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Even-Odd Harmonious Labeling of Certain Family of Cyclic Graphs

M. Kalaimathi, B. J. Balamurugan

***Abstract*: An even-odd harmonious labeling of a graph** *G with p vertices and q edges is a process of assigning odd numbers $1,3,5,...,2q-1$ to the vertices through a 1-1 computing function f and even numbers $0,2,4,...,2(p-1)$ to the edges through a bijective computing function f^* with the condition that $f^*(e=uv)=(f(u)+f(v))(\text{mod } 2q)$ where u and v are the vertices of G. This type of labelled graph is called as even-odd harmonious graph.*

Keywords*: *Graphs, Even-Odd Harmonious Labeling, Injective Function, Bijective Function.

I. INTRODUCTION

Labeling of a graph is an immense and vast area of research in the field of graph theory. It deals on how vertices and edges of a graph are labelled with respect to certain mathematical condition [1]. The recent research advancements in the field of graph labeling have been taken from Gallian [5]. Rosa [10] introduced the labelled graphs and N. Lakshmi Prasana et al. stated the applications of graph labeling in [8].The concepts and terminology of graphs used in this paper are referred to the textbooks Harary [4] and West [12]. In the year 1980, Graham and Sloane [3] introduced the harmonious graph and later Z. Liang et al. [9] and P.B. Sarasija et al. [11] introduced odd and even harmonious graphs respectively in the years 2009 and 2011 and in the later years even-odd harmonious labeling of a graph was introduced by Adalin Beatress and Sarasija [2]. Subsequently, in the year 2019, we have proved in [6] the existence of the even-odd harmonious labeling of certain graphs which are obtained through certain graph operations. Further in the same year in [7], we also shown the existence of this labeling to certain family of acyclic graphs. Now in this article, we prove the existence of this labeling to certain family of cyclic graphs.

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II. PRELIMINARIES

In this section, we recall the definitions of certain graphs pertaining to this paper.

Definition 2.1

A fan graph $F_n\,(n\geq 2)$ is obtained by joining all vertices of the path P_n to a common vertex called the center of F_n, and contains $n+1$ vertices and $2n-1$ edges.

Definition 2.2

A ladder graph $L_n = P_n \times K_2$ is a planar, undirected, connected graph with $2n$ vertices and $n+2(n-1)=3n-2$ edges.

Definition 2.3

The product graph of path P_n and cycle C_m is called prism graph and it is symbolized by $P_n \times C_m$.

Definition 2.4

The total graph $T(G)$ of G is the graph with the vertex set $V \cup E$ and two vertices are adjacent whenever they are either adjacent or incident in G. For example, when $G=P_n$, total graph of path $T(P_5)$ is given in Fig. 1.

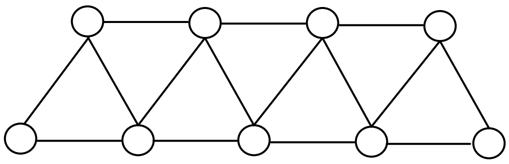

Fig. 1 $T(P_5)$ graph

Definition 2.5

The braid graph $B(n)$ is obtained from the pair of paths $P_n^{'}$ and $P_n^{''}$. Let $v_1, v_2, \ldots, v_n$ are the vertices of path $P_n^{'}$ and $u_1, u_2, \ldots, u_n$ are the vertices of path $P_n^{''}$. The braid graph is defined as a join i^{th} vertex of path $P_n^{'}$ with $(i+1)^{th}$ vertex of path $P_n^{''}$ and the i^{th} vertex of path $P_n^{''}$ with $(i+2)^{th}$ vertex of path $P_n^{'}$ with the new edges for all $1 \leq i \leq n-2$.

Definition 2.6

The jellyfish graph $J(m,n)$ is a graph with the vertex set

$$V\left(J\left(m,n\right)\right)=\left\{u,v,x,y\right\}\cup\left\{u_1,u_2,\ldots,u_m\right\}\cup\left\{v_1,v_2,\ldots,v_n\right\}$$

and the edge set

$$\left(J\left(m,n\right)\right)=\left\{(u,x),(u,y),(v,x),(v,y),(x,y)\right\}\cup\left\{(u_i,u):i=1,2,\ldots,m\right\}\cup\left\{(v_i,v):i=1,2,\ldots,n\right\}.$$

Definition 2.7

The $P_{2n}(+)N_m$ is a graph obtained by joining the endpoints of path P_n with m further vertices so that $V\left(P_{2n}(+)N_m\right)=\left\{u_1,u_2,\ldots,u_{2n},v_1,v_2,\ldots,v_m\right\}$ where $V\left(P_{2n}\right)=\left\{u_1,u_2,\ldots,u_{2n}\right\}$ and $V\left(N_m\right)=\left\{v_1,v_2,\ldots,v_m\right\}$ and $E\left(P_{2n}(+)N_m\right)=E\left(P_{2n}\right)\cup\left\{(u_1,v_1),(u_1,v_2),\ldots,(u_1,v_m),(u_{2n},v_1),(u_{2n},v_2),\ldots,(u_{2n},v_m)\right\}$

For example $P_4(+)N_2$ is shown in Fig. 2.

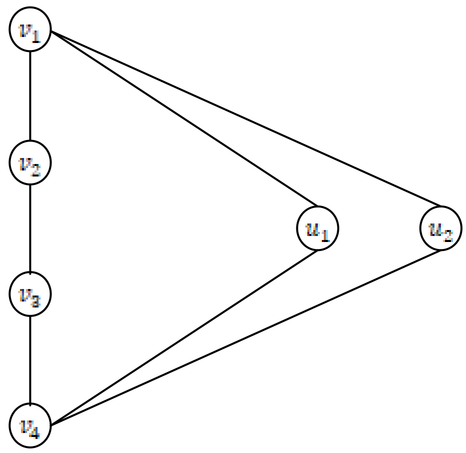

Fig. 2 $P_4(+)N_2$ graph

Definition 2.8

The wheel W_n is the graph obtained by joining every vertex of the cycle C_n to exactly one vertex called the center. The edges incident to the center are called spokes.

III. EVEN-ODD HARMONIOUS LABELING OF GRAPHS

In this section, we recall the definition of even-odd harmonious labeling of graphs [2].

Definition 3.1

Let $G(V,E)$ be a graph with order p and size q. An injective function $f:V\rightarrow\left\{1,3,5,\ldots,2p-1\right\}$ is called an even-odd harmonious labeling of the graph G if the induced edge function $f^*:E\rightarrow\left\{0,2,\ldots,2(q-1)\right\}$ defined by $f^*(e=uv)=\left(f(u)+f(v)\right)(mod\ 2q)$ is bijective function.

Definition 3.2

An even-odd harmonious graph is a graph which admits the even-odd harmonious labeling.

Remark

The even-odd harmonious labeling of a graph is called as EOH labeling of a graph for simplicity in this paper.

Example 3.1

An EOH labeling of G is shown in Fig. 3

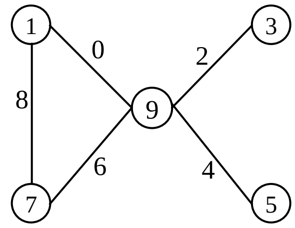

Fig. 3 EOH labeling of G.

IV. EVEN-ODD HARMONIOUS LABELING OF CYCLIC GRAPHS

Theorem 4.1

The fan graph F_n admits an EOH labeling when $2\le n\le 6$.

Proof

Let $V=\{u\}\cup\{v_i:1\le i\le n\}$ be the vertex set of fan graph where u be the common vertex and v_i are the vertices of the path P_n.

Let $E=\{e_{ui}=uv_i:1\le i\le n\}\cup\{e_i=v_iv_{i+1}:1\le i\le n-1\}$ be the edge set of the fan graph F_n. Here the fan graph has $p=n+1$ vertices and $q=2n-1$ edges.

Define an injective function $f:V\rightarrow\{1,3,\ldots,2(n+1)-1\}$ such that

Case (i) $n=2$

$$f(u)=1$$

$$f(v_i)=2i+1,\ 1\le i\le 2$$

Case (ii) $n=3$

$$f(u)=2n+1$$

$$f(v_1)=n+2,\ f(v_2)=1,\ f(v_3)=n$$

Case (iii) $n=4$

$$f(u)=n+3$$

$$f(v_1)=2n+1, f(v_2)=n+1, f(v_3)=1, f(v_4)=n-1$$

Case (iv) $n=5$

$$f(u)=n+2$$

$$f(v_1)=2n+1, f(v_2)=2n-1, f(v_3)=n,$$
$$f(v_4)=1, f(v_5)=n-2$$

Case (v) $n=6$

$$f(u)=n-1$$

$$f(v_1)=2n-1, f(v_2)=2n+1, f(v_3)=n+3,$$
$$f(v_4)=n-1, f(v_5)=1, f(v_6)=n-3$$

and an induced edge function $f^*:E\rightarrow\{0,2,4\ldots,\ 2(2n-1)-2\}$ such that

Case (i) $n=2$

$$f^*(e_1)=2$$
$$f^*(uv_1)=4, f^*(uv_2)=0$$

Case (ii) $n=3$

$$f^*(e_1)=6, f^*(e_2)=4$$
$$f^*(uv_1)=2, f^*(uv_2)=8, f^*(uv_3)=0$$

Case (iii) $n=4$

$$f^*(e_1)=0, f^*(e_2)=6, f^*(e_3)=4$$
$$f^*(uv_1)=2, f^*(uv_2)=12, f^*(uv_3)=8, f^*(uv_4)=10$$

Case (iv) $n=5$

$$f^*(e_1)=2, f^*(e_2)=14, f^*(e_3)=6, f^*(e_4)=4$$
$$f^*(uv_1)=0, f^*(uv_2)=16, f^*(uv_3)=12,$$
$$f^*(uv_4)=8, f^*(uv_5)=10$$

Case (v) $n=6$

$$f^*(e_1)=2, f^*(e_2)=0, f^*(e_3)=14,$$
$$f^*(e_4)=6, f^*(e_5)=4$$
$$f^*(uv_1)=18, f^*(uv_2)=20, f^*(uv_3)=16,$$
$$f^*(uv_4)=12, f^*(uv_5)=8, f^*(uv_6)=10$$

Where f^* is a 1-1 and onto function. The computing functions defined in the proof of the theorem are giving the EOH labeling of G.

Hence, the fan graph F_n is an EOH graph when $2\le n\le 6$.

Theorem 4.2

The ladder graph L_n admits an EOH labeling when $n\equiv 1(mod\ 2)$.

Proof

Let $V=\{u_i:1\le i\le n\}\cup\{v_j:1\le j\le n\}$ be the vetex set of the ladder graph L_n. Let $E=\{e_i=u_iu_{i+1}:1\le i\le n-1\}\cup\{e_j^{'}=v_jv_{j+1}:1\le j\le n-1\}\cup\{e_{ij}=u_iv_j:1\le i\le n\,and\,1\le j\le n\}$ be the edge set of the ladder graph L_n. Here the ladder graph has $p=2n$ vertices and $q=3n-2$ edges.

Define an injective function $f:V\rightarrow\{1,3,\ldots,2(2n)-1\}$ such that

$$f^*:E\rightarrow\{0,2,4,\ldots 2(3n-2)-2\}$$

$$f(u_i)=\begin{cases} i, & i\,is\,odd \\ n+i, & i\,is\,even\end{cases}$$

$$f(v_i)=\begin{cases} 2n+i-1, & i\,is\,even \\ 2n+n-1+i, & i\,is\,odd\end{cases}$$

and an induced edge function $f^*:E\rightarrow\{0,2,4,\ldots 2(3n-2)-2\}$ such that

$$f^*(e_i)=f^*(u_iu_{i+1})=(n+2i+1)(mod\ 2q), 1\le i\le n-1$$
$$f^*(e_j^{'})=f^*(v_jv_{j+1})=(5n+2j-1)(mod\ 2q), 1\le j\le n-1$$
$$f^*(e_{ij})=f^*(u_iv_j)=(3n+2i-1)(mod\ 2q),$$
$$1\le i\le n\ and\ 1\le j\le n$$

Where f^* is a 1-1 and onto function. The computing functions defined in the proof of the theorem are giving the EOH labeling of G.

Hence, the ladder graph L_n is an EOH graph when $n\equiv 1(mod\ 2)$.

Example 4.1

An EOH labeling of L_7 is shown in Fig. 4

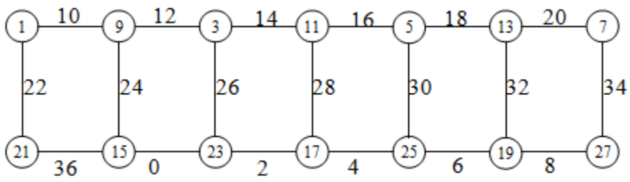

Fig. 4 EOH labeling of L_7.

Theorem 4.3

The prism graph $P_n\times C_{2m+1}$ admits an EOH labeling

Proof

Let v_{11} be any fixed vertex of the innermost C_{2m+1} and $v_{12},v_{13},\ldots,v_{2m+1}$ be the other vertices of the cycle taken in the clockwise direction. For $2\le i\le n$, let v_{i1} be the vertex of the i^{th} copy of C_{2m+1} adjacent to the vertex $v_{(i-1)(2m+1)}$ in the $(i-1)^{th}$ copy of C_{2m+1} and take the other vertices v_{ij} in the clockwise direction as in the first copy of C_{2m+1}.

Where the vertex set is $V=\{v_{ij} :1\le i\le n \, and\, 1\le j\le 2m+1\}$ and the edge set of G is $E=\{e_{ij}=v_{ij}v_{ij+1} :1\le i\le n \, and\, 1\le j\le 2m\}\cup$ $\{e_{ij}=v_{ij}v_{i1} : j=2m+1 \, and\, 1\le j\le n\}\cup$ $\{e_{ij}=v_{ij}v_{i+1j+1} :1\le i\le n \, and\, 1\le j\le 2m\}\cup$ $\{e_{ij}=v_{ij}v_{i+1,1} :1\le i\le n \, and\, j=2m+1\}$.

Define an injective function $f:V\to\{1,3,\ldots,2(nm)-1\}$ such that

$$f(v_{ij})=\begin{cases}(i-1)(2m)+j, & j\, is\, odd\\ (i-1)(2m+1)+m+j, & j\, is\, even\end{cases}$$

and an induced edge function $f^*:E\to\{0,2,4,\ldots 2(nm+(n-1)m)-2\}$ such that

$$f^*(e_{ij})=((i-1)(4m)+m+1+2j)(mod\ 2q)$$

Where f^* is a 1-1 and onto function. The computing functions defined in the proof of the theorem are giving the EOH labeling of G.

Hence, the prism graph $P_n\times C_{2m+1}$ is an EOH graph.

Example 4.2

An EOH labeling of $P_3\times C_5$ is shown in Fig. 5

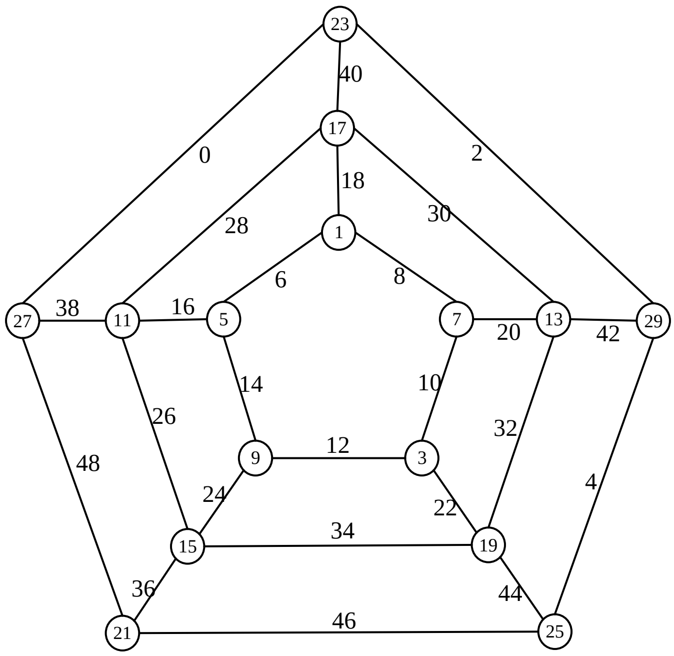

Fig. 5 EOH labeling of $P_3\times C_5$

Theorem 4.4

The total graph $T(P_n)$ admits an EOH labeling when $n\ge 2$.

Proof

Let $V=\{u_i :1\le i\le n\}\cup\{v_j :1\le j\le n-1\}$ be the vertex set of the total graph where u_i are the vertex of the path P_n $E=\{e_i=u_iu_{i+1} :1\le i\le n-1\}\cup$ $\{e_j^{'}=v_jv_{j+1} :1\le j\le n-1\}\cup$ $\{e_{ij}=u_iv_j :1\le i\le n-1 \, and\, 1\le j\le n-1\}\cup$ $\{e_{ij}^{'}=u_{i+1}v_j : i=j, 1\le i\le n-1 \, and\, 1\le j\le n-1\}$ be the edge set of the total graph $T(P_n)$. Here the total graph has $p=2n-1$ vertices and $q=4n-5$ edges.

Define an injective function $f:V\to\{1,3,\ldots,2(2n-1)-1\}$ such that

$$f(u_i)=4(i-1)+1,\ 1\le i\le n$$

$$f(v_j)=4(j-1)+3,\ 1\le j\le n-1$$

and an induced edge function $f^*:E\to\{0,2,4,\ldots,2(4n-5)-2\}$ such that

$$f^*(e_i)=f^*(u_iu_{i+1})=(6+8(i-1))(mod\ 2q),\ 1\le i\le n-1$$

$$f^*(e_j^{'})=f^*(v_jv_{j+1})=(10+8(j-1))(mod\ 2q),\ 1\le j\le n-2$$

$$f^*(e_{ij})=f^*(u_iv_j)=(4+(i-1)8)\ (mod\ 2q),\ 1\le i\le n-1, 1\le j\le n-1$$

$$f^*(e_{ij}^{'})=f^*(u_{i+1}v_j)=(8+(i-1)8)(mod\ 2q),\ 1\le i\le n-1, 1\le j\le n-1$$

Where f^* is a 1-1 and onto function. The computing functions defined in the proof of the theorem are giving the EOH labeling of G.

Hence, the total graph $T(P_n)$ is an EOH graph when $n\ge 2$.

Example 4.3

An EOH labeling of $T(P_6)$ is shown in Fig. 6

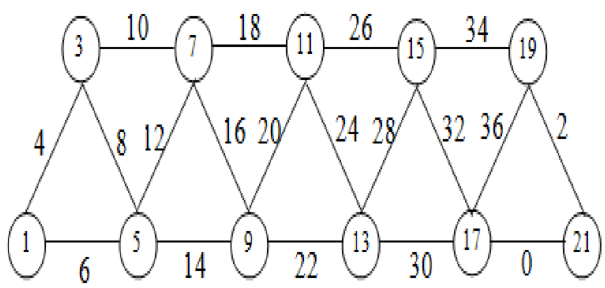

Fig. 6 EOH labeling of $T(P_6)$

Theorem 4.5

The braid graph $B(n)$ admits an EOH labeling, when $n\ge 3$.

Proof

Let $V=\{u_i :1\le i\le n\}\cup\{v_j :1\le j\le n\}$ be the vertex set of the braid graph.

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Let
$$E=\{e_i=u_iu_{i+1}:1\le i\le n-1\}\cup$$
$$\{e_j^{'}=v_jv_{j+1}:1\le j\le n-1\}\cup$$
$$\{e_{ij+1}=u_iv_{j+1}:1\le i\le n-1 and 1\le j\le n-1\}\cup$$
$$\{e_{(i+2)j}^{'}=u_{i+2}v_j:1\le i\le n-2 and 1\le j\le n-2\}$$
be the edge set of the braid graph $B(n)$. Here the braid graph has $p=2n$ vertices and $q=4(n-1)$ edges.

Define an injective function $f:V\to\{1,3,\ldots,2(2n)-1\}$ such that

$$f(u_i)=4i-3,\ 1\le i\le n$$
$$f(v_j)=4j-1,\ 1\le j\le n$$

and an induced edge function $f^*:E\to\{0,2,4,\ldots 2(4n-4)-2\}$ such that

$$f^*(e_i)=f^*(u_iu_{i+1})=(6+8(i-1))(mod\ 2q),\ 1\le i\le n-1$$
$$f^*(e_j^{'})=f^*(v_jv_{j+1})=(10+8(i-1))(mod\ 2q),\ 1\le i\le n-1$$
$$f^*(e_{i(j+1)})=f^*(u_iv_{j+1})=(8i)(mod\ 2q),\ 1\le i\le n-1,1\le j\le n-1$$
$$f^*(e_{(i+2)j}^{'})=f^*(u_{i+2}v_j)=(12i+8i)(mod\ 2q),\ 1\le i\le n-2,1\le j\le n-2$$

Where f^* is a 1-1 and onto function. The computing functions defined in the proof of the theorem are giving the EOH labeling of G.

Hence, the braid graph $B(n)$ is an EOH graph, when $n\ge 3$.

Example 4.4

An EOH labeling of $B(5)$ is shown in Fig. 7

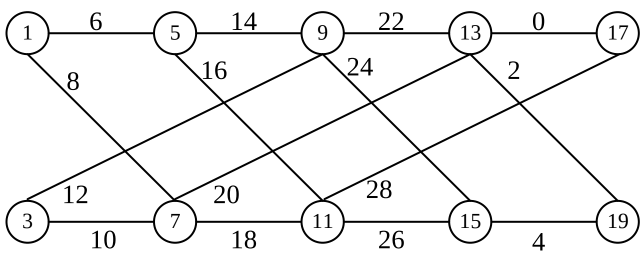

Fig. 7 EOH labeling of $B(5)$

Theorem 4.6

The graph $P_{2n}(+)N_m$ admits an EOH labeling, when $n\ge 1$ and $m\ge 1$.

Proof

Let $V=\{u_i:1\le i\le 2n\}\cup\{v_j:1\le j\le m\}$ be the vertex set of the graph $P_{2n}(+)N_m$.

Let $E=\{e_i=u_iu_{i+1}:1\le i\le 2n\}\cup\{e_{1j}=u_1v_j:1\le j\le m\}\cup\{e_{nj}=u_nv_j:1\le j\le m\}$ be the edge set of the graph $P_{2n}(+)N_m$. Here the graph $P_{2n}(+)N_m$ has $p=2n+m$ vertices and $q=2n+2m-1$ edges.

Define an injective function $f:V\to\{1,3,\ldots,2(2n+m)-1\}$ such that

$$f(v_j)=2n+2j-1,\ 1\le j\le m$$
$$f(u_i)=\begin{cases} i,\ i\ is\ odd \\ m+2n+i,\ i\ is\ even\end{cases}$$

and an induced edge function $f^*:E\to\{0,2,4,\ldots 2(2n+2m-1)-2\}$ such that

$$f^*(e_i)=f^*(u_iu_{i+1})=(2n+2m+2i)(mod\ 2q),\ 1\le i\le n$$
$$f^*(e_{1j})=f^*(u_1v_j)=(2n+2j)(mod\ 2q),\ 1\le j\le m$$
$$f^*(e_{nj})=f^*(u_nv_j)=(2m+3(2n-1)+2j+1)(mod\ 2q),\ 1\le j\le m$$

Where f^* is a 1-1 and onto function. The computing functions defined in the proof of the theorem are giving the EOH labeling of G.

Hence, the graph $P_{2n}(+)N_m$ is an EOH graph, when $n\ge 1$ and $m\ge 1$.

Example 4.5

An EOH labeling of $P_4(+)N_3$ is shown in Fig. 8

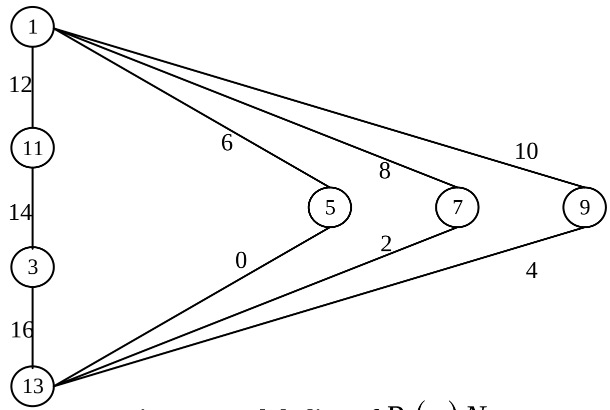

Fig. 8 EOH labeling of $P_4(+)N_3$.

Theorem 4.7

The jellyfish graph $J(m,n)$ admits an EOH labeling, when $m,n\ge 1$.

Proof

$V=\{u_i:1\le i\le m\}\cup\{v_j:1\le j\le n\}\cup\{u\}\cup\{v\}\cup\{x\}\cup\{y\}$ be the vertex set of the jellyfish graph.

Let $E=\{e_i=u_iu:1\le i\le m\}\cup\{e_j^{'}=v_jv:1\le j\le n\}\cup\{e_{xy}=xy\}\cup\{e_{ux}=ux\}\cup\{e_{uy}=uy\}\cup\{e_{vx}=vx\}\cup\{e_{vy}=vy\}$ be the edge set of the jellyfish graph $J(m,n)$. Here the jellyfish graph has $p=m+n+4$ vertices and $q=m+n+5$ edges.

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Define an injective function $f:V \to \{1,3,\ldots,2(m+n+4)\}-1$ such that

$f(u_i)=2i-1,\ 1\le i\le m$

$f(v_j)=2(m+4)+2j-1,\ 1\le j\le n$

$f(u)=2m+3$

$f(x)=2m+1$

$f(v)=2m+5$

$f(y)=2m+7$

and an induced edge function $f^*:E\to\{0,2,4,\ldots 2(m+n+5)-2\}$ such that

$f^*(e_i)=f^*(u_iu)=(2(m+i+1))\ (mod\ 2q),\ 1\le i\le m$

$f^*(e_j^{'})=f^*(v_jv)=(4(m+3)+2j)(mod\ 2q),$
$1\le j\le n$

$f^*(e_{xy})=f^*(xy)=(4(m+2))(mod\ 2q)$

$f^*(e_{ux})=f^*(ux)=(4(m+1))(mod\ 2q)$

$f^*(e_{uy})=f^*(uy)=(4n+10)(mod\ 2q)$

$f^*(e_{vx})=f^*(vx)=(4n+6)(mod\ 2q)$

$f^*(e_{vy})=f^*(vy)=(4(n+3))(mod\ 2q)$

Where f^* is a 1-1 and onto function. The computing functions defined in the proof of the theorem are giving the EOH labeling of G.

Hence, the jellyfish graph $J(m,n)$ is an EOH graph, when $m,n\ge 1$.

Example 4.6

An EOH labeling of $J(4,5)$ is shown in Fig. 9

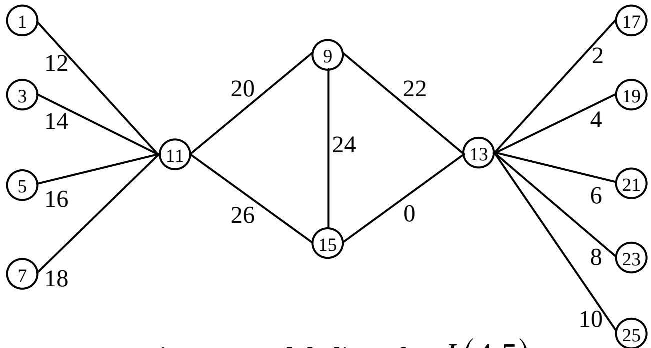

Fig. 9 EOH labeling of $J(4,5)$

Theorem 4.8

The planar graph $(P_2\cup mK_1)+N_2$ admits an EOH labeling, when $k\ge 1$.

Proof

Let $V=\{u_i:i=1,2\}\cup\{v_j:1\le j\le m\}\cup\{x\}\cup\{y\}$ be the vertex set of the planar graph $(P_2\cup mK_1)+N_2$. Let $E=\{e_1=u_1u_2\}\cup\{e_{1x}=u_1x\}\cup\{e_{2x}=u_2x\}\cup\{e_{1y}=u_1y\}\cup\{e_{2y}=u_2y\}\cup\{e_{jx}^{'}=v_jx:1\le j\le m\}\cup\{e_{jy}^{'}=v_jy:1\le j\le m\}$ be the edge of the planar graph $(P_2\cup mK_1)+N_2$. Here the graph $(P_2\cup mK_1)+N_2$ has $p=m+4$ vertices and $q=2m+5$ edges.

Define an injective function $f:V\to\{1,3,\ldots,2(m+4)-1\}$ such that

$f(u_i)=3,\ i=1$

$f(u_i)=2k+5,\ i=2$

$f(x)=1$

$f(y)=2k+7$

$f(v_j)=2j+3,\ 1\le j\le m$

and an induced edge function $f^*:E\to\{0,2,4,\ldots 2(2m+5)-2\}$ such that

$f^*(e_1)=f^*(u_1u_2)=(8+2k)(mod\ 2q)$

$f^*(e_{1x})=f^*(u_1x)=4$

$f^*(e_{2x})=f^*(u_2x)=(2k+6)(mod\ 2q)$

$f^*(e_{1y})=f^*(u_1y)=(2k+10)(mod\ 2q)$

$f^*(e_{2y})=f^*(u_2y)=(8k)(mod\ 2q)$

$f^*(e_{jx}^{'})=f^*(v_jx)=(4+2j)(mod\ 2q),\ 1\le j\le m$

$f^*(e_{jy}^{'})=f^*(v_jy)=(2k+2j+10)(mod\ 2q),\ 1\le j\le m$

Where f^* is a 1-1 and onto function. The computing functions defined in the proof of the theorem are giving the EOH labeling of G.

Hence, the planar graph $(P_2\cup mK_1)+N_2$ is an EOH graph, when $k\ge 1$

Example 4.7

An EOH labeling of $(P_2\cup 3K_1)+N_2$ is shown in Fig. 10

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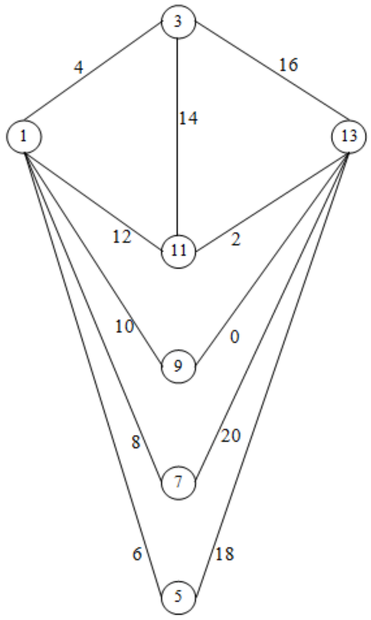

Fig. 10 EOH labeling of $(P_2 \cup 3K_1) + N_2$

Theorem 4.9

The Peterson graph admits an EOH labeling.

Proof

Let $V = \{u_1, u_2, u_3, u_4, u_5, v_1, v_2, v_3, v_4, v_5\}$ be the vertex set of Peterson graph. Let $E = \{v_i v_{i+1} | 1 \le i \le 5\} \cup \{v_5 v_1\} \cup \{u_i v_i | 1 \le i \le 5\}$ $\cup \{u_1u_3, u_3u_5, u_5u_2, u_2u_4, u_4u_1\}$ be the edge set. Here the peterson graph has 10 vertices and 15 edges.

Define an injective function $f : V \to \{1,3,5,7,9,11,13,15,17,19\}$ such that

$f(u_i) = 2i - 1, 1 \le i \le 5$

$f(v_1) = 15$, $f(v_2) = 19$, $f(v_3) = 13$, $f(v_4) = 17$, $f(v_5) = 11$

and an induced edge function $f^* : E \to \{0,2,4,6,8,10,12,14,16,18,20,22,24,26,28\}$ such that $f^*(v_1v_2) = 4$, $f^*(v_2v_3) = 2$, $f^*(v_3v_4) = 0$ $f^*(v_4v_5) = 28$, $f^*(v_5v_1) = 26$. $f^*(u_1u_3) = 18$, $f^*(u_3u_5) = 14$, $f^*(u_5u_2) = 12$, $f^*(u_2u_4) = 10$, $f^*(u_4u_1) = 8$. $f^*(v_1u_1) = 16$, $f^*(v_2u_2) = 22$, $f^*(v_3u_3) = 18$, $f^*(v_4u_4) = 24$, $f^*(v_5u_5) = 20$.

Where f^* is a 1-1 and onto function. The computing functions defined in the proof of the theorem are giving the EOH labeling of G . Hence, Peterson graph is an EOH graph.

Example 4.8

Peterson graph is shown in Fig. 11

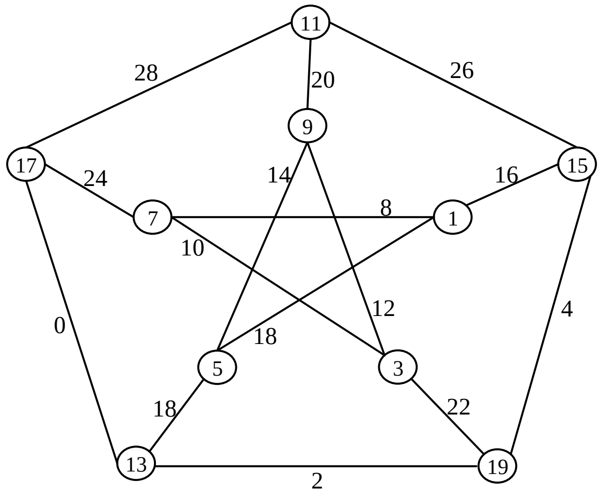

Fig. 11 EOH labeling of Peterson graph

Theorem 4.10

The wheel graph W_n does not admit an EOH labeling when $n \ge 3$.

Proof

We prove this theorem by giving a counter example which is shown in Fig.12.

Let us consider a wheel graph W_5 .

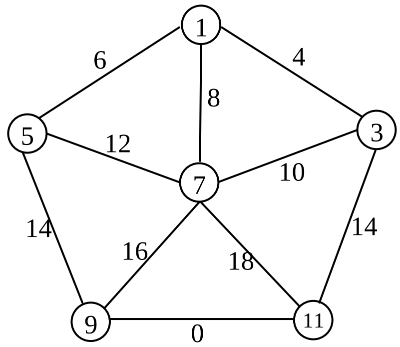

Fig. 12 Labeling of W_5

Let $V = \{v, v_1, v_2, v_3, v_4, v_5\}$ be the vertex set of wheel graph W_5 . Let $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1\}$ $\cup \{vv_1, vv_2, vv_3, vv_4, vv_5\}$ be the edge set of wheel W_5 .

As per the definition (3.1) of the even-odd harmonious labeling, we assign the labels to the vertices of W_5 as follows: $f(v) = 7$, $f(v_1) = 1$, $f(v_2) = 3$, $f(v_3) = 11$, $f(v_4) = 9$, $f(v_5) = 5$. The edge labels can be computed using the edge function f^* given in the definition (3.1). That is, $f^* : E \to \{0,4,6,8,10,12,14,14,16,18\}$ computes the labels as follows:

$f^*(vv_1) = 8$, $f^*(vv_2) = 10$, $f^*(vv_3) = 18$, $f^*(vv_4) = 16$, $f^*(vv_5) = 12$, $f^*(v_1v_2) = 4$, $f^*(v_2v_3) = 14$, $f^*(v_3v_4) = 0$ $f^*(v_4v_5) = 14$, $f^*(v_5v_1) = 6$.

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Here the number "14" is repeated twice. Hence the function f^* is not bijective. Therefore the wheel graph W_5 is not admitting a EOH labeling.

V. CONCLUSION

In this paper, we have proved the existence of even-odd harmonious labeling to certain interesting family of cyclic graphs such as fan graph, ladder graph, prism graph, total graph, braid graph, $P_{2n}(+)N_m$, jellyfish graph, $(P_2 \cup 3K_1)+N_2$, peterson graph. Interestingly, we have proved that the wheel graph W_n when $n \geq 3$ is not admitting this labeling.

REFERENCES

1. B.D. Acharya , S.M Hegde, Arithmetic Graphs. J. Graph Theory, Vol. 14, No.3, 275-299, (1990).
2. N. Adalin Beatress, P.B Sarasija, Even-Odd Harmonious Graphs, International Journal of Mathematics and Soft Computing, Vol. 5, No. 1, 23-29, (2015).
3. R.L Graham, N.J.A Sloane, On Additive Bases and Harmonious Graphs, SIAM Journal on Algebraic and Discrete Methods, Vol. 1, No. 4, 382-404, (1980).
4. F. Harary, Graph Theory, Addison-Wesley, Reading Mass, (1972).
5. Joseph A. Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, DS6, (2018).
6. M.Kalaimathi, B.J Balamurugan, Computation of Even-Odd Harmonious Labeling of Graphs Obtained by Graph Operations, Recent Trends in Pure and Applied Mathematics, AIP Conf. Proc. 2177, 020030-1 - 020030-9, (2019).
7. M.Kalaimathi, B.J Balamurugan, Computation of Even-Odd Harmonious Labeling of Acyclic Graphs, International Journal of engineering and advanced technology, Vol. 9, No. 1S3, 414-419, (2019).
8. N. Lakshmi Prasana, K. Saravanthi, Nagalla Sudhakar, Applications of Graph Labeling in Major Areas of Computing Science. International Journal of Research in Computer and Communication Technology, Vol. 3, No. 8, (2014).
9. Z. Liang, Z. Bai Z, On The Odd Harmonious Graphs with Applications, J. Appl. Math. Comput., 29, 105-116, (2009).
10. A. Rosa, On Certin Valuations of the Vertices of a Graph, In Theory of Graphs(Internat.Sympos. Rome. 1966), Gordan and Breach, Newyork, Dunod, Paris, 349-359, (1967).
11. P. B. Sarasija, R. Binthiya, Even Harmonious Graphs with Applications, International Journal of Computer Science and Information Security, Vol. 9, No. 7, 161-163, (2011).
12. D. B. West, Introduction to Graph Theory, PHI Learning Private Limited, 2nd Edition, (2009).

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