New Family of Parity Combination Cordial Labeling of Graph

A. Muthaiyan, M. Kathiravan

Abstract: *Let G be a (p, q) graph. Let f be an injective map from V(G) to {1, 2, …, p}. For each edge xy, assign the label* $\binom{x}{y}$ *or* $\binom{y}{x}$ *according as x > y or y > x. f is called a parity combination cordial labeling (PCC-labeling) if f is a one to one map and* $|e_f(0) - e_f(1)| \le 1$ *where* $e_f(0)$ *and* $e_f(1)$ *denote the number of edges labeled with an even number and odd number respectively. A graph with a parity combination cordial labeling is called a parity combination cordial graph (PCC-graph). In this paper we investigate the PCC- labeling of the graph G′, It is obtained by identifying a vertex* v_k *in G and a vertex of degree n in* H_n*, where G is a PCC graph with p vertices and q edges under f with* $f(v_k) = 1$*.*

Keywords : Parity combination cordial labeling, Parity combination cordial graph, identifying a vertex and Helm H_n***.***

I. INTRODUCTION

In this paper we consider only finite, undirected and simple graphs. For standard notations and terminology related to Graph theory, we refer Harary [2], Graph labeling, we refer Gallian[1]. The concept of Parity combination cordial labeling is introduced by Ponraj et al [3]. We present the Parity combination cordial of the graph G′, which is obtained by identifying a vertex v_k in G and a vertex of degree n in H_n, where G is a Parity combination cordial graph with p vertices and q edges under f with $f(v_k) = 1$.

II. BASIC RESULTS AND DEFINITIONS

A. Definition 2.1

Let G be a (p, q) graph. Let f be an injective map from V(G) to {1, 2, …, p}. For each edge xy, assign the label $\binom{x}{y}$ or $\binom{y}{x}$ according as x>y or y> x. f is called a parity combination cordial labeling (PCC-labeling) if f is a one to one map and $|e_f(0) - e_f(1)| \le 1$ where $e_f(0)$ and $e_f(1)$ denote the number of edges labeled with an even number and odd number respectively. A graph with a parity combination cordial labeling is called a parity combination cordial graph (PCC-graph).

B. Result 2.1

$\binom{n}{n-1} = \binom{n}{1}$ is even if n is even and odd if n is odd.

C. Result 2.2

$\binom{n}{2}$ is even if $n \equiv 0, 1 \pmod 4$ and odd if $n \equiv 2, 3 \pmod 4$.

D. Result 2.3

$\binom{n}{k}$ is even when n is even and k is odd.

III. MAIN RESULT

A. Theorem 3.1

G is a PCC graph with p vertices and q edges under f with $f(v_k) = 1$, then the graph is obtained by identifying a vertex v_k in G and a vertex of degree n in helm H_n admits PCC labeling for $n \ge 3$.

Proof.

G is a PCC graph with $v_1, v_2, ..., v_p$ vertices and $e_1, e_2, ..., e_q$ edges and f is PCC labeling of G. Then $f : V(G) \to \{1,2,\dots,p\}$ with $f(v_k) = 1$ and $|e_f(0) - e_f(1)| \le 1$.

Let H_n be a helm graph. Let $w, w_1, w_2, ..., w_{2n}$ be the vertices and $e_{11}, e_{12}, ..., e_{1n}, e_{21}, e_{22}, ..., e_{2n}, e_{31}, e_{32}, ..., e_{3n}$ be the edges of H_n. The graph obtained by identifying a vertex v_k in G and a vertex w of degree n in H_n. The resultant graph is denoted by G′. Here $|V(G')| = p+2n$ and $|E(G')| = q+3n$.

Now define $g : V(G') \to \{1,2,\dots,p+2n\}$ as follows:

$g(v_i) = f(v_i)$, for all $v_i \in V(G)$.

Consider $g(v_k) = g(w) = 1$. Since v_k is identified with w in G′.

Case (1) : p and q are even and n = 3.

$g(w_i) = p+2i+1$, for i = 1, 2.

$g(w_3) = p+6$,

$g(w_{n+i}) = p+2i$, for i = 1,2.

$g(w_6) = p+1$.

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Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is even, then $e_f(1) = e_f(0) = \frac{q}{2}$. Thus $e_g(1) = e_g(0) = \frac{q}{2}$ for G in G′.

$g'(ww_i) = g'(e_{1i}) = 1$, for i = 1, 2.
$g'(ww_3) = g'(e_{13}) = 0$,
$g'(w_1w_2) = g'(e_{21}) = 0$, for $p \equiv 0 \pmod 4$.
$g'(w_1w_2) = g'(e_{21}) = 1$, for $p \equiv 2 \pmod 4$,
$g'(w_2w_3) = g'(e_{22}) = 0$,
$g'(w_3w_1) = g'(e_{23}) = 0$,
$g'(w_iw_{n+i}) = g'(e_{3i}) = 1$, for i = 1, 2.
$g'(w_3w_{n+3}) = g'(e_{33}) = 0$.

When $p \equiv 0 \pmod 4$, then $e_g(1) = \frac{q}{2}$ for G in G′+4 for H_n in G′, $e_g(0) = \frac{q}{2}$ for G in G′+5 for H_n in G′ and $|e_g(0)-e_g(1)| = 1$.

When $p \equiv 2 \pmod 4$, then $e_g(1) = \frac{q}{2}$ for G in G′+5 for H_n in G′, $e_g(0) = \frac{q}{2}$ for G in G′+4 for H_n in G′ and $|e_g(0)-e_g(1)| = 1$.

Case (2) : p and q are even and n > 3.
$g(w_i) = p+2i-1$, for i = 1, 2, …, n–1.
$g(w_n) = p+2n$,
$g(w_{n+1}) = p+6$,
$g(w_{n+2}) = p+2$,
$g(w_{n+3}) = p+4$,
$g(w_{n+i}) = p+2i$, for i = 4, 5, …, n–1.
$g(w_{2n}) = p+2n-1$,
Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is even, then $e_f(1) = e_f(0) = \frac{q}{2}$. Thus $e_g(1) = e_g(0) = \frac{q}{2}$ for G in G′.

$g'(ww_i) = g'(e_{1i}) = 1$, for i = 1, 2, …, n–1.
$g'(ww_n) = g'(e_{1n}) = 0$,
$g'(w_iw_{n+i}) = g'(e_{3i}) = 0$, for i = 1, 4,5, …, n.
$g'(w_2w_{n+2}) = g'(e_{32}) = 1$,
$g'(w_3w_{n+3}) = g'(e_{33}) = 1$.

For $p \equiv 0 \pmod 4$
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for i = 1, 2, …, n–2 and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$, for i = 1, 2, …, n–2 and i is even.
$g'(w_{n-1}w_n) = g'(e_{2(n-1)}) = 0$,
$g'(w_nw_1) = g'(e_{2n}) = 0$.

For $p \equiv 2 \pmod 4$
$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$, for i = 1, 2, …, n–2 and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for i = 1, 2, …, n–2 and i is even.
$g'(w_{n-1}w_n) = g'(e_{2(n-1)}) = 0$,
$g'(w_nw_1) = g'(e_{2n}) = 0$.

When $p \equiv 0 \pmod 4$, q is even and n is odd, then $e_g(1) = \frac{q}{2}$ for G in G′ + $\frac{3n+1}{2}$ for H_n in G′, $e_g(0) = \frac{q}{2}$ for G in G′ + $\frac{3n-1}{2}$ for H_n in G′ and $|e_g(0) - e_g(1)| = 1$.

When $p \equiv 2 \pmod 4$, q is even and n is odd, then $e_g(1) = \frac{q}{2}$ for G in G′ + $\frac{3n-1}{2}$ for H_n in G′, $e_g(0) = \frac{q}{2}$ for G in G′ + $\frac{3n+1}{2}$ for H_n in G′ and $|e_g(0) - e_g(1)| = 1$.

When p, q and n are even, then $e_g(1) = \frac{q}{2}$ for G in G′ + $\frac{3n}{2}$ for H_n in G′, $e_g(0) = \frac{q}{2}$ for G in G′ + $\frac{3n}{2}$ for H_n in G′ and $|e_g(0) - e_g(1)| = 0$.

Case (3) : p and q are odd, n is even and $n \geq 4$.
$g(w_1) = p+2$,
$g(w_i) = p+2i-1$, for i = 2,3, …, n.
$g(w_{n+1}) = p+1$,
$g(w_{n+i}) = p+2i$, for i = 2, 3, …, n.
Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$. Thus $e_g(1) = \frac{q+1}{2}$ and $e_g(0) = \frac{q-1}{2}$ for G in G′.

For q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$. Thus $e_g(1) = \frac{q-1}{2}$ and $e_g(0) = \frac{q+1}{2}$ for G in G′.

$g'(ww_1) = g'(e_{11}) = 1$;
$g'(ww_i) = g'(e_{1i}) = 0$, for i = 2, 3, …, n.
$g'(w_iw_{n+i}) = g'(e_{3i}) = 1$, for i = 1, 2, …, n.

For $p \equiv 1 \pmod 4$
$g'(w_1w_2) = g'(e_{21}) = 0$,
$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$, for i = 2, 3, …, n–1 and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for i = 2, 3, …, n–1 and i is even.
$g'(w_nw_1) = g'(e_{2n}) = 0$.

For $p \equiv 3 \pmod 4$
$g'(w_1w_2) = g'(e_{21}) = 0$,
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for i = 2, 3, …, n–1 and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$ for i = 2, 3, …, n–1 and i is even.
$g'(w_{n-1}w_n) = g'(e_{2(n-1)}) = 0$,
$g'(w_nw_1) = g'(e_{2n}) = 0$.

When n is even, p is odd, q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$, then $e_g(1) = \frac{q+1}{2}$ for G in G′ + $\frac{3n}{2}$ for H_n in G′, $e_g(0) = \frac{q-1}{2}$ for G in G′

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$+\frac{3n}{2}$ for H_n in G' and $|e_g(0) - e_g(1)| = 1$.

When n is even, p is odd, q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$, then $e_g(1) = \frac{q-1}{2}$ for G in G' + $\frac{3n}{2}$ for H_n in G', $e_g(0) = \frac{q+1}{2}$ for G in G' + $\frac{3n}{2}$ for H_n in G' and $|e_g(0) - e_g(1)| = 1$.

Case (4) : $p \equiv 1 \pmod 4$, q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$, n is odd and $n \geq 3$.

$g(w_1) = p+4$,
$g(w_i) = p+2i-1$, for $i = 2,3, \ldots, n$.
$g(w_{n+1}) = p+1$,
$g(w_{n+1}) = p+2$,
$g(w_{n+i}) = p+2i$, for $i = 3, 4, \ldots, n$.

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$. Thus $e_g(1) = \frac{q+1}{2}$ and $e_g(0) = \frac{q-1}{2}$ for G in G'.

$g'(ww_1) = g'(e_{11}) = 1$;
$g'(ww_i) = g'(e_{1i}) = 0$, for $i = 2, 3, \ldots, n$.
$g'(w_1w_{n+1}) = g'(e_{31}) = 0$,
$g'(w_2w_{n+2}) = g'(e_{32}) = 0$,
$g'(w_iw_{n+i}) = g'(e_{3i}) = 1$, for $i = 3, 4, \ldots, n$.

For $p \equiv 1 \pmod 4$
$g'(w_1w_2) = g'(e_{21}) = 1$,
$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$, for $i = 2, 3, \ldots, n-1$ and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for $i = 2, 3, \ldots, n-1$ and i is even.
$g'(w_nw_1) = g'(e_{2n}) = 0$.

When n is odd, $p \equiv 1 \pmod 4$, q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$, then $e_g(1) = \frac{q+1}{2}$ for G in G' + $\frac{3n-1}{2}$ for H_n in G', $e_g(0) = \frac{q-1}{2}$ for G in G' + $\frac{3n+1}{2}$ for H_n in G' and $|e_g(0) - e_g(1)| = 0$.

Case (5) : $p \equiv 1 \pmod 4$, q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$, n is odd and $n \geq 3$.

$g(w_1) = p+2$,
$g(w_i) = p+2i-1$, for $i = 2,3, \ldots, n$.
$g(w_{n+1}) = p+1$,
$g(w_{n+i}) = p+2i$, for $i = 2, 3, \ldots, n$.

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$. Thus $e_g(1) = \frac{q-1}{2}$ and $e_g(0) = \frac{q+1}{2}$ for G in G'.

$g'(ww_1) = g'(e_{11}) = 1$;
$g'(ww_i) = g'(e_{1i}) = 0$, for $i = 2, 3, \ldots, n$.
$g'(w_iw_{n+i}) = g'(e_{3i}) = 1$, for $i = 1, 2, \ldots, n$.

For $p \equiv 1 \pmod 4$
$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$, for $i = 1, 2, \ldots, n$ and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for $i = 1, 2, \ldots, n$ and i is even.

When n is odd, $p \equiv 1 \pmod 4$, q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$, then $e_g(1) = \frac{q-1}{2}$ for G in G' + $\frac{3n+1}{2}$ for H_n in G', $e_g(0) = \frac{q+1}{2}$ for G in G' + $\frac{3n-1}{2}$ for H_n in G' and $|e_g(0) - e_g(1)| = 0$.

Case (6) : $p \equiv 0 \pmod 4$, q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$, n is odd and $n = 3$.

$g(w_3) = p+3$,
$g(w_3) = p+4$,
$g(w_3) = p+5$,
$g(w_4) = p+1$,
$g(w_5) = p+6$,
$g(w_6) = p+2$.

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$. Thus $e_g(1) = \frac{q-1}{2}$ and $e_g(0) = \frac{q+1}{2}$ for G in G'.

$g'(ww_1) = g'(e_{11}) = 1$,
$g'(ww_2) = g'(e_{12}) = 0$,
$g'(ww_3) = g'(e_{13}) = 1$,
$g'(w_1w_2) = g'(e_{21}) = 0$,
$g'(w_2w_3) = g'(e_{22}) = 1$,
$g'(w_3w_1) = g'(e_{23}) = 0$,
$g'(w_1w_{n+1}) = g'(e_{31}) = 1$,
$g'(w_2w_{n+2}) = g'(e_{32}) = 1$,
$g'(w_3w_{n+3}) = g'(e_{33}) = 0$.

When $p \equiv 0 \pmod 4$, then $e_g(1) = \frac{q-1}{2}$ for G in G' + 5 for H_n in G', $e_g(0) = \frac{q+1}{2}$ for G in G' + 4 for H_n in G' and $|e_g(0) - e_g(1)| = 0$.

Case (7) : $p \equiv 0 \pmod 4$, q is odd with $e_f(1) = \frac{q+1}{2}$ and

$e_f(0) = \frac{q-1}{2}$, n is odd and n = 3.
$g(w_i) = p+2i+1$, for i = 1, 2.
$g(w_3) = p+6$,
$g(w_4) = p+2$,
$g(w_5) = p+4$,
$g(w_6) = p+1$.
Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$. Thus $e_g(1) = \frac{q+1}{2}$ and $e_g(0) = \frac{q-1}{2}$ for G in G′.
$g'(ww_i) = g'(e_{1i}) = 1$, for i = 1, 2.
$g'(ww_3) = g'(e_{13}) = 0$,
$g'(w_1w_2) = g'(e_{21}) = 0$,
$g'(w_2w_3) = g'(e_{22}) = 0$,
$g'(w_3w_1) = g'(e_{23}) = 0$,
$g'(w_1w_{n+1}) = g'(e_{31}) = 1$,
$g'(w_2w_{n+2}) = g'(e_{32}) = 1$,
$g'(w_3w_{n+3}) = g'(e_{33}) = 0$.

When $p \equiv 0 (\text{mod } 4)$, then $e_g(1) = \frac{q+1}{2}$ for G in G′ + 4 for H_n in G′, $e_g(0) = \frac{q-1}{2}$ for G in G′ + 5 for H_n in G′ and $|e_g(0) - e_g(1)| = 0$.

Case (8) : $p \equiv 0 (\text{mod } 4)$, q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$, n is odd and n > 3.
$g(w_i) = p+2i-1$, for i = 1, 2, …, n–1.
$g(w_n) = p+2n$,
$g(w_{n+1}) = p+4$,
$g(w_{n+2}) = p+2$,
$g(w_{n+i}) = p+2i$, for i = 3, 4, …, n–1.
$g(w_{2n}) = p+2n-1$,
Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$. Thus $e_g(1) = \frac{q+1}{2}$ and $e_g(0) = \frac{q-1}{2}$ for G in G′.
$g'(ww_i) = g'(e_{1i}) = 1$, for i = 1, 2, …, n–1.
$g'(ww_n) = g'(e_{1n}) = 0$,
$g'(w_iw_{n+i}) = g'(e_{3i}) = 0$, for i = 1, 3,4, …, n.
$g'(w_2w_{n+2}) = g'(e_{32}) = 1$,
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for i = 1, 2, …, n–2 and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$, for i = 1, 2, …, n–2 and i is even.
$g'(w_{n-1}w_n) = g'(e_{2(n-1)}) = 0$,
$g'(w_nw_1) = g'(e_{2n}) = 0$.

When n is odd, $p \equiv 0 (\text{mod } 4)$, q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$, then $e_g(1) = \frac{q+1}{2}$ for G in G′ + $\frac{3n-1}{2}$ for H_n in G′, $e_g(0) = \frac{q-1}{2}$ for G in G′ + $\frac{3n+1}{2}$ for H_n in G′ and $|e_g(0) - e_g(1)| = 0$.

Case (9) : $p \equiv 0 (\text{mod } 4)$, q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$, n is odd and n > 3.
$g(w_i) = p+2i-1$, for i = 1, 2, …, n–1.
$g(w_n) = p+2n$,
$g(w_{n+1}) = p+6$,
$g(w_{n+2}) = p+2$,
$g(w_{n+3}) = p+4$,
$g(w_{n+i}) = p+2i$, for i = 4, 5, …, n–1.
$g(w_{2n}) = p+2n-1$,
Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$. Thus $e_g(1) = \frac{q-1}{2}$ and $e_g(0) = \frac{q+1}{2}$ for G in G′.
$g'(ww_i) = g'(e_{1i}) = 1$, for i = 1, 2, …, n–1.
$g'(ww_n) = g'(e_{1n}) = 0$,
$g'(w_iw_{n+i}) = g'(e_{3i}) = 0$, for i = 1, 4,5, …, n.
$g'(w_2w_{n+2}) = g'(e_{32}) = 1$,
$g'(w_3w_{n+3}) = g'(e_{33}) = 1$,
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for i = 1, 2, …, n–2 and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$, for i = 1, 2, …, n–2 and i is even.
$g'(w_{n-1}w_n) = g'(e_{2(n-1)}) = 0$,
$g'(w_nw_1) = g'(e_{2n}) = 0$.

When n is odd, $p \equiv 0 (\text{mod } 4)$, q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$, then $e_g(1) = \frac{q-1}{2}$ for G in G′ + $\frac{3n+1}{2}$ for H_n in G′, $e_g(0) = \frac{q+1}{2}$ for G in G′ + $\frac{3n-1}{2}$ for H_n in G′ and $|e_g(0) - e_g(1)| = 0$.

Case (10) : $p \equiv 2 (\text{mod } 4)$, q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$, n is odd and n = 3.
$g(w_i) = p+2i+1$, for i = 1, 2.
$g(w_3) = p+6$,
$g(w_4) = p+4$,
$g(w_5) = p+2$,
$g(w_6) = p+1$.
Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

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For q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$. Thus $e_g(1) = \frac{q+1}{2}$ and $e_g(0) = \frac{q-1}{2}$ for G in G′.

$g'(ww_i) = g'(e_{1i}) = 1$, for $i = 1, 2$.
$g'(ww_3) = g'(e_{13}) = 0$,
$g'(w_1w_2) = g'(e_{21}) = 1$,
$g'(w_2w_3) = g'(e_{22}) = 0$,
$g'(w_3w_1) = g'(e_{23}) = 0$,
$g'(w_1w_{n+1}) = g'(e_{31}) = 0$,
$g'(w_2w_{n+2}) = g'(e_{32}) = 1$,
$g'(w_3w_{n+3}) = g'(e_{33}) = 0$.

When $p \equiv 2 \pmod 4$, then $e_g(1) = \frac{q+1}{2}$ for G in G′ + 4 for H_n in G′, $e_g(0) = \frac{q-1}{2}$ for G in G′ + 5 for H_n in G′ and $|e_g(0) - e_g(1)| = 0$.

Case (11) : $p \equiv 2 \pmod 4$, q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$, n is odd and n = 3.

$g(w_i) = p+2i+1$, for $i = 1, 2$.
$g(w_3) = p+6$,
$g(w_4) = p+2$,
$g(w_5) = p+4$,
$g(w_6) = p+1$.

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$. Thus $e_g(1) = \frac{q-1}{2}$ and $e_g(0) = \frac{q+1}{2}$ for G in G′.

$g'(ww_i) = g'(e_{1i}) = 1$, for $i = 1, 2$.
$g'(ww_3) = g'(e_{13}) = 0$,
$g'(w_1w_2) = g'(e_{21}) = 1$,
$g'(w_2w_3) = g'(e_{22}) = 0$,
$g'(w_3w_1) = g'(e_{23}) = 0$,
$g'(w_1w_{n+1}) = g'(e_{31}) = 1$,
$g'(w_2w_{n+2}) = g'(e_{32}) = 1$,
$g'(w_3w_{n+3}) = g'(e_{33}) = 0$.

When $p \equiv 2 \pmod 4$, then $e_g(1) = \frac{q-1}{2}$ for G in G′ + 5 for H_n in G′, $e_g(0) = \frac{q+1}{2}$ for G in G′ + 4 for H_n in G′ and $|e_g(0) - e_g(1)| = 0$.

Case (12) : $p \equiv 2 \pmod 4$, q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$, n is odd and n > 3.

$g(w_i) = p+2i-1$, for $i = 1, 2, \ldots, n-1$.
$g(w_n) = p+2n$,
$g(w_{n+1}) = p+6$,
$g(w_{n+2}) = p+2$,
$g(w_{n+3}) = p+4$,
$g(w_{n+i}) = p+2i$, for $i = 4, 5, \ldots, n-1$.
$g(w_{2n}) = p+2n-1$,

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$. Thus $e_g(1) = \frac{q+1}{2}$ and $e_g(0) = \frac{q-1}{2}$ for G in G′.

$g'(ww_i) = g'(e_{1i}) = 1$, for $i = 1, 2, \ldots, n-1$.
$g'(ww_n) = g'(e_{1n}) = 0$,
$g'(w_iw_{n+i}) = g'(e_{3i}) = 0$, for $i = 1, 4, 5, \ldots, n$.
$g'(w_2w_{n+2}) = g'(e_{32}) = 1$,
$g'(w_3w_{n+3}) = g'(e_{33}) = 1$,
$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$, for $i = 1, 2, \ldots, n-2$ and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for $i = 1, 2, \ldots, n-2$ and i is even.
$g'(w_{n-1}w_n) = g'(e_{2(n-1)}) = 0$,
$g'(w_nw_1) = g'(e_{2n}) = 0$.

When n is odd, $p \equiv 2 \pmod 4$, q is odd with $e_f(1) = \frac{q+1}{2}$ and $e_f(0) = \frac{q-1}{2}$, then $e_g(1) = \frac{q+1}{2}$ for G in G′ + $\frac{3n-1}{2}$ for H_n in G′, $e_g(0) = \frac{q-1}{2}$ for G in G′ + $\frac{3n+1}{2}$ for H_n in G′ and $|e_g(0) - e_g(1)| = 0$.

Case (13) : $p \equiv 2 \pmod 4$, q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$, n is odd and n > 3.

$g(w_i) = p+2i-1$, for $i = 1, 2, \ldots, n-1$.
$g(w_n) = p+2n$,
$g(w_{n+1}) = p+8$,
$g(w_{n+2}) = p+2$,
$g(w_{n+3}) = p+4$,
$g(w_{n+4}) = p+6$,
$g(w_{n+i}) = p+2i$, for $i = 5, 6, \ldots, n-1$.
$g(w_{2n}) = p+2n-1$,

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$. Thus $e_g(1) = \frac{q-1}{2}$ and $e_g(0) = \frac{q+1}{2}$ for G in G′.

$g'(ww_i) = g'(e_{1i}) = 1$, for $i = 1, 2, \ldots, n-1$.
$g'(ww_n) = g'(e_{1n}) = 0$,
$g'(w_iw_{n+i}) = g'(e_{3i}) = 0$, for $i = 1, 5, 6, \ldots, n$.
$g'(w_2w_{n+2}) = g'(e_{32}) = 1$,
$g'(w_3w_{n+3}) = g'(e_{33}) = 1$,
$g'(w_4w_{n+4}) = g'(e_{34}) = 1$,

$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$, for i = 1, 2, …, n–2 and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for i = 1, 2, …, n–2 and i is even.
$g'(w_{n-1}w_n) = g'(e_{2(n-1)}) = 0$,
$g'(w_nw_1) = g'(e_{2n}) = 0$.

When n is odd, $p \equiv 2 \pmod 4$, q is odd with $e_f(1) = \frac{q-1}{2}$ and $e_f(0) = \frac{q+1}{2}$, then $e_g(1) = \frac{q-1}{2}$ for G in G′ + $\frac{3n+1}{2}$ for H_n in G′, $e_g(0) = \frac{q+1}{2}$ for G in G′ + $\frac{3n-1}{2}$ for H_n in G′ and $|e_g(0) - e_g(1)| = 0$.

Case (14) : $p \equiv 1 \pmod 4$, q is even, n is even and $n \geq 4$.
$g(w_1) = p+2$,
$g(w_i) = p+2i-1$, for i = 2,3, …, n.
$g(w_{n+1}) = p+1$,
$g(w_{n+i}) = p+2i$, for i = 2, 3, …, n.
Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is even, then $e_f(1) = e_f(0) = \frac{q}{2}$. Thus $e_g(1) = e_g(0) = \frac{q}{2}$ for G in G′.

$g'(ww_1) = g'(e_{11}) = 1$;
$g'(ww_i) = g'(e_{1i}) = 0$, for i = 2, 3, …, n.
$g'(w_iw_{n+i}) = g'(e_{3i}) = 1$, for i = 1, 2, …, n.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$, for i = 1, 2, …, n–1 and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for i = 1, 2, …, n–1 and i is even.
$g'(w_nw_1) = g'(e_{2n}) = 0$,

When n is even, $p \equiv 1 \pmod 4$, q is even with $e_f(1) = e_f(0) = \frac{q}{2}$, then $e_g(1) = \frac{q}{2}$ for G in G′ + $\frac{3n}{2}$ for H_n in G′, $e_g(0) = \frac{q}{2}$ for G in G′ + $\frac{3n}{2}$ for H_n in G′ and $|e_g(0) - e_g(1)| = 0$.

Case (15) : $p \equiv 3 \pmod 4$, q is even, n is even and $n \geq 4$.
$g(w_1) = p+2$,
$g(w_i) = p+2i-1$, for i = 2,3, …, n.
$g(w_{n+1}) = p+1$,
$g(w_{n+i}) = p+2i$, for i = 2, 3, …, n.
Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is even, then $e_f(1) = e_f(0) = \frac{q}{2}$. Thus $e_g(1) = e_g(0) = \frac{q}{2}$ for G in G′.

$g'(ww_1) = g'(e_{11}) = 1$;
$g'(ww_i) = g'(e_{1i}) = 0$, for i = 2, 3, …, n.
$g'(w_iw_{n+i}) = g'(e_{3i}) = 1$, for i = 1, 2, …, n.
$g'(w_1w_2) = g'(e_{21}) = 0$,
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for i = 2, 3, …, n–1 and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$, for i = 2, 3, …, n–1 and i is even.
$g'(w_nw_1) = g'(e_{2n}) = 0$,

When n is even, $p \equiv 3 \pmod 4$, q is even with $e_f(1) = e_f(0) = \frac{q}{2}$, then $e_g(1) = \frac{q}{2}$ for G in G′ + $\frac{3n}{2}$ for H_n in G′, $e_g(0) = \frac{q}{2}$ for G in G′ + $\frac{3n}{2}$ for H_n in G′ and $|e_g(0) - e_g(1)| = 0$.

Case (16) : $p \equiv 1 \pmod 4$, q is even, n is odd and $n \geq 3$.
$g(w_1) = p+2$,
$g(w_i) = p+2i-1$, for i = 2,3, …, n.
$g(w_{n+1}) = p+1$,
$g(w_{n+i}) = p+2i$, for i = 2, 3, …, n.
Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is even, then $e_f(1) = e_f(0) = \frac{q}{2}$. Thus $e_g(1) = e_g(0) = \frac{q}{2}$ for G in G′.

$g'(ww_1) = g'(e_{11}) = 1$;
$g'(ww_i) = g'(e_{1i}) = 0$, for i = 2, 3, …, n.
$g'(w_iw_{n+i}) = g'(e_{3i}) = 1$, for i = 1, 2, …, n.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$, for i = 1, 2, …, n–1 and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for i = 1, 2, …, n–1 and i is even.
$g'(w_nw_1) = g'(e_{2n}) = 0$,

When n is even, $p \equiv 1 \pmod 4$, q is even with $e_f(1) = e_f(0) = \frac{q}{2}$, then $e_g(1) = \frac{q}{2}$ for G in G′ + $\frac{3n+1}{2}$ for H_n in G′, $e_g(0) = \frac{q}{2}$ for G in G′ + $\frac{3n-1}{2}$ for H_n in G′ and $|e_g(0) - e_g(1)| = 1$.

Case (17) : $p \equiv 3 \pmod 4$, q is even, n is odd and $n \geq 3$.
$g(w_1) = p+2$,
$g(w_i) = p+2i-1$, for i = 2,3, …, n.
$g(w_{n+1}) = p+1$,
$g(w_{n+i}) = p+2i$, for i = 2, 3, …, n.
Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For q is even, then $e_f(1) = e_f(0) = \frac{q}{2}$. Thus $e_g(1) = e_g(0) = \frac{q}{2}$ for G in G′.

$g'(ww_1) = g'(e_{11}) = 1$;
$g'(ww_i) = g'(e_{1i}) = 0$, for i = 2, 3, …, n.
$g'(w_iw_{n+i}) = g'(e_{3i}) = 1$, for i = 1, 2, …, n.
$g'(w_1w_2) = g'(e_{21}) = 0$,
$g'(w_iw_{i+1}) = g'(e_{2i}) = 1$, for i = 2, 3, …, n–1 and i is odd.
$g'(w_iw_{i+1}) = g'(e_{2i}) = 0$, for i = 2, 3, …, n–1 and i is even.
$g'(w_nw_1) = g'(e_{2n}) = 0$,

When n is even, $p \equiv 3 \pmod 4$, q is even with $e_f(1) = e_f(0) = \frac{q}{2}$, then $e_g(1) = \frac{q}{2}$ for G in G′ + $\frac{3n-1}{2}$ for H_n in G′, $e_g(0) = \frac{q}{2}$ for G in G′ + $\frac{3n+1}{2}$ for H_n in G′ and $|e_g(0) - e_g(1)| = 1$.

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From the above all cases, we have $| e_g(0) - e_g(1) | \leq 1$.
Hence G is a PCC graph with p vertices and q edges under f with $f(v_k) = 1$, then the graph is obtained by identifying a vertex v_k in G and a vertex of degree n in H_n admits PCC labeling for $n \geq 3$.

IV. CONCLUSION

In this paper we study the PCC- labeling of the graph G′. The graph G′ is obtained by identifying a vertex v_k in G and a vertex of degree n in H_n, where G is a PCC graph with p vertices and q edges under f with $f(v_k) = 1$.

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