

Enhancing the Security in ElGamal Cryptosystem using Paring Functions



Mani K., Barakath Begam A.

Abstract: The potential disadvantage of ElGamal cryptosystem is the ciphertext produced is always twice as long as the plaintext i.e., the message expansion by a factor of two takes place during encryption. When the message is too long the ciphertext produced by the ElGamal cryptosystem is also too long. i.e., when the ciphertexts are transmitted through the communication channel which lead to provide less security because if anyone of the ciphertext from two ciphertexts for each character of the plaintext is intercepted by the adversary, the other may be retrieved easily because there is a relationship between the two ciphertexts. If two ciphertexts are reduced to one, the adversary may not be able to predict the two ciphertexts from one. To enhance the security of ElGamal cryptosystem, the binary Cantor function $c: N \times N \rightarrow N$, Rosenberg pairing function and Elegant pairing functions are used in this paper. When the said functions are used, the two ciphertexts produced by each plaintext character are reduced to one, so that the adversary cannot easily be recovered the plaintext. Experimental results clearly revealed enhancing the security of ElGamal cryptosystem after incorporating the pairing functions into it.

Keywords: ElGamal Cryptosystem, Cantor Pairing, Rosenberg pairing function, Elegant pairing function, Security.

I. INTRODUCTION

ElGamal cryptosystem is a kind of public key cryptosystem which utilizes randomization in the encryption process. It was proposed by Taher ElGamal in 1984 [1]. It is used in the free GNU Privacy Guard System, recent version of PGP. The security of ElGamal is based on the intractability of the Discrete Logarithm Problem (DLP) i.e., Z_p^* in the group G and the Diffie-Hellman Problem (DHP). The G should satisfy the two important conditions viz., (i) the G should be relatively easy to apply for efficiency and (ii) the G should be computationally infeasible because of DLP for security. This is because the ElGamal encryption scheme can be viewed as simply comprising a DH key exchange to determine a session key $(\alpha^a)^k$, and then encrypting the message by multiplication with that session key [2]. It is a non-deterministic-cryptosystem because the random key K is chosen each time which results in for the same plaintext (M) multiple times produce different ciphertext (C).

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The padding scheme and the properties of underlying G play a vital role in determining the security of ElGamal cryptosystem. It is noted that ElGamal cryptosystem produces double $c \in C$ for each $m \in M$. Suppose, the length of the M is too long, the C obtained from the M using ElGamal cryptosystem is also too long.

When it is transmitted through the communication channel, it will take more time which leads to customer dissatisfaction and impatience. In order to avoid it, pairing is performed before and after applying the ElGamal encryption. For that, three well known pairing schemes viz., (i) Cantor pairing (ii) Elegant pairing and (iii) Rosenberg pairing are taken in this paper.

The rest of the paper is organized as follows. The various works related to different pairing schemes are discussed in section 2. Mathematical preliminaries related to pairing functions are presented in section 3. Section 4 describes the proposed methodology of ElGamal encryption with pairing schemes. The result of the proposed methodology is discussed in section 5. Finally, section 6 ends with conclusion.

II. RELATED WORKS

Arnold L. Rosenberg [3], provided a short tour about the pairing functions for computation situations and then they discussed about computationally simplest pairing functions-Cauchy-Cantor “diagonal” polynomial. They described the usage of pairing functions based on two specific computational situations in the paper viz., (i) in storage mappings for rectangular arrays/tables, pairing functions could be used for expand and shrink them dynamically and (ii) pairing function was used for instilling accountability mechanism into web-computing projects.

In [4], R.Reddaiah discussed a different types of pairing functions viz., Cantor paring, Elegant pairing, Tate Pairing, Weil pairing, which have a unique nature of handling real numbers (N) while processing the data. The advantages and disadvantages of pairing functions were also discussed in this paper.

B. Harikrishna et al. [5] proposed a symmetric-key encryption method for encrypting two messages at the same time using pairing function which reduced the original length of the M into half. Using the transposition cipher, the K was embedded within the C . Then, it was decrypted with the help of depairing function which produce the original M . The problem of squeezing multiple ciphertexts without losing original information of M was dealt by Myungsun Kim in [6]. For that purpose, they formalized the notion of decomposability for public-key encryption and investigated the reason for adding the decomposability of challenging.



They constructed an ElGamal encryption scheme over extension field and illustrated the support for efficient decomposition. They analyzed the security scheme under the standard DH assumption and evaluated the program too. Steven D. Galbraith et al. [7], aimed to outline the basic choices and usages of pairing functions available for cryptographic schemes in a simple way.

They summarized the main properties and efficiency issues of pairing functions. The work intended the non-specialists to make use of the pairing, who are interested in using pairings for designing the cryptographic schemes. In [8], Mehmet Sabir Kiraz and Osmanbey Uzunkol mainly focused and gave the correct use of the pairing based cryptography in an understandable, informative and most up-to-date manner and they gave a compact and a correct usage of the pairings.

In [9], an identity-based encryption from Weil pairing was proposed. A variant of the computational DHP was assumed for choosing the C security in the random oracle model. The proposed scheme was based on the bilinear maps between the groups. They used the Weil pairing on elliptic curves. For security, the identity based encryption schemes was précised the definitions and also they gave several applications.

Neal Koblitz and Alfred Menezes [10], examined the need of security impact based on pairing function in cryptosystem. At first they described three reasons, why the users may have the long-term viability concerned about high-security of those systems. Then, two families of elliptic curves were discussed for pairing. At first, the pairing values over the prime field F_p has been taken from the defined curve. Now, the super singular curves with embedding degree $k = 2$ were exists in this family. Finally, the efficiency of the Weil pairing opposed to the Tate pairing were examined and a range of choices of embedding degree k was compared.

III. MATHEMATICAL PRELIMINARIES

This section describes the various pairing schemes viz., Cantor, Rosenberg's and Elegant pairing functions which are useful to pair the numbers in this paper.

Definition 1: Pairing Function

It is a bijection between $N \times N$ and N that is also strictly monotone in each of its arguments.

$$p : N \times N \rightarrow N \quad (1)$$

where N is a natural number.

An arbitrary pairing function can be denoted as $f(x, y)$ with pointed brackets $\langle x, y \rangle$. If $\langle x, y \rangle$ be some pairing function, then the pairing function has to recover each x and y arguments from $\langle x, y \rangle$. Hence, it is called as two function projection and it can be written as $\pi_1(z) = x$ and $\pi_2(z) = y$. If $z < \langle x, y \rangle$ then, $\pi_1(z) = x$ and $\pi_2(z) = y$ [11,12].

A. Cantor Pairing Function

Given any set B , a pairing function for B is a 1-1 correspondence from the set of ordered pairs B^2 to the set B . The set B is said to be finite with pairing functions, it has fewer than two elements. A pairing function for B necessarily exists, if B is infinite. The Cantor's pairing function [12, 13] for the integers is of the form

$$c(x, y) = z = \frac{1}{2}(x^2 + 2xy + y^2 - x - 3y + 2) \quad (2)$$

It maps each pair (x, y) of positive integers to a single positive integer $c(x, y)$ or z exists in the literature. There are several variant of Cantor's exist. They are viz., (i) given any pairing function $f(x, y)$ for the positive integers, then the function $f(x + 1, y + 1) - 1$ is also a Cantor's pairing function for the non-negative integers. It is defined as $z = f(x + 1, y + 1) - 1 = \frac{1}{2}(x^2 + 2xy + y^2 + 3x + y)$

(3)

(ii) Given any pairing function $f(x, y)$ for a set B , the Cantor's pairing function is also obtained by exchanging x and y in $c(x, y)$.

Hence,

$$c(x, y) = z = \frac{1}{2}(x + y)(x + y + 1) + y \quad (4)$$

The inverse function of the Cantor's pairing function $c(x, y)$ is defined as

$$c^{-1}(z) = \left(z - \frac{w(w+1)}{2}, \frac{w(w+3)}{2} \right) - z \quad (5)$$

where $w = \left\lfloor \frac{-1 + \sqrt{1+8z}}{2} \right\rfloor$

Cantor Pairing Function- An Example

Consider the two numbers $x = 47$ and $y = 32$. Then using eqn. (2), we have

$$\begin{aligned} c(x, y) = z &= \frac{1}{2}(47^2 + (2 \times 47 \times 32) + (3 \times 47) + 32) \\ &= 3207. \end{aligned}$$

To recover the original numbers, inverse of Cantor function shown in eqn. (5) is used. $w = \left\lfloor \frac{\sqrt{1+(8 \times 3207)}-1}{2} \right\rfloor = 79$ and $c^{-1}(z) = \left(3207 - \frac{79(79+1)}{2}, \frac{79(79+3)}{2} - 3207 \right) = (3207 - 3160, 3239 - 3207) = (47, 32)$.

B. Rosenberg-Strong Pairing Function

The Rosenberg-Strong pairing function [14] for the non-negative integers is defined by the formula

$$r(x, y) = (\max(x, y))^2 + \max(x, y) + x - y \quad (6)$$

In the context of the Rosenberg-Strong pairing function, the quantity $\max(x, y)$ is said to be the shell number of the point (x, y) . The inverse of the Rosenberg-Strong pairing function $r(x, y)$ is given by the formula,

$$r^{-1}(z) = \begin{cases} (z - m^2, m) & \text{if } z - m^2 < m \\ (m, m^2 + 2m - z) & \text{otherwise} \end{cases} \quad (7)$$

where $m = \lfloor \sqrt{z} \rfloor$

Rosenberg-Strong Pairing Function- An Example

Again consider the two numbers $x = 47, y = 32$, using eqn. (6), we have $r(x, y) = z = (47)^2 + 47 + 47 - 32 = 2271$

To recover the original numbers we have $m = \lfloor \sqrt{z} \rfloor = 47$.

Using condition 2 in eqn. (7) we get

$$r^{-1}(z) = 47, (47)^2 + (2 \times 47) - 2271 = (47, 32)$$

C. Elegant Pairing Function

If x and y are non-negative integers of elegant pairing function. Then $E(x, y)$ outputs a single non-negative integer that is uniquely associated with that pair [15].

$$E(x, y) = z = \begin{cases} y^2 + x & x \neq \max(x, y) \\ x^2 + x + y & x = \max(x, y) \end{cases} \quad (8)$$

The inverse function $E^{-1}(z)$ outputs the pair associated with each non-negative integer z is defined as



$$E^{-1}(z) = \begin{cases} \{z - \lfloor \sqrt{z} \rfloor^2, \lfloor \sqrt{z} \rfloor\}, & z - \lfloor \sqrt{z} \rfloor^2 < \lfloor \sqrt{z} \rfloor \\ \{\lfloor \sqrt{z} \rfloor, z - \lfloor \sqrt{z} \rfloor^2 - \lfloor \sqrt{z} \rfloor\}, & z - \lfloor \sqrt{z} \rfloor^2 \geq \lfloor \sqrt{z} \rfloor \end{cases} \quad (9)$$

Elegant Pairing Function – An Example

Consider the two numbers $x = 47$ and $y = 32$. Since, $x = \max(x, y)$ then, using eqn. (8), we have $E(x, y) = z = x^2 + x + y = 47^2 + 47 + 32 = 2288$. To recover the original numbers, inverse of Elegant pairing function defined in eqn. (9), condition 2 is used as $E^{-1}(z) = (47, 2288 - 2209 - 47) = (47, 32)$.

D. ElGamal Encryption

The ElGamal encryption [16] has its security on the basis of difficulty of calculating DL in a finite field. It consists of three steps. They are viz., key generation, performing encryption using public-key, performing decryption using private-key. To Generate a key pair, i.e., private and public-key, first choose a prime p and two random numbers g and x , where both g and x are less than p . Then, compute

$$y = g^x \mod p \quad (10)$$

The public key for encryption process is y, g, p and x is the private key. To perform encryption, choose a random k ,

such that $\gcd(k, p-1) = 1$. Then, encryption is performed by computing

$$a = g^k \mod p \quad (11)$$

$$b = y^k M \mod p \quad (12)$$

Then, a and b are the ciphertexts pair. For decrypting ciphertexts, compute

$$M = \frac{b}{a^x} \mod p \quad (13)$$

IV. PROPOSED METHODOLOGY

For the given M , first find the length of the M denoted as $l(M) = l$. Suppose, M is an odd number, append either 0 or a dummy character in M . In order to perform pairing, for each $m_i \in M$, $ASCII(m_i), i = 1, 2, \dots, l$ is found. Then, the adjacent m_i are paired as $f_i(m_k, m_{k+1}), i = 1, 2, \dots, \frac{l}{2}$ and $k = 1, 3, 5, \dots, l-1$. Pairing of m_i is performed in two ways using said three pairing methods. viz., (i) for two adjacent $m_i, i = 1, 3, \dots, k$, $ASCII(m_i)$ values are paired and encryption is performed using ElGamal encryption (ii) encryption is performed using ElGamal first and the resultant c is paired using pairing methods. The proposed methodology is shown in fig. 1 and 2. It is noted that the ciphertexts are not transmitted to the receiver, instead pairing is performed and the paired numbers are transmitted to the receiver.

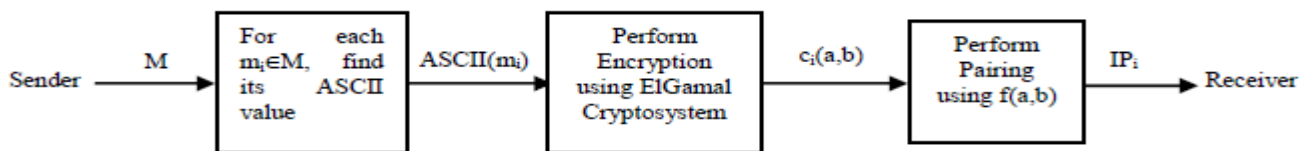


Fig. 1(a): Encryption of plaintext before Pairing Function

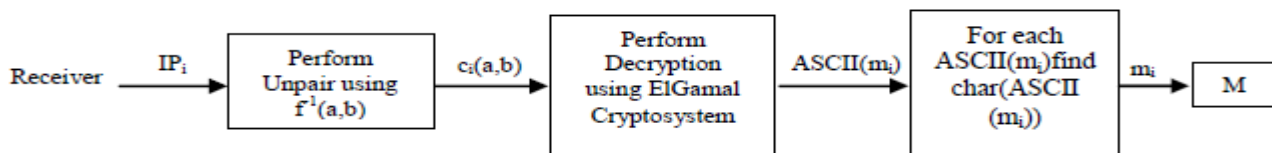


Fig. 1(b): Decryption of IP_i before Pairing Function

Fig. 1: Encryption and Decryption of plaintext before Pairing Function

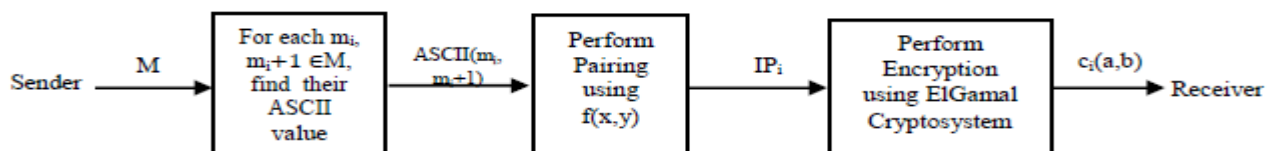


Fig. 2(a): Encryption of Plaintext after Pairing Function

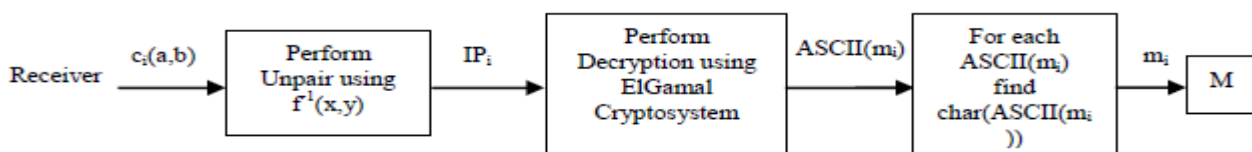


Fig. 2(b): Decryption of $c_i(a,b)$ after Pairing Function

M -Plaintext, m_i -Each Character of the M , $ASCII(m_i)$ -ASCII value of each m_i , $c_i(a,b)$ - Ciphertext of each m_i , IP -Intermediate Plaintext, $f(x,y)$ -Pairing Function and $f^{-1}(x,y)$ -Inverse of Pairing Function.,

Fig. 2: Encryption and Decryption of Plaintext after Pairing Function

Proposed Methodology- An Example

In order to understand the relevance of the work, let M is taken as "KANNAN BABA". Now, $m_1 = "K", m_2 = "A", m_3 = "N", \dots, m_{11} = "A"$ i.e., $m_i \in M$ and $c_i \in C$, where $i=1,2,3,\dots,11$. Then, $ASCII(m_1) = 75$. For ElGamal encryption, let $p = 100003$; $ASCII(m_1) = 75$; $g = 34$; $x = 13$ and $k = 25$. Using eqn. (10), y is computed as $y = 34^{13} \bmod 100003 = 51195$. Using eqns. (11) and (12), m_1 is encrypted as $a = 34^{25} \bmod 100003 = 57337$ and $b = 51195^{25} \times 75 \bmod 100003 = 68229$. Thus, $c_i(a, b) = c_1(57337, 68229)$. It is noted that the $c_i(a, b)$ is not transmitted directly to the receiver. Instead, it is converted into single integer using the pairing functions. Let the (a, b) is considered as $c(x, y)$.

A. ElGamal Encryption Before Pairing Function

Based on the ElGamal encryption, the ciphertext for m_1 of M is $c_1(57337, 68229)$.

(i) Cantor Pairing

Cantor pairing is performed using eqn. (2). Let $c_1(57337, 68229) = c(57337, 68229) = z = \frac{1}{2}(57337^2 + (2 \times 57337 \times 68229) + 68229^2 - 57337 - 3 \times 68229 + 2) = 7883279167$. This is called IP_1 for calculating $c^{-1}(z)$ first, w is calculated using eqn. (5) as $w = \left\lfloor \frac{\sqrt{1+8 \times 7883279167} - 1}{2} \right\rfloor = 125564$. After calculating w , unpair of $c(x, y)$ is performed using eqn. (5). Now, $c^{-1}(z) = 7883279167 - \frac{125564(125564+1)}{2}, \frac{125564(125564+3)}{2} - 7883279167 = (57337, 68229)$. Decryption is performed using eqn. (14), $m_1 = \frac{68229}{57337^{13}} \bmod 100003 = 68229 \times 76912^{-1} \bmod 100003 = 68229 \times 76439 \bmod 100003 = 75$.

(ii) Rosenberg Strong Pairing

Let c_1 from the ElGamal encryption is taken as $r(x, y) = (57337, 68229)$. Then, using eqn. (6), $r(x, y) = (\max(57337, 68229)^2 + \max(57337, 68229) + 57337 - 68229 = 68229^2 + 68229 + 57337 - 68229 = 4655253670$. This is called IP_1 . For unpairing the paired $r(x, y)$, it must satisfy the condition using eqn. (7). Then, calculate $m = 68229$. Here, for unpairing $r(x, y)$ the two conditions are applied and condition 1 is satisfied so, calculate using eqn. (7) $r(x, y)^{-1} = (4655253670 - 68229^2, 68229) = (57337, 68229)$.

(iii) Elegant Pairing

Let c_1 from the ElGamal encryption is taken as $E(x, y) = (57337, 68229)$. Using eqn. (8), then $E(x, y) = 68229^2 + 57337 = 4655253778$. Now, $z = IP_1 = 4655253778$ and for unpairing $E(x, y)$ using eqn. (9), $E^{-1}(x, y) = (4655253778 - \lfloor \sqrt{4655253778} \rfloor^2, \lfloor \sqrt{4655253778} \rfloor) = (57337, 68229)$. Decryption is performed using eqn. (14), $M = \frac{68229}{57337^{13}} \bmod 100003 = 68229 \times 76912^{-1} \bmod 100003 = 68229 \times 76439 \bmod 100003 = 75$.

B. ElGamal Encryption after Pairing Function

Before performing pairing, consider $x = ASCII(m_1) = 75$ and $y = ASCII(m_2) = 65$.

(i) Cantor Pairing

Cantor pairing is performed for (x, y) using the eqn. (2) i.e., $c(x, y) = c(75, 65)$. Now, $c(75, 65) = z = \frac{1}{2}(75^2 + (2 \times 75 \times 65) + 65^2 - 75 - (3 \times 65) + 2) = 9945$. The value of z is taken as IP_1 for performing ElGamal encryption instead of original plaintext m_1, m_2 . Using eqns. (10), (11) and (12), we have $y = 34^{13} \bmod 100003 = 51195$ $a = 34^{25} \bmod 100003 = 57337$. $b = 51195^{25} \times 9945 \bmod 100003 = 66896$. The ciphertexts $(57337, 66896)$ are transmitted to the receiver. After receiving the ciphertexts, the receiver decrypts using eqn. (13) so that he/she gets the IP_1 i.e., $IP_1 = \frac{66896}{57337^{13}} \bmod 100003 = 66896 \times 76912^{-1} \bmod 100003 = 66896 \times 76439 \bmod 100003 = 9945$. In order to get the $ASCII(m_1)$ and $ASCII(m_2)$, IP_1 is unpaired using eqn. (5) i.e., $w = \left\lfloor \frac{\sqrt{1+8 \times 9945} - 1}{2} \right\rfloor = 140$ and $c^{-1}(z) = 9945 = (75, 65)$.

(ii) Rosenberg Pairing with ElGamal

For the same $(x, y) = (75, 65)$, using eqn. (7), we have $r(x, y) = IP_1 = (\max(75, 65)^2 + \max(75, 65) + 75 - 65 = 75^2 + 75 - 65 = 5710 = 5710$. To perform encryption, we have $y = 34^{13} \bmod 100003 = 51195$. $a = 34^{25} \bmod 100003 = 57337$. $b = 51195^{25} \times 5710 \bmod 100003$. This ciphertexts $(57337, 54347)$ are transmitted to the receiver. In order to recover the IP_1 , we have $IP_1 = \frac{54347}{57337^{13}} \bmod 100003 = 54347 \times 76912^{-1} \bmod 100003 = 54347 \times 76439 \bmod 100003 = 5710$. To recover the original $ASCII(m_1)$ and $ASCII(m_2)$, unpairing is performed using eqn. (8) as $m_i = 75$.

$$r(x, y)^{-1}(IP)_1 = (75, 75^2 + 2 \times 75 - 5710) = (75, 5625 + 150 - 5710) = (75, 65).$$

(iii) Elegant Pairing with ElGamal

Let, $E(x, y) = (75, 65)$ and $E(x, y) = 75^2 + 75 + 65 = 5765$. $IP_1 = 5765$ and for unpairing $M = 5765$; $p = 100003$; $g = 34$; $x = 13$; $k = 25$. Encryption is performed using the eqns. (12) and (13) $y = 34^{13} \bmod 100003 = 51195$ and $a = 34^{25} \bmod 100003 = 57337$. $b = 51195^{25} \times 5765 \bmod 100003 = 84381$. $M = \frac{84381}{57337^{13}} \bmod 100003 = 84381 \times 76912^{-1} \bmod 100003 = 84381 \times 76439 \bmod 100003 = 5765$. $E^{-1}(x, y) = (\lfloor \sqrt{5765} \rfloor, 5765 - 5765^2, 5765) = (75, 65)$.

Similar computations can also be performed for other $m_i \in M$, the resultant are tabulated in table I and II for before and after pairing respectively.

Table I: Encryption of Plaintext before Pairing

m _i	ASCII(m _i)	Encryption of Plaintext before Pairing								
		ElGamal			Cantor		Rosenberg		Elegant	
		a	b	No. of bits	z	No. of bits	z	No. of bits	z	No. of bits
K	75	57337	68229	16+17=33	7.88E+09	33	4.655E+09	33	4.66E+09	33
A	65	57337	99133	16+17=33	1.22E+10	34	9.827E+09	34	9.83E+09	34
N	78	57337	98959	16+17=33	1.22E+10	34	9792941018	33	9.79E+09	34
N	78	57337	98959	16+17=33	1.22E+10	34	9792941018	33	9.79E+09	34
A	65	57337	99133	16+17=33	1.22E+10	34	9.827E+09	34	9.83E+09	34
N	78	57337	98959	16+17=33	1.22E+10	34	9792941018	33	9.79E+09	34
BS	32	57337	44423	16+16=32	5.18E+09	33	3287601820	32	3.29E+09	32
B	66	57337	76042	16+17=33	8.9E+09	34	5782443101	33	5.78E+09	33
A	65	57337	99133	16+17=33	1.22E+10	34	9.827E+09	34	9.83E+09	34
B	66	57337	76042	16+17=33	8.9E+09	34	5782443101	33	5.78E+09	33
A	65	57337	99133	16+17=33	1.22E+10	34	9.827E+09	34	9.83E+09	34
		Total No. of bits			362		372		366	
										369

Table II: Encryption of Plaintext after Pairing

m _i	ASCII (m _i)	Cantor with ElGamal				Rosenberg with ElGamal				Elegant with ElGamal			
		Cantor	ElGamal			Rosenberg	Elgamal			Elegant	ElGamal		
		z	a	b	No. of Bits	z	a	b	No. of Bits	z	a	b	No. of Bits
K	75	9945	57337	66896	16+17 =33	5710	57337	54347	16+16 =32	5765	57337	84381	16+17 =33
A	65				=32				=31				=33
N	78	12324	57337	35054	16+16 =32	6162	57337	17527	16+15 =31	6240	57337	16483	16+15 =31
N	78				=32				=31				=31
A	65	10361	57337	61328	16+16 =32	6149	57337	17701	16+15 =31	6149	57337	17701	16+15 =31
N	78				=32				=31				=31
BS	32	4883	57337	50031	16+16 =32	4388	57337	79734	16+17 =33	4388	57337	79734	16+17 =33
B	66				=32				=33				=33
A	65	8711	57337	60335	16+16 =32	4421	57337	17752	16+15 =31	4421	57337	17752	16+15 =31
B	66				=32				=31				=31
A	65	2210	57337	70423	16+17 =33	4355	57337	41713	16+16 =32	65	57337	99133	16+17 =33
Total No. of Bits					194				190				192

BS-Blank Space

V. RESULTS AND DISCUSSION

The proposed methodology is implemented in VC++ with IDE 5.0. The said three pairing functions are applied in the M before and after performing encryption using ElGamal cryptosystem with different file size. The security level is measured using All Block Cipher (ABC) Universal Hackman tool which uses dictionary attack. The encryption time, decryption time and the security level are recorded in table 3, 4 and 5 respectively. The graphical representation of the said parameters is also shown in fig. 3, 4 and 5 respectively. It is evident from tables III and IV that the time taken for performing encryption and decryption is increasing before pairing than after pairing.

In before pairing, for each m_i two different ciphertexts are produced when ElGamal encryption is applied and the magnitude of c_i depends on the modulus p and its value ranging from 0 to $p - 1$. Suppose, if M has n number of m_i , then $2n$ number of ciphertexts are produced. While applying pairing, it is reduced to again n . The number of bits involved in pairing is increasing for all pairing schemes when it is compared with conventional ElGamal. It is observed from the table I that even though the number of bits producing the ciphertext before pairing is larger than the conventional ElGamal encryption, the security is increasing before pairing because, the ciphertexts obtained from the ElGamal encryption is converted into single ciphertext for each character and it is not able to predictable by the adversary.

In the case of after pairing, $ASCII(m_i)$ and $ASCII(m_i + 1)$ are combined which is reduced to one z_i . It is evident from table II, that the number of bits required for ciphertexts are reduced approximately by half of the bits required for the plaintext when conventional and before ElGamal encryption which results in decreasing the encryption and decryption time. Further, it is noticed from table 2 that the number of ciphertexts are exactly equal to the number of characters in M contrary to the $2c_i$ produced by the conventional ElGamal. It also enhances the security because original $m_i \in M$ are not used for encryption but it uses only the paired value.

Table III: Encryption Time Produced by ElGamal Encryption Before and After Pairing

File Size (in mb)	Existing Method (in ms)	Proposed Method (in ms)					
		Before Pairing			After Pairing		
		EIG	CPEL	RPEL	EPEL	ELCP	ELRP
1	2633	3703	3360	3241	3716	3346	3315
2	4710	6463	5897	5825	6212	5835	5676
3	7302	9892	9051	8763	9735	9125	8803
4	9769	13374	12260	11949	13110	12248	11861
5	12323	16751	15547	14978	16623	15484	14978
Avg.	7347.4	10037	9223	8951.2	9879.2	9207.6	8926.6

Table IV: Decryption Time Produced by ElGamal Encryption Before and After Pairing

File Size (in mb)	Existing Method (in ms)	Proposed Method (in ms)					
		Before Pairing			After Pairing		
		ELG	CPEL	RPEL	EPEL	ELCP	ELRP
1	2741	3736	3344	3316	3706	3295	3203
2	4643	6507	5931	5769	6394	5973	5724
3	7172	9922	9114	8813	9784	9133	8803
4	9945	13408	12384	11987	13264	12271	11812
5	12209	16509	15573	15098	16766	15605	15137
Avg.	7342	10016.4	9269.2	8996.6	9982.8	9255.4	8935.8

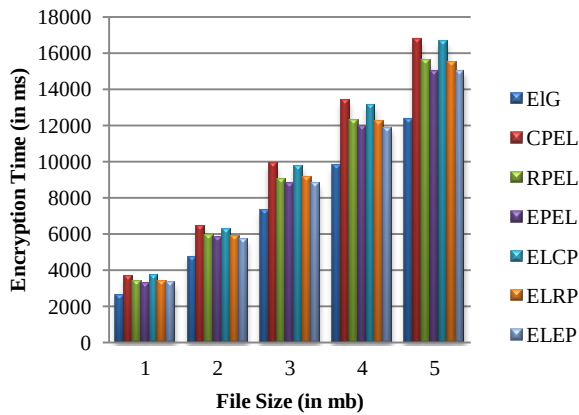


Fig. 3: Encryption Time Produced by ElGamal Encryption Before and After Pairing

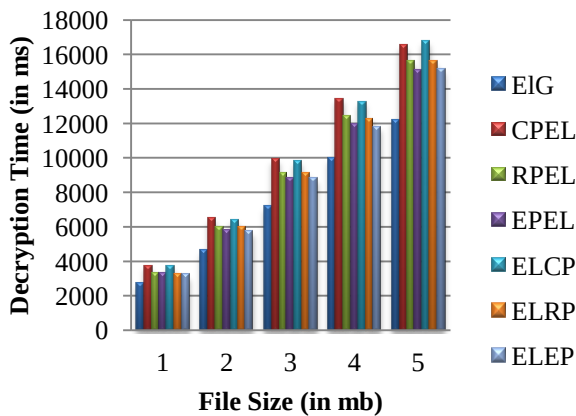
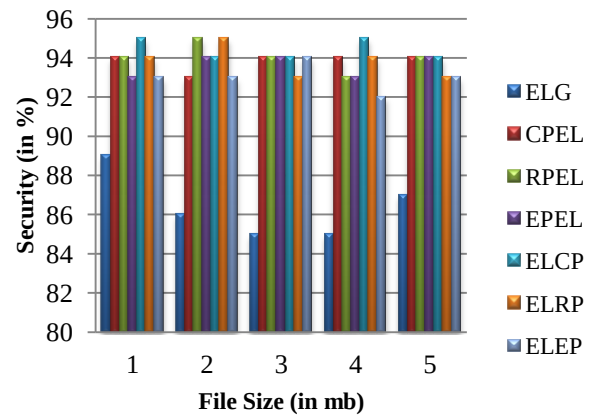


Fig. 4: Decryption Time Produced by ElGamal Encryption Before and After Pairing

Table V: Security Level Produced by ElGamal Encryption before and After Pairing

File Size (in mb)	Existing Method (in %)	Proposed Method (in %)					
		Before Pairing			After Pairing		
		ELG	CPEL	RPEL	EPEL	ELCP	ELRP
1	89	94	94	93	95	94	93
2	86	93	95	94	94	95	93
3	85	94	94	94	94	93	94
4	85	94	93	93	95	94	92
5	87	94	94	94	94	93	93
Avg.	86.4	93.8	93.2	93.6	94.4	94.2	94



ELG- ElGamal, CPEL-Cantor Pairing with ElGamal, RPEL- Rosenberg's Pairing with ElGamal, EPEL-Elegant Pairing with ElGamal, ELCP- ElGamal with Cantor Pairing, ELRP-ElGamal with Rosenberg's Pairing, ELEP- ElGamal with Elegant Pairing

Fig. 5: Security Level Produced by ElGamal Encryption before and After Pairing

VI. CONCLUSION

An enhanced version of ElGamal encryption with three different pairing methods is thought of and implemented them successfully. The experimental results clearly indicated that there is an increase in the security level of ElGamal encryption when pairing function are used in it. It is noted that the security level is increasing in ElGamal by 94.4%, 94.2% and 94% after pairing than before pairing by 93.8%, 93.2% and 93.6% but the conventional ElGamal produces only 86.4% security level. It is observed from the experimental results that that the number of bits has been reduced when pairing the plaintext is performed after encryption rather than before pairing. Further, it is evident from the tables that the security is increasing when the encryption is performed after pairing the plaintext than before pairing for all the three pairing methods taken. Among the three pairing methods, Cantor pairing provides more security than other two counterparts when encryption is performed before and after pairing. Also, Rosenberg's Strong pairing scheme is the best because it reduces the number of bits with respect to all pairing which in turn increases the transmission speed of ciphertext when it is transmitted through the communication channel.

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