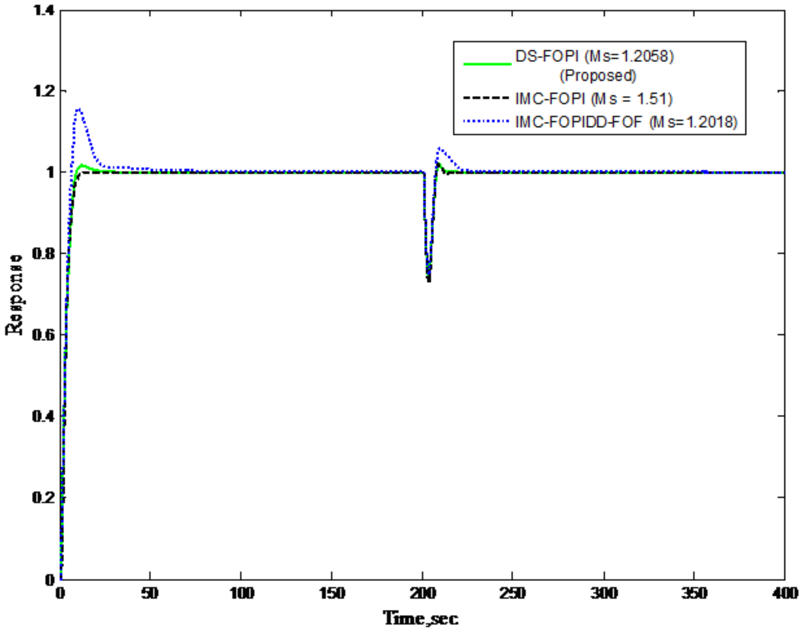

Fig. 3: Servo and regulatory response for case study 2.

From Figure 3, it is evident that the servo response with proposed method is slightly oscillatory when compared with IMC-FOPID controller [3] but better than IMC-FOPID-FOF [4].

But the rise time and settling time are faster than the reported methods. Similarly for regulatory response, proposed DS-FOPID controller is better when compared with literature reported methods. Robustness analysis is given in Table 3 and Performance analysis is carried out in terms of ISE, IAE and ITAE and indicated in Table 2. Also proposed controller is robust when compared to literature reported methods (refer Table 3).

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Table 3: FOPID controller settings and Ms value for case study 2 (1 < α < 2)

Case Study	Method	Controller Settings								Tuning Parameter	Ms
		k_c	τ_I	τ_D	τ_{DD}	τ_f	μ	N	υ		
2	DS FO-PID	0.3	1.5	-	-	#	0.5	-		λ=6 q=0.5	1.21
	IMC-FOPID	0.3	1.49	-	-	##	1.5	-		Ms=1.51 k = 2	1.51
	*IMC-PID-FOF	0.2	0.5	3	1.5	###	1	0.5	1.2018	$\Phi_m=60^0$, ω_c=0.3 λ=0.33,τ_c=4.959	1.21

$*G_c(s) = K_c\left(1+\frac{1}{\tau_I s^{\mu}}+\tau_D s^{N}+\tau_{DD}s^{\upsilon}\right)\tau_f$ # $\tau_f = \frac{(12s^{0.5})(1+0.5s)}{s^{0.5}*(18s+12s^{0.5}+1)}$ ## $\tau_f = \frac{s^{0.5}(1+0.5s)}{0.79s^2+2.85s+2.52}$ ### $\tau_f = \frac{1}{1+9.918s^{0.33}}$

C. Two Non-Integer Order Systems with Time Delay (Case Study 3)

Two non-integer order transfer function $G_p(s) = \dfrac{5.069}{s^{1.9954}+6.0645s^{0.9997}+5.069}e^{-0.0518s}$ is considered [3] and Fractional Order PID controller parameters for various methods are given in Table 4. Closed loop response for both Servo and Regulatory problems is shown in Figure 4. At time t=0 sec, Servo problem is studied by incorporating changes in set point and at time t = 30secs, disturbance rejection is analyzed with D = -0.5 as final value. Robustness analysis in terms of Ms is given in Table 4 and Performance analysis in terms of ISE, IAE and ITAE are given in Table 2.

From Figure 4, it can be concluded that servo response with proposed method [DS-FOPID] is slightly oscillatory when compared with IMC-FOPID [3] but better than IMC-FOPID-FOF [4]. But the rise time and settling time are faster than the reported methods. Similarly for regulatory response, the proposed DS-FOPID controller is better when compared with literature reported methods. Robustness and performance analysis for designed controller is carried out and compared with that of literature.

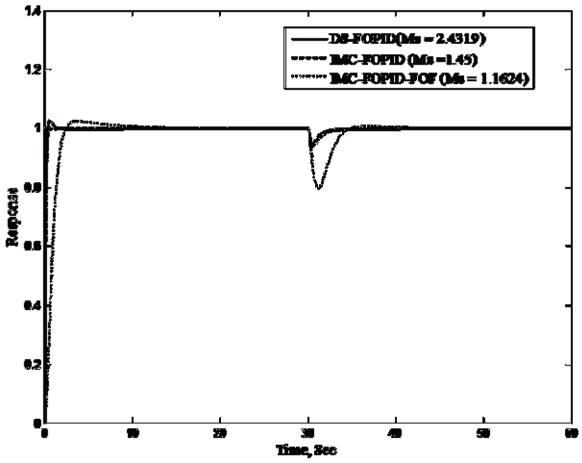

Fig. 4: Servo and regulatory response for case study 3

Table 4: FOPID controller settings and Ms value for case study 3 (1 < α < 2)

Case Study	Method	Controller settings						Tuning parameter	Ms
		k_c	τ_I	τ_D	τ_f	μ	N		
3	DS-FOPID	23.09	1.1963	0.197	*	0.999	0.996	λ=0.5 q = 0.05	2.43
	IMC- FOPID	1.126	0.9999	0.197	**	0.999	0.996	Ms = 1.45 k = 2	1.45
	IMC-FOPID-FOF	23.08	1.196	0.033	***	0.999	0.996	$\Phi_m= 80^0$ ωc =1rad/sec	1.16

$*\frac{(2s^{0.1}+1)(1+0.0259s))}{s^{0.0003}*(0.5s^{0.2}+2s^{0.1}+19.2s^{-0.8}+1)}$ $**\frac{1+0.0259s}{s^{0.0003}((1.2287*10^{-4})s^2+0.0082s+0.1898)}$ $***\frac{(s^{0.003})}{(1+19.205s^{0.11})}$

D. D. Two Non-Integer Order Systems with Time Delay (Case Study 4)

Two non-integer order transfer function $G_p = \dfrac{4.47}{s^{2.47}+5.23s^{1.02}+4.47}e^{-0.12s}$ is considered [3, 4] and Fractional Order PID controller parameters for various methods are given in Table 5. Closed loop response for both servo and regulatory problems is shown in Figure 5. At time t=0 sec servo problem is studied by incorporating changes in set point and at time t = 15 seconds disturbance rejection is analyzed with D = -0.5 as final value.

From Figure 5, it can be concluded that servo response with proposed method [DS-FOPID] is slightly oscillatory when compared with IMC-FOPID [3] but better than IMC-FOPID-FOF [4]. But the rise time and settling time are faster than the reported methods. Similarly regulatory response, with the proposed DSFOPID, it is better when compared with literature reported methods. Robustness analysis is given in Table 5 and performance analysis is given in Table 2.

Retrieval Number: D1494029420 /2020©BEIESP
DOI: 10.35940/ijitee.D1494.029420
Journal Website: www.ijitee.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

Table 5: FOPID controller settings and Ms value for case study 4 ($2 < \alpha < 3$)

Case Study	Method	Controller settings						Tuning parameter	Ms
		k_c	τ_I	τ_D	τ_f	μ	ν		
4	DS-FOPID	1.17	1.17	0.191	†	1.02	1.45	λ=1.25, q=0.12	1.81
	IMC-FOPID	1.17	1.17	0.195	††	1.02	1.45	Ms=1.45 k = 3	1.45
	IMC-FOPID-FOF	1.170	1.17	0.19	†††	1	1.45	$\Phi_m=80^0$ ωc =1 λ=0.11 τ_c=1	1.65

† $\frac{s^{0.02}(2s^{0.1}+1)(1+0.06s)}{(0.096s^{0.2}+0.24s^{0.1}+s^{-0.8}+0.12)}$ †† $\frac{s^{0.02}(1+0.06s)}{(2.76*10^{-4}s^3+0.009s^2+0.11s+0.62)}$ ††† $\frac{1}{s^{0.11}+0.1216}$

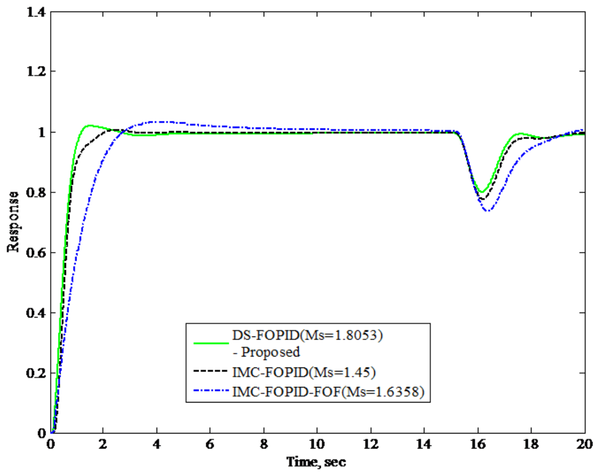

Fig. 5: Servo and regulatory response for case study 4

IV. RESULT ANALYSIS

In this work, direct synthesis algorithm is used in obtaining the FOPID controller settings in terms of tuning parameters and model parameters. After closely observing the results of closed loop behavior, performance analysis and robustness analysis through simulation study with the proposed controller, tuning guidelines are laid down.

Performance analysis reveals that the proposed DS-FOPID controller shows a comparable performance in terms of ITAE for servo problem and good performance in terms of IAE and ITAE for regulatory problem

These tuning guidelines are with respect to q and λ as function of higher fractional order and time delay viz., q = 1*$\boldsymbol{\alpha}$, 0.33*$\boldsymbol{\alpha}$, 0.5*$\boldsymbol{\alpha}_1$, 0.11*$\boldsymbol{\alpha}_1$ and λ = 2.07*$\boldsymbol{\tau}$, 6*$\boldsymbol{\tau}$, 9.65*$\boldsymbol{\tau}$, 10.41*$\boldsymbol{\tau}$ for case studies 1, 2, 3 and 4 respectively.

Tuning parameters with respect to q (adjustable tuning parameter in assumed closed loop transfer function) and $\boldsymbol{\lambda}$ are arrived at for different case studies. For case study-1 and case study-2, they are tuned as q = 0.5 and $\boldsymbol{\lambda}$ = 4, q = 0.5and $\boldsymbol{\lambda}$ = 6 respectively. For the systems in case study 3 and 4, they are tuned as q = 0.05 and $\boldsymbol{\lambda}$ = 0.5, q = 0.12 and $\boldsymbol{\lambda}$ = 1.25 respectively.

V. CONCLUSION

Design of FOPID controller for NIOS systems based on Direct Synthesis method is presented in this work. In literature, design of FOPID controllers based on DS method is at low extent when compared to IMC and Stability Analysis methods. Three types of Non-Integer Order Systems are considered in this work, namely, One Non – Integer order system with Time Delay, Two Dissimilar Non – Integer Order systems with time delay. DS-FOPID controllers are designed for all the two types of Non-Integer Order Systems. 'q' appears as a non integer in the desired closed loop trajectory (y/yr) which is assumed. The closed loop behavior of proposed controller thus obtained is compared with IMC-FOPID [3] and IMC-FOPID-FOF [4] for case studies 1 through 4. Performance analysis in terms of ISE, IAE, ITAE is reported and on comparison with literature reported methods it can be concluded that the proposed controller method can also be adopted. Maximum Sensitivity value is evaluated to understand the robustness of the proposed controller. Simulation studies are carried out using FOMCON toolbox and ninteger toolbox through *Simulink* of MATLAB software for generating closed loop response plots.

REFERENCES

1. Igor Podulbny, Fractional - Order Systems And Fractional - Order Controllers, 1994.
2. T. Vinopraba, N. Sivakumaran, and S. Narayanan, "*IMC Based Fractional Order PID Controller*", IEEE, 2001.
3. M. Bettayeb and R. Mansouri, "*Fractional IMC-PID-filter controllers design for non integer order systems*", Journal of Process Control, 24 2014), pp.261–271.
4. Dazi Li, Lang Liu, Qibing Jin and Kotaro Hirasawa, "*Maximum sensitivity based fractional IMC–PID controller design for non-integer order system with time delay*", Journal of Process Control, 31 (2015), pp.17–29.
5. Truong Nguyen Luan Vu and Moonyong Lee, *"Analytical design of fractional-order proportional-integral controllers for time-delay processes*", ISA Transactions, 52 (2013), pp.583–591.
6. Ying Luo, Yang Quan Chen, Chun Yang Wang and You Guo Pi, *"Tuning fractional order proportional integral controllers for fractional order systems"*, Journal of Process Control , 20 (2010), pp. 823–831.
7. Ying Luo and Yang Quan Chen, *"Fractional order [Proportional derivative] controller for a class of fractional order systems"*, Automatica, 45 (2009), pp. 2446-2450.
8. Fabrizio Padula and Antonio Visioli, *"On the fragility of fractional order PID- controllers for FOPDT processes"*, ISA Transactions (Article in Press) (2015).
9. Concepcio'n A. Monje, Vicente Feliu, Blas M. Vinagre and Yang Quan Chen, *"Tuning and Auto- Tuning Of Fractional Order Controllers For Industry Applications"*, Control Engineering Practice, 16 (2008), pp.798–812.
10. Abdullah Ateş and Celaleddin Yeroglu, *"Optimal Fractional order PID design via Tabu Search based algorithm"*, ISA Transactions, (Article in Press) (2015).
11. Richa Sharma, Prerna Gaur and A.P.Mittal, *"Performance analysis of two-degree of freedom fractional order PID controllers for robotic manipulator with payload"*, ISA Transactions, 58 (2015), pp. 279–291.
12. Nooshin Bigdeli, *"The design of a non-minimal state space fractional order predictive functional controller for fractional systems of arbitrary order"*, Journal of Process Control, 29(2015), pp. 45–56.
13. Fernando Martín, Concepción A. Monje, Luis Moreno and Carlos Balaguer, *"DE-based tuning of $PI^{\lambda}D^{\mu}$ controllers"*, ISA Transactions, 59 (2015), pp. 398 -407.
14. Indranil Pan and Saptarshi Das, *"Fractional order fuzzy control of hybrid power system with renewable generation using chaotic PSO"*, ISA Transactions, (Article in Press), (2015).

15. Bi Tao Zhang and You Guo Pi, *"Enhanced robust fractional order proportional-plus-integral controller based on neural network for velocity control of permanent magnet synchronous motor"*, ISA Transactions, 52 (2013), pp. 510–516.
16. Shiqi Zheng, Xiaoqi Tang and Bao Song, *"A Graphical Tuning Method of Fractional Order Proportional Integral Derivative Controllers for Interval Fractional - Order Plant"*, Journal of Process Control, 24 (2014), pp. 1691–1709.
17. Cheon Yu Jin , Kyung Hwan Ryu, Jietae Lee, Su Whan Sung and In-Beum Lee, *"PID auto-tuning using new model reduction method and explicit PID tuning rule for a fractional order plus time delay model"*, Journal of Process Control, 24 (2014), pp. 113–128.
18. Ch. Anil and R. Padma Sree., *"Tuning of PID controllers for integrating systems using direct synthesis method"*, ISA Transactions, 57 (2015), pp. 211–219.
19. A. Seshagiri Rao, V. S. R. Rao, and M. Chidambaram., *"Direct synthesis-based controller design for integrating processes with time delay"*, Journal of the Franklin Institute, 346 (2009), pp.38–56.
20. C.V. Nageswara Rao, A. Seshagiri Rao, and R. Padma Sree., *"Design of PID controller for Pure Integrating Systems with Time Delay"*, International Journal of Applied Science and Engineering, 9(4) (2011), pp. 241-260.
21. Anuj Narang, Identification and control of fractional and integer order systems, PhD Thesis, University of Alberta, spring, 2012.
22. Aleksei Tepljakov, Eduard Petlenkov and JuriBelikov, FOMCON: a MATLAB Toolbox for Fractional-order System Identification and Control, International Journal of Microelectronics and Computer Science, 2(2), pp. 51-62, 2011.
23. H. Parastand, and M.J. Khosrowjerdi, *"Stability and Performance Attainment with Fixed Order Controller Using Frequency Response"*, IJE Transactions A: Basics, 28(4) (2015), pp. 553-560.

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