The Unsteady Free Convective Flow over Rotating Disk under the Existence of Chemical Reaction, Soret and Dufour Effects

Bismeeta Buragohain, Nabajyoti Dutta, B. R. Sharma

Abstract: ***In this article, the unsteady free convective flow through a rotating disk was discussed under the influence of the chemical reaction, Dufour effect and Soret effect. Similarity transformation is used to change the unsteady non-linear boundary equations to a set of ordinary differential equations. Then we used the MATLAB's bvp4c solver to solve these differential equations. Temperature as well as concentration profiles are drawn for different flow parameters of flow such as Soret Effect, Dufour Effect and also parameter of Chemical Reaction. We have also discussed the Nusselt number graph and values are given in a tabular form for different kinds of flow parameters.***

Keywords **:** ***Chemical Reaction, Dufour effect, Rotating Disk. Soret effect, Unsteady Flow.***

I. INTRODUCTION

Lots of research works have been done on free convection due to its vast applications in many areas like thermal insulation, solar power collectors, drying processes etc. Free convection is observed in nature due to difference of temperature and due to difference of concentration.

Prakash et. al.[1] presented their study of free convective MHD flow on a vertical cone in the year 2013. Heat and Mass flux were taken as variables. Pullepu et. al.[2] also reported free convective flow in a vertical cone under the existence of chemical reaction and the effect of heat generation and absorption. They carried out their study in non-uniform surface temperature and concentration and prepared a mathematical model through the use of finite difference scheme of Crank-Nicholson in the year 2014. In the year 2015, Malapati and et. al.[3] carried out their study i.e. MHD free convective flow. They used a vertical plate which is permeable in nature.

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They investigated the flow, by taking into consideration of chemical reaction and thermal radiation along with transfer of heat and mass. In a vertical plate that is porous in nature, Mahender et. al. [4] reported the MHD free convection and mass transfer too. The basis of their mathematical model was Finite Element Method. The MHD free convection was noted under different temperatures by Paul [5] on an exponentially accelerated porous plate. Laplace transform technique was used by him. Uwanta et. al.[6] reported the free convective MHD flow over a vertical channel under viscous dissipation and constant suction. Soret and Dufour effect were taken as a measure of their study. Reddy et. al.[7] presented their study of free convection flow with mass and heat transfer on a porous vertical surface. Investigation was carried out under the existence of Soret and Dufour effects and also viscous dissipation. Talla et. al [8] have considered the effect of viscous dissipation on the chemically reacting fluid across the vertical plate.

The analysis was carried out under the influence of Dufour and Soret effect. Freidoonimehr et. al. [9] carried out their investigation of MHD flow on a rotating porous disk under Soret and Dufour effect. Homotopy analysis method was used by them for their mathematical calculation. Under influence of chemical reaction, Ibrahim [10] researched the dissipating radiative MHD flow in a non-isothermal stretching sheet. Reddy et. al.[11] recorded MHD flows between the two rotating plates with non-linear thermal radiation and homogeneous-heterogeneous reaction.

This paper contains the numerical analysis of unsteady free convective flow over a rotating disk. It is done by taking into consideration of chemical reaction and Dufour effects. To transform the partial differential equations into non-dimensional ordinary differential equations we used similarity transformation and such equations are attempted to solve numerically by using MATLAB's built in solver bvp4c. For various parameters, the numerical results are shown graphically.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

In our investigation, we consider axially symmetric and incompressible flow on a rotating disk. The disk rotates with a steady angular velocity Ω and cylindrical polar coordinates $\left(r,\theta,z\right)$ around the $z-$axis.

The symbol r indicates the radial direction, θ indicates tangential direction and z indicates axial direction respectively. u, v and w are taken as velocity components in radial, tangential and axial direction. These components are independent on temperature T, pressure P and also on θ.

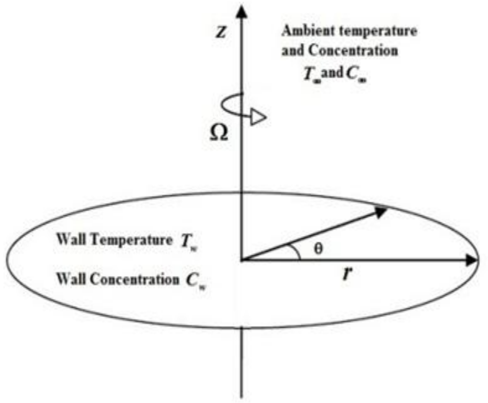

Fig. 1: Geometrical representation of the problem

The governing partial differential equations are

$$\frac{(ru)_r}{r}+w_z=0 \qquad (1)$$

$$u_t+wu_z+uu_r-\frac{1}{r}v^2 =-\frac{1}{\rho}p_r+\nu\left(u_{rr}+\frac{1}{r}u_r+u_{zz}-\frac{u}{r^2}\right) \qquad (2)$$

$$v_t+wv_z+uv_r+\frac{1}{r}uv=\nu\left(v_{rr}+\frac{1}{r}v_r+v_{zz}-\frac{v}{r^2}\right) \qquad (3)$$

$$w_t+ww_z+uw_r=-\frac{1}{\rho}p_z+\nu\left(\frac{1}{r}w_r w_{rr}+w_{zz}\right) \qquad (4)$$

$$T_t+wT_z+uT_r=\frac{k}{\rho C_P}\left(T_{rrr}+T_{zz}+\frac{1}{r}T\right)+\frac{D_m k_t}{C_S C_P}\left(C_{rr}+\frac{1}{r}C_r+C_{zz}\right) \qquad (5)$$

$$C_t+uC_r+wC_z=D_m\left(C_{rr}+\frac{1}{r}C_r+C_{zz}\right)+\frac{D_m k_t}{T_m}\left(T_{rr}+\frac{1}{r}T_r+T_{zz}\right)-k_c\left(C-C_\infty\right) \qquad (6)$$

The corresponding boundary conditions are given below:

$$\left.\begin{aligned}&u(r,z,t)=0, v(r,z,t)=\Omega r, w(r,z,t)=0,\\&u_{zz}(r,z,t)=0, v_z(r,z,t)=0.\\&T(r,z,t)=T_w, C(r,z,t)=C_w, p=0 \ as\ z=0\\&u(r,z,t)=0, w(r,z,t)=0,\\&u_z(r,z,t)=0, v_z(r,z,t)=0.\\&T(r,z,t)=T_\infty, C(r,z,t)=C_\infty \ as\ z\to\infty.\end{aligned}\right\} \qquad (7)$$

where D_m is the mass diffusivity, C_p denotes the specific heat at constant pressure, C_s represents susceptibility of concentration, T_m implies mean value of the fluid temperature, k_t defines the ratio of thermal diffusivity, k indicates the thermal conductivity, ρ is the density, ν denotes the kinematic viscosity of the fluid.

The shear stress τ_{w1}, τ_{w2} and the heat flux q_w are defined as

$$\tau_{w1}=-\mu\left(u_z\right)_{z=0}, \tau_{w2}=-\mu\left(v_z\right)_{z=0}, q_w=-k\left(T_z\right)_{z=0} \qquad (8)$$

Both the local skin friction $\left(C_f, C_g\right)$ and Nusselt number are defined as follows:

$$C_f=\frac{2\tau_{w1}}{\rho\left(\frac{ar}{t}\right)^2}, C_g=\frac{\tau_{w2}}{\rho\left(\frac{ar}{t}\right)^2}, Nu=\frac{-r\left(T_z\right)_{z=0}}{k\left(T_w-T_\infty\right)} \qquad (9)$$

Similarity transformations are

$$\left.\begin{aligned}&v=\frac{ar}{t}g(\eta), w=a\sqrt{\frac{\nu}{t}}h(\eta), p=\mu\frac{a}{t}P(\eta), \eta=\frac{z}{\sqrt{\nu t}}\\&\Omega=\frac{a}{t}, S_c=\frac{\nu}{D_m}, \Pr=\frac{\nu}{\alpha}, T=T_\infty+\left(T_w-T_\infty\right)\theta(\eta),\\&R=\frac{kk_s}{4\sigma_s T_\infty^3}, Sr=\frac{D_m k_t\left(T_w-T_\infty\right)}{T_m\nu\left(C_w-C_\infty\right)}, q_r=\frac{-4\sigma_s}{3k_s}\frac{\partial T^4}{\partial z},\\&Du=\frac{D_m k_t\left(C_w-C_\infty\right)}{C_S C_P\nu\left(T_w-T_\infty\right)}, \gamma=\frac{k_c\nu Re}{u^2}, Re=\frac{ar^2}{t\nu},\\&T^4=4T_\infty^3 T-3T_\infty^4, C=C_\infty+\left(C_w-C_\infty\right)\phi(\eta).\end{aligned}\right\} \qquad (10)$$

Using the equation (10) into equations (1) – (6), we get

$$2f+h'=0 \qquad (11)$$

$$h'''+\left(h'+\frac{\eta}{2}h''\right)-a\left(\frac{h'^2}{2}-\frac{hh''}{2}-2g^2\right)=0 \qquad (12)$$

$$g''+a\left(h'g-hg'\right)+\left(g+\frac{\eta}{2}g'\right)=0 \qquad (13)$$

$$P'-h''+a\,hh'-\frac{h}{2}-\frac{\eta h'}{2}=0 \qquad (14)$$

$$\theta''\left(1+\frac{4}{3}R\right)+PrDu\phi''+Pr\theta'\frac{\eta}{2}-Prah\theta'=0 \qquad (15)$$

$$a\gamma h'^2 Sc(1+\phi)-4\phi''-4ScSr\theta''-2Sc\phi'(\eta-2ah)=0 \qquad (16)$$

The boundary conditions become

$$\begin{aligned}&h=0, h'=0, h'''=0, g=1, g'=0, \theta=1, \varphi=1, P=0 \ at \ \eta=0\\&h=0, h'=0, h''=0, g'=0, \theta=0, \varphi=0 \ at\ \eta\to\infty\end{aligned} \qquad (17)$$

Using equations (8) and (10), from equation (9) we get

$$C_f Re^{\frac{1}{2}}=a^{\frac{-1}{2}}h''(0), C_g Re^{\frac{1}{2}}=-a^{\frac{-1}{2}}g'(0), Nu=-Re^{\frac{1}{2}}a^{\frac{-1}{2}}\theta'(0) \qquad (18)$$

Pressure P can be determined from equation (14)

$$P = \left(\frac{\partial h}{\partial z} - \frac{ah^2}{2}\right) + \int\left(\frac{h}{2} + \frac{\eta h'}{2}\right)dz \tag{19}$$

III. NUMERICAL METHOD

The differential equations from (11) to (16) are solved by using MATLAB's built in solving under certain boundary conditions. The impact of Sr on the distributions of temperatures and concentrations are measured. The research is also undertaken to study the effects of Du, R and γ for the distribution of temperature and concentration.

The Nusselt number is also studied for these mentioned parameters.

IV. RESULTS AND DISCUSSIONS

The free convection flow over a rotating disk is studied numerically under the existence of Soret, Dufour, thermal radiation and Chemical reaction effect. For the study, we considered some constant values like $Pr = 0.71, Sr = 0.3, Du = 0.4,$ $R = 0.2, Sc = 2.6, \gamma = 0.2$ and $Re = 0.1$. These values are taken as some throughout our study except the values shown in the respective figure.

(a) Temperature distribution:

Fig. 2 shows that with increase of temperature the values of Du increases. This shows that an increase in Du value causes the temperature to increase. From the definition, it is clear that with increase in value of thermal diffusivity, thermal conductivity also increases and it increases the molecular vibrations of the fluid particles and ultimately overall temperature increases. Figure 3 demonstrates Sr 's effect on temperature. The figure clearly indicates, increases in value of Sr , rises the temperature. Figure 4 shows that the temperature decreases as the radiation parameter value increases. In general, temperature increases with an increase in the radiation parameter value. But, in our present case temperature decreases and this may be due to the geometry.

Figure 5 indicates that the increase in the value of the parameter for chemical reaction contributes to a temperature drop.

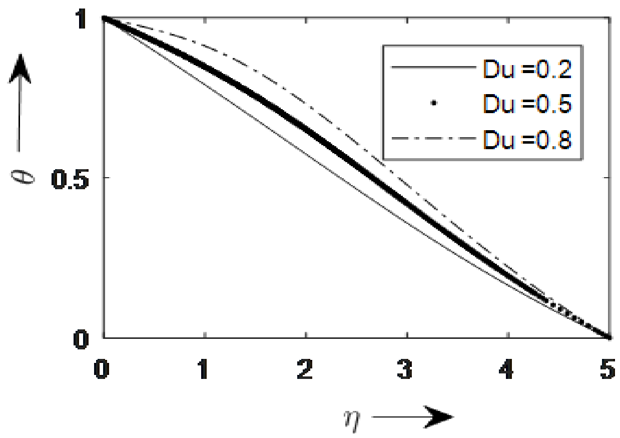

Fig. 2: Temperature with Du variation

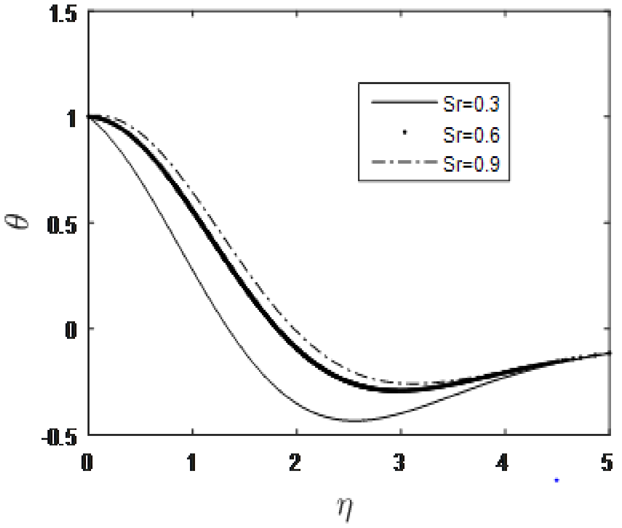

Fig. 3: Temperature with Sr variation

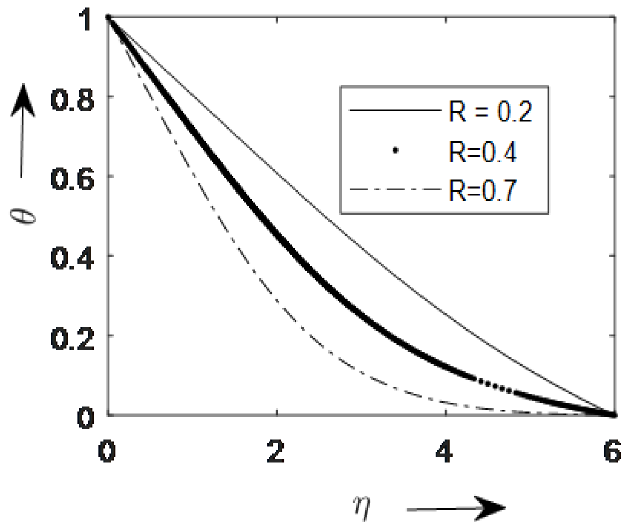

Fig. 4: Temperature with R variation

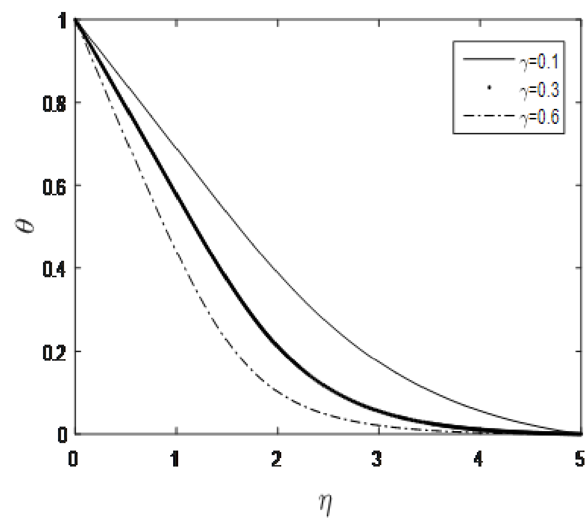

Fig. 5: Temperature with γ variation

(b) Concentration distribution:

Figure 6 indicates that the concentration declines as the Du value increases. Since Soret effect is the reverse of Dufour effect and hence its results are also reverse.

Figure 7 shows that due to the increase in Sr values, the concentration decreases. As a consequence, the mass diffusivity is increased. When the concentration difference is large, there is high mass diffusivity and it probably represents the greater value of mass diffusivity, greater is the concentration gradient and this leads to increase of concentration.

The concentration is enhanced with the rise in R from figure 8. The concentration increases with the increase of γ which is shown in figure 9. Fluid concentration falls by raising the chemical reaction parameter and hence greater the value of γ greater is the decrease in molecular diffusivity. But in our investigation, we obtain concentration increase with increase in value of γ and this may be due to geometry.

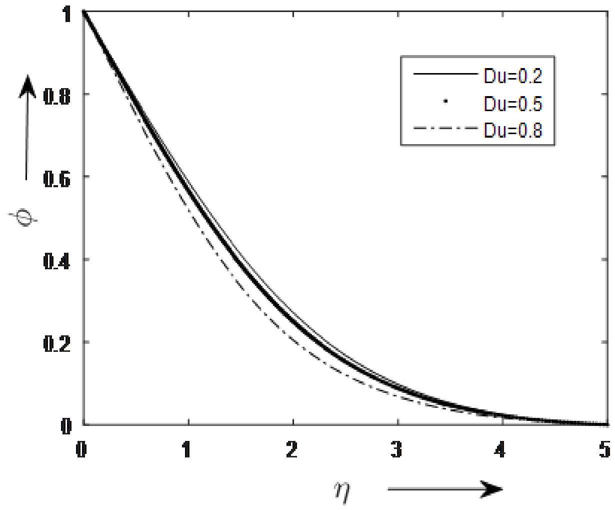

Fig. 6: Concentration with Du variation

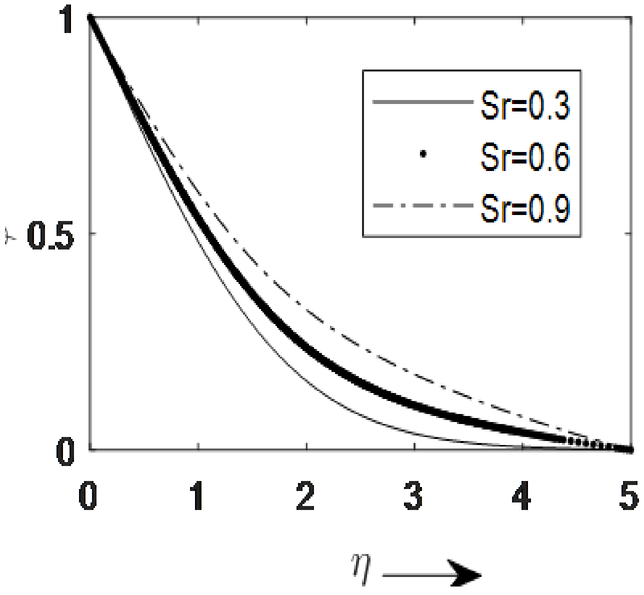

Fig. 7: Concentration with Sr variation

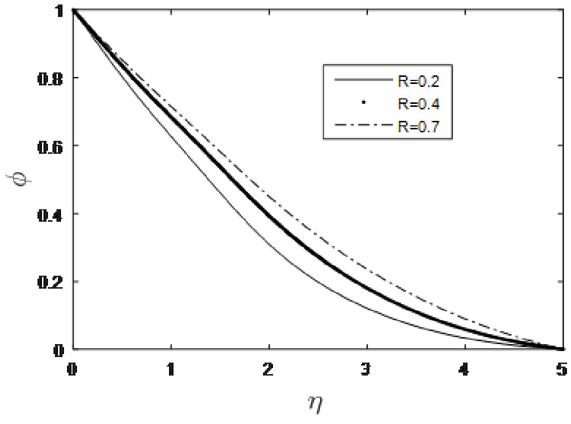

Fig. 8: Concentration with R variation

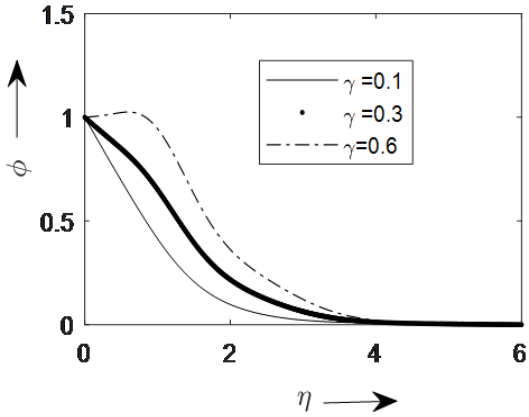

Fig. 9: Concentration with γ variation

The local Nusselt number varies with Sc at different Du values in figure 10. It is observed in fig. 10 that the local Nusselt number as well as Sc increases due to increase in the value of Du. When there is increase in the movement of particles, rate of heat transfer increases.

Fig. 11 indicates that increase in value of Sr and Pr enhance to increase in local Nusselt number. But the local Nusselt number decreases as R and Sc increase and it is shown by figure 12. Again the local Nusselt number decreases with the rise of the value of Pr and γ which is indicated by the figure 13.

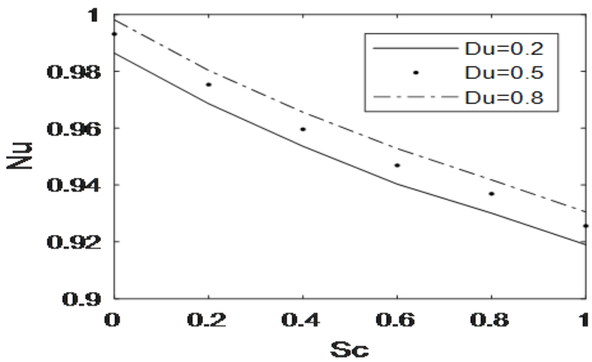

Fig. 10 : Local Nusselt number variance with Sc for various Du values

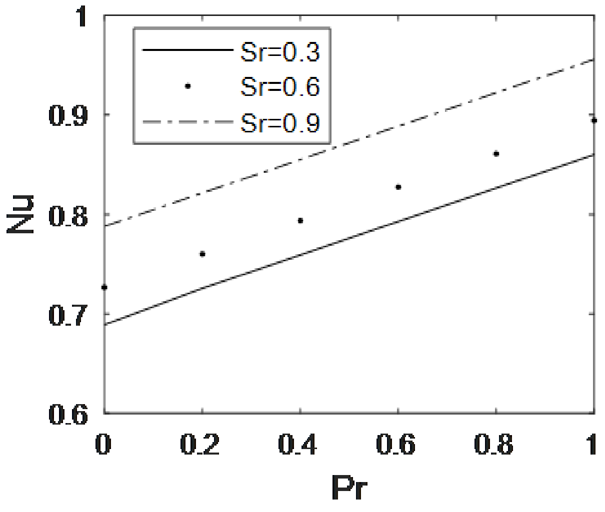

Fig. 11: *Local Nusselt number variance with Pr for different Sr values*

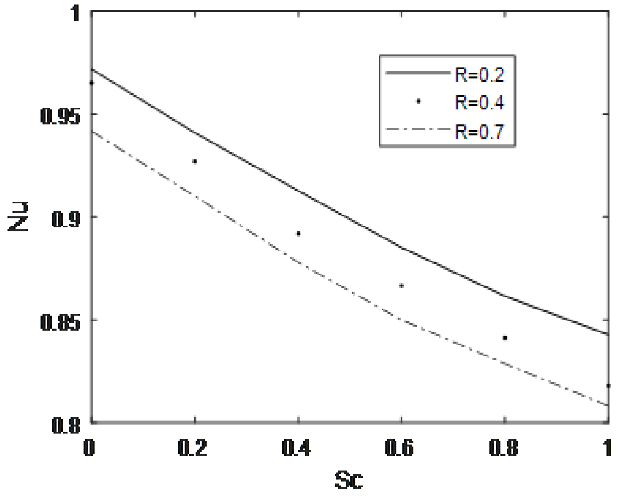

Fig. 12: *Variation of the local Nusselt number to Sc for various R values*

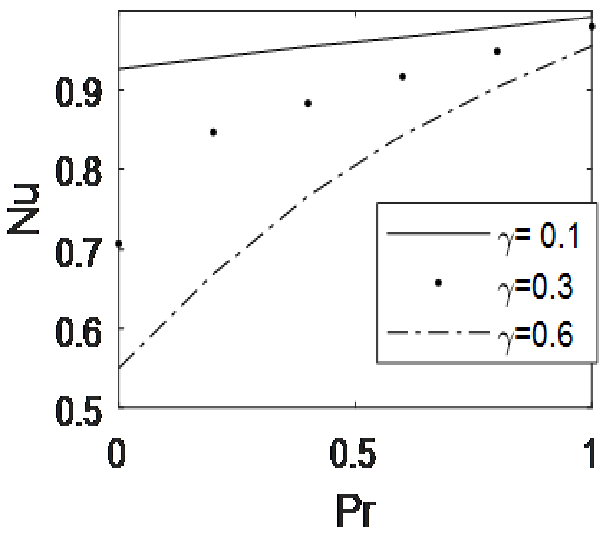

Fig. 13: *Local Nusselt number variation with Pr for different values of* γ

In the result, the heat transfer rate values i.e. Nusselt number is tabulated in the following way in relation to different parameters:

The constant values of different parameters are considered here as $Pr = 0.71, Sr = 0.3, Du = 0.4, R = 0.2, Sc = 2.6, \gamma = 0.2$ and $Re = 0.1$.

Table 1: Values of Nusselt number for different flow parameters.

Parameters	**Values**	**Nusselt Number** (Nu)
Du	0.2	0.9446
	0.5	0.9636
	0.8	0.9837
Sr	0.3	0.7531
	0.6	0.8097
	0.9	0.8502
R	0.2	0.8964
	0.4	0.8646
	0.7	0.8452
γ	0.1	0.9347
	0.3	0.8895
	0.6	0.8264

V. CONCLUSIONS

The following are the major findings of the discussions referred to the above:

(1) When there is increase in the values of Dufour as well as Soret number, temperature of the fluid raises to a high level. On the other hand temperature will be decreased if we increased the parameter of chemical reaction and thermal radiation.

(2) When there is raise of the value of Soret number and parameters of chemical reaction and thermal radiation, concentration of the considered fluid decreases.

(3) When there is raise in the value of local Nusselt number, it helps to increase the value of Dufour as well as Soret number.

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