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LQR based Control Strategies for a Spherical Tank Level Process

Ka. Suriyaprabha, D. Rathikarani

Abstract: ***Spherical tank finds wide application in gas plants and process industries, petro chemical industries, paper manufacturing and water treatment industries. In a spherical tank, controlling the level is a critical entity. Due to change in shape, the system displays high nonlinear behavior which in turn exhibits the non-linear characteristics. The system's nonlinearity complicates in the design of conventional PI controllers. In this work, The PI controller gains are formulated as the optimal state-feedback gains. A PSO has been accustomed optimally discover the weighting matrices, related to the various optimum state-feedback regulator design whereas minimizing ISE and the controller effort. The performance of the proposed PSO based LQRPI controller is contrasted with those of GA-LQRPI and LQRPI controllers.***

Keywords: spherical tank, PI controller, LQR and PSO.

I. INTRODUCTION

Spherical tank finds wide application in gas plants and process industries, petrochemical industries, paper manufacturing, water treatment industries, etc.,In a spherical tank, controlling the level is a critical entity. Due to change in shape, the system displays high nonlinear behavior which in turn exhibits the non -linear characteristics. The chemical process industries require process liquids to be stored in a process vessel. In most of the time, the process liquid is processed by mixing treatment in tanks, but then again the level of the process liquid will be controlled at some preferred value as per the set point or requirement. The variations in the process parameters can be trounced by persistent tuning of the controller parameters using LQR.
Pradeepkannan [1] implemented a gain scheduled PID Controller for a Nonlinear Coupled Spherical Tank Process. T.P Febin,et. al., [2] developed the GA based Fuzzy Logic Controller for a level control of spherical tank process at lower region. Simulated results showed a considerable improvement in rising time and settling time besides reduces overshoot, IAE and ISE.

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S.Nithya, et al., [3] developed PI controller using Skogestad tuning rule, Fuzzy Logic Controller (FLC) for the control of liquid level in spherical tank. Moghadam and Amir Alizadeh [4] planned a boundless dimensional LQR based structure for a nonlinear process. Ochi and Kondo [5] have demonstrated that the vital sort ideal servo for second request framework can be diminished to a LQR issue and an ideal I-PD controller can be planned with this system. However, selection of rho and R in the design of LQR is not reported. In this work, PSO-LQRPI is proposed for a spherical tank system. The parameters rho and R of LQRPI are optimized using PSO and GA methods. The performance of the proposed control strategy is compared with those of other control strategies like GA-LQRPI and LQRPI.

II. PROCESS DESCRIPTION AND MATHEMATICAL MODELING

In many aspects, spherical tank is considered as an optimum shaped vessel to process chemicals and to carry out chemical reactions. Depending on mass and energy balances, the first phase is to design a mathematical model. The spherical tank level process model is showed in Figure 1. The variables of the system are given below

- Control input (fin) is the input flow rate (m3/s)
- Output (x) is the fluid level (m).
- r be the radius of spherical tank(m).
- d0thickness (diameter) of pipe (m).
- x0, initial liquid level height (m).
- Assume 'rsurface' is radius on the surface of the fluid. It will vary according to the level of fluid in the tank.

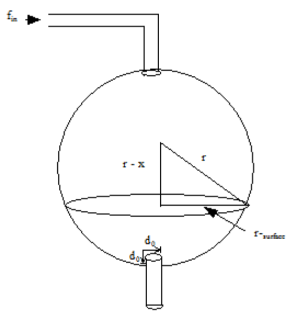

Figure 1. Spherical Tank Level Process

Where,
Length = rsurface;
Height = radius of tank (r) – fluid level (x)
Hypotenuse = radius of tank (r)

$$r_{surface} = \sqrt{2rx - x^2}$$

Now the Dynamic model is given by

$$\frac{\delta}{\delta t}\left[\int_0^{x_1} A(x)\delta x\right] = \text{fin}\,(t) - a\sqrt{2gx - x_0} \qquad (1)$$

where $A(x)$ = area of cross section of tank =

$$\pi r^2_{surface} = \pi\,(2rx - x^2)$$

a = area of cross section of pipe = $\pi\left(\frac{d_0}{2}\right)^2$

Where, $A(x)\delta x$ = Amount of water

$f_{in}\delta t$ = Input flow rate and $a\sqrt{2g(x - x_0)}\ \delta t$ = Output flow rate

$$\frac{\delta x}{\delta t} = \frac{f_{in}\delta t - \frac{\pi.d_0^2}{4}\sqrt{2g\,(x-x_0)}}{\pi\,(2rx - x^2))} \qquad (2)$$

By applying

$lim_{d\to 0}$ in equation (2), we have $\frac{\delta x}{\delta t} = \frac{dx}{dt}$

Therefore

$$\frac{dx}{dt} = \frac{f_{in}\delta t - \frac{\pi.d_0^2}{4}\sqrt{2g\,(x-x_0)}}{\pi\,(2rx - x^2)} \qquad (3)$$

Equation (3) shows the dynamic model of the Spherical Tank System and this model representation is considered for simulation studies.

The entire operating region of spherical tank level process is divided into three regions. In the first region the level of the spherical tank level process is brought to steady state condition. Then an increase as well as decrease in inflow rate as of equal magnitude is applied. The change in level of the spherical tank level process is recorded with respect to time for both cases. The process parameters like process gain (Kp), time constant (τ) and dead time (td) are estimated for both cases and average value is considered.In this work, PI controller parameters are designed based on Z-N tuning method. The obtained worst case transfer function of the spherical tank system is given by

$$G(s) = \frac{1.76}{92.45S+1}e^{-17.85s} \qquad (4)$$

Time delay in equation (8) is converted into transfer function using pade approximation method. The obtained model is given in equation (9)

$$Gt_d(s) = \frac{-s+0.112}{s+0.112} \qquad (5)$$

The overall transfer function model of the spherical tank system is given in equation (10)

$$G(s) = \frac{-1.76S+0.1972}{92.45S^2+11.81S+0.112} \qquad (6)$$

III. DESIGN OF LQRPI AND ITS IMPLEMENTATION

Linear-quadratic-regulator (LQR) is a part of optimal control strategy which has been widely developed and used in various applications. LQR design is based on the selection of feedback gains K such that the cost function J is minimized. This ensures that the gain selection is optimal for the cost function specified.

A continuous system is expressed in a form of

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (7)$$

A state variable feedback control is given by

$$u(t) = -Kx(t) \qquad (8)$$

The performance index (PI)is defined as

$$J = \int_0^{\alpha}\left(x^T QX + u^T Ru\right)dt \qquad (9)$$

where, Q is Real symmetric positive definite or semi-definite matrix. The feedback control function can de written as

$$K=R^{-1}BTP \qquad (10)$$

and it will result in

$$A^T P + PA + PBR^{-1}B^T P + Q = 0 \qquad (11)$$

P matrix can be expressed as

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \qquad (12)$$

Substitute the values of R, B and equation (12) in equation (10), the Feedback gain matrix Kis given below

$$K = R[0\ 1]\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \qquad (13)$$

The equation of PI controller is presented in following equation

$$[-K_i \quad -K_c] = [R * p_{12} \qquad R * p_{22}] \qquad (14)$$

Equation (8) can be expressed as

$$u(t) = -[-K_i \quad -K_c]\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \qquad (15)$$

$$= K_c e(t) + K_i \int e(t)dt \qquad (16)$$

The above formulation clearly shows that with judicious choice of weighting matrices {Q, R} a PI controller can easily be tuned which preserves the achievable performance of an LQR i.e. minimum deviation in the state trajectories with minimum time domain performance index. The PSO and GA based choice of Q, R and its advantages are discussed in the next section. For the design of LQR, the worst case transfer function model given in equation (17) is considered and it is converted into state space model. The obtained state space model is given in equation (17) and (18)

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -0.1224 & -0.0012 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u(t) \qquad (17)$$

$$y(t) = \begin{bmatrix} -0.0182 & 0.0020 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + 0 \quad (18)$$

IV. DESIGN OF LQRPI AND ITS IMPLEMENTATION

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Dr. Eberhart and Dr. Kennedy. The flow chart of PSO algorithm is presented in Figure 2.

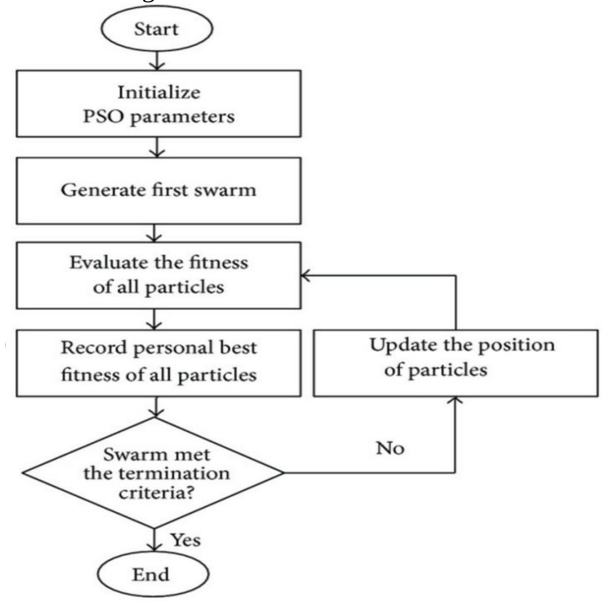

Figure 2. Flow diagram of PSO algorithm

V. SIMULATION RESULTS AND DISCUSSION

The simulated servo responses for step change in setpoint of level from 35 to 40 % , 50-55% and 60 to 65% as shown in Figures 3,5 and 7 respectively. Initially, the level is maintained at 35 % and then the step change with magnitude of 5% is applied. It is observed that controllers take necessary action to alter the inflow rate (Fin) and maintain corresponding level of spherical tank. Also the corresponding changes in manipulated variable are shown in Figures 4,6 and 8 respectively.

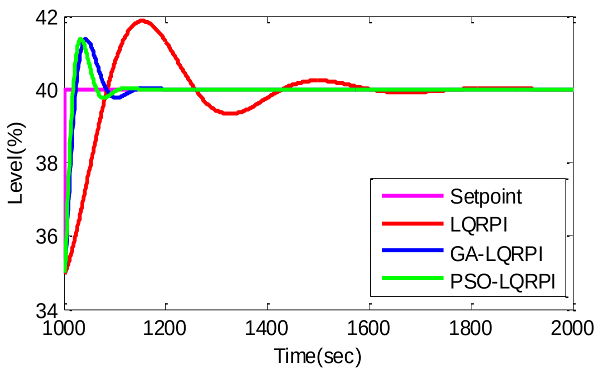

Figure 3. Servo response of spherical tank level process at the operating point of 40% with LQRPI, GA-LQRPI and PSO-LQRPI .

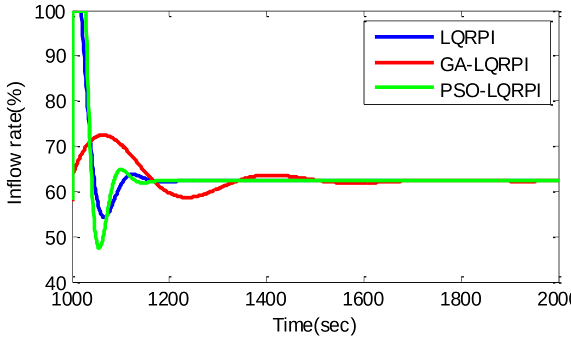

Figure 4. Controller output of spherical tank level process at the operating point of 40% with LQRPI, GA-LQRPI and PSO-LQRPI .

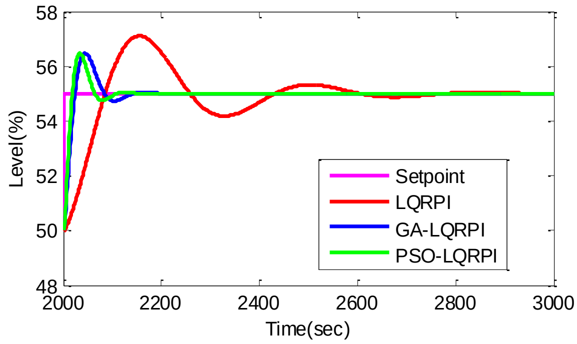

Figure 5. Servo response of spherical tank level process at the operating point of 55% with LQRPI, GA-LQRPI and PSO-LQRPI .

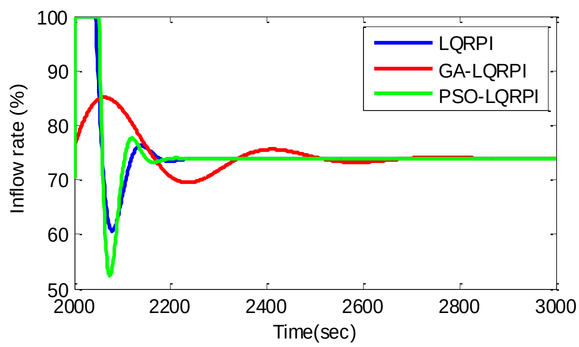

Figure 6. Controller output of spherical tank level process at the operating point of 40% with LQRPI, GA-LQRPI and PSO-LQRPI .

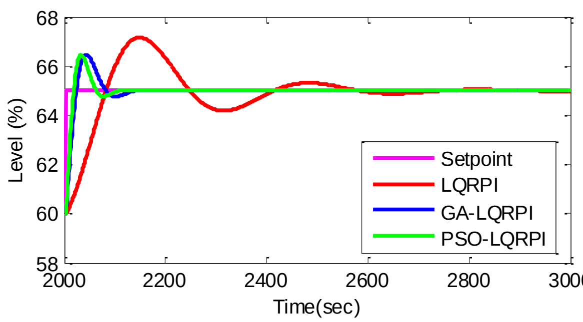

Figure 7. Servo response of spherical tank level process at the operating point of 65% with LQRPI, GA-LQRPI and PSO-LQRPI .

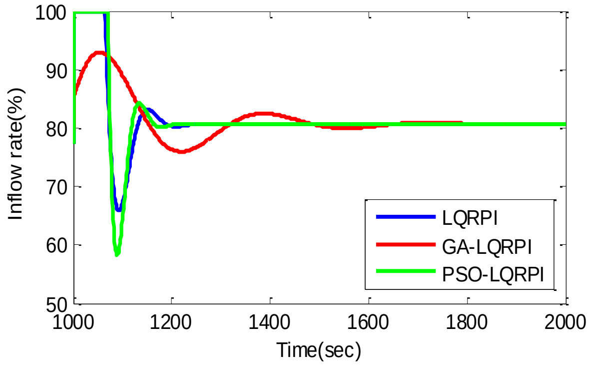

Figure 8. Controller output of spherical tank level process at the operating point of 40% with LQRPI, GA-LQRPI and PSO-LQRPI .

Table 1. Performance measure for servo response of spherical tank level process.

Controller	ISE			IAE		
	ΔSP			ΔSP		
	35- 40 %	50-55 %	60-65%	35- 40 %	50-55%	60-65%
LQRPI	710	770	740	340	477	361
GA-LQRPI	205	220	210	105.9	113.8	107.9
PSO- LQRPI	154	167	160	79.9	85.6	80.7

The regulatory responses of spherical tank level process obtained by increase in inflow rate of 10% applied at 2000th second for the operating points 45%, 55% and 65%. Due to increase in inflow rate of 10% at the operating point the level of the spherical tank is increased from nominal value as seen from Figures 9, 11 and 13. LQRPI, GA-LQRPI and PSO-LQRPI controllers takes the necessary action and bring back the level into nominal operating point. Also the manipulate variable are shown in Figures 10,12 and 14.

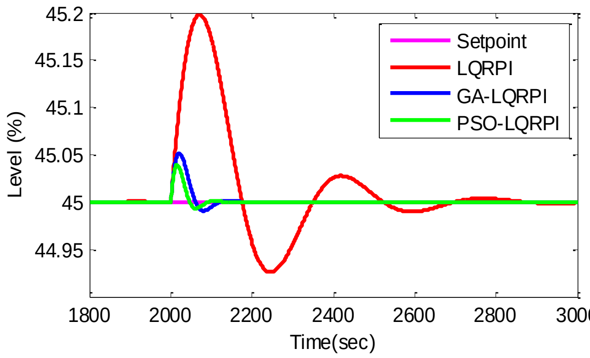

Figure 9. Regulatory response of spherical tank level process at the operating point of 45%.

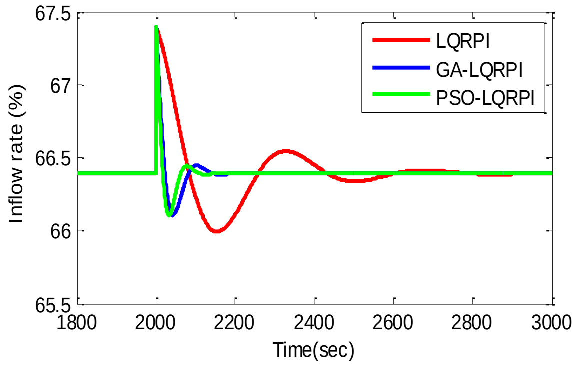

Figure 10. Controller output of spherical tank level process at the operating point of 45%.

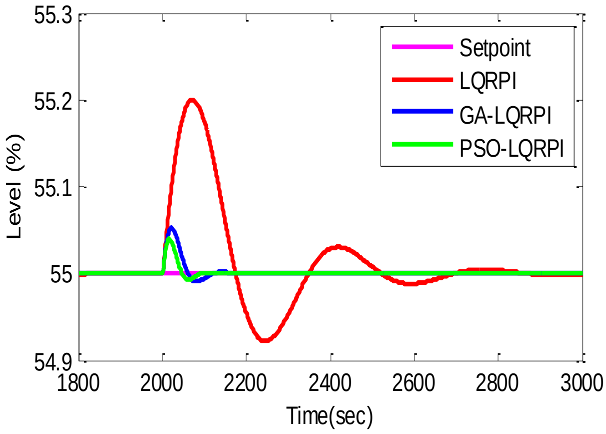

Figure11.Regulatory response of spherical tank level process at the operating point of 55%.

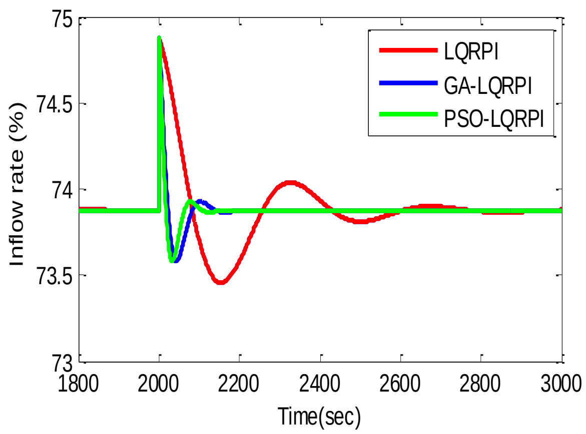

Figure 12.Controller output of spherical tank level process at the operating point of 55%.

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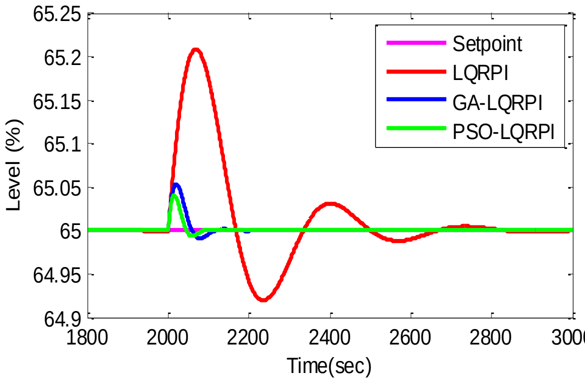

Figure 13. Regulatory response of spherical tank level process at the operating point of 65%.

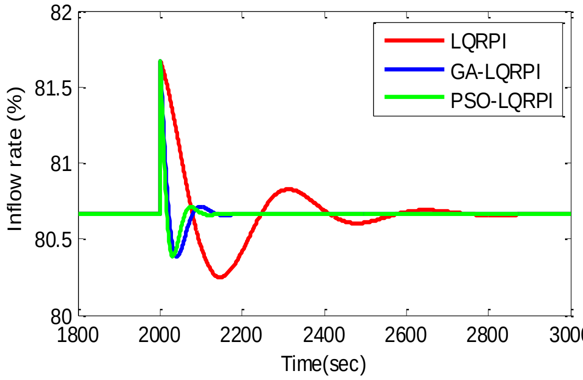

Figure 14. Controller output of spherical tank level process at the operating point of 65%.

The performances of all control strategies are tabulated in Table 2. From the performances it is observed that PSO based LQRPI produces better result with those of GA based LQRPI and LQRPI controllers.

Table 2. Performance measure for regulatory response of spherical tank level process.

Controller	ISE			IAE		
	Operating point (%)			Operating point (%)		
	45	55	65	45	55	65
LQRPI	5.1	10.1	6.5	34	35	35
GA-LQRPI	0.32	0.5	0.24	2.3	2.2	2
PSO-LQRPI	0.28	0.31	0.15	1.3	1.3	1.2

VI. CONCLUSION

In this paper, PSO based LQRPI controller is proposed and implemented for a spherical tank level process. The response curve for different operating level of the system thus obtained using PSO based LQRPI is compared with response obtained by a other control strategies like GA based LQRPI and LQRPI. For servo and regulatory responses, the proposed controller gives minimum ISE and IAE than the other control strategies. Comparing the performance of responses, PSO based LQRPI performs well and it can be used for nonlinear time varying processes.

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