Narayana Prime Cordial Labeling of Cycle Related and Tree Related Graphs

S Venkatesh, B J Balamurugan

Abstract*: The Narayana prime cordial labeling of a graph $G(U,E)$ is a process of assigning the binary numbers 0 and 1 to the edges satisfying the cordiality condition through a $1-1$ function on U and an induced function on E. In this manuscript, we compute this labeling to certain cycle related graphs and tree related graphs viz. (i) ladder graph, (ii) wheel graph, (iii) fan graph, (iv) double star graph, (v) bistar graph and (vi) generalized star graph..

Keywords*: NP-cordial graphs, Prime numbers, Narayana numbers.

I. INTRODUCTION

Let $G(U,E)$ be a graph with vertex set U and edge set E. A mathematical scheme [1] of allotting numbers or labels to the vertices or edges of G is said to be a graph labeling of G. A first labeled graph was introduced by A.Rosa [10] in the year 1967. A list of applications of graph labeling can be found in [6]. The developments in the research field of graph labeling are updated by J.A.Gallian [4] in day to day basis. The graph terminologies and notions in this paper are referred to Harary [5]. B J Murali et al. introduced the Narayana prime cordial graphs using Narayana numbers [8]. We refer the research articles [7, 9 and 12] for Narayana numbers and their properties. Subsequently in [2, 3 and 11] one can find the Narayana prime cordial labeling of some interesting family of graphs. In this research article, we prove the existence of this Narayana prime cordial labeling to the ladder graph, wheel graph, fan graph, double star graph, bistar graph, and generalized star graph.

II. NP-CORDIAL LABELING OF GRAPHS

We recall the Narayana prime cordial graphs introduced by Murali B.J et al [8] in this section, with an appropriate example.

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Definition 2.1: [8]

Let $G(U,E)$ be a simple graph and $\mathbb{N}_0$ be the set of non-negative integers. A $1-1$ mapping $f:U\rightarrow\mathbb{N}_0$ of the graph G is called a Narayana prime cordial labeling if the induced edge mapping $f^*:E\rightarrow\{0,1\}$ defined for all $uv\in E$ and $u,v\in U$ such that

$$\begin{aligned}
f^*(uv) &= 1 \textit{ if } p \mid N(f(u),f(v)), \textit{ where } f(u)>f(v) \textit{ and } f(u)=p^m \textit{ for some } m\in\mathbb{N}_0; 1\le f(v)\le f(u)-2 \textit{ where } p \textit{ is a prime number}\\
&= 1 \textit{ if } p \mid N(f(v),f(u)), \textit{ where } f(v)>f(u) \textit{ and } f(v)=p^m \textit{ for some } m\in\mathbb{N}_0; 1\le f(u)\le f(v)-2 \textit{ where } p \textit{ is a prime number}\\
&= 0 \textit{ if } p \nmid N(f(u),f(v)), \textit{ where } f(u)>f(v) \textit{ and } f(u)=p^m-1 \textit{ for some } m\in\mathbb{N}_0; 0\le f(v)\le f(u)-1 \textit{ where } p \textit{ is a prime number}\\
&= 0 \textit{ if } p \nmid N(f(v),f(u)), \textit{ where } f(v)>f(u) \textit{ and } f(v)=p^m-1 \textit{ for some } m\in\mathbb{N}_0; 0\le f(u)\le f(v)-1 \textit{ where } p \textit{ is a prime number}
\end{aligned}$$

satisfying the condition $|e_{f^*}(0)-e_{f^*}(1)|\le 1$ where $e_{f^*}(0)$ and $e_{f^*}(1)$ represents respectively the number of edges with the label 0 and 1.

Definition 2.2

A graph $G(U,E)$ which admits a Narayana prime cordial labeling is called as a Narayana prime cordial graph.

Remark

In this paper, we call the Narayana prime cordial labeling of a graph as NP-cordial labeling of a graph for simplicity.

Example 2.1

Fig. 1 is an example of NP-cordial graph.

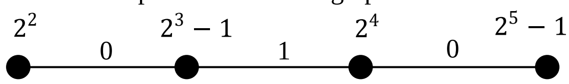

Fig. 1 NP-cordial labeling of P_4

III. NP-CORDIAL LABELING OF CYCLE RELATED GRAPHS

In this section, the existence of the NP-cordial labeling of some cycle related graphs such as ladder graph, wheel graph and fan graph are proved.

Theorem 3.1

The ladder graph L_n admits a NP-cordial labeling for $n \geq 2$.

Proof

Let $V = \{u_i | 1 \leq i \leq n\} \cup \{v_j | 1 \leq j \leq n\}$ be the vertex set and $E = \{u_i u_{i+1} | 1 \leq i \leq n-1\} \cup \{v_i v_{i+1} | 1 \leq i \leq n-1\} \cup \{u_i v_i | 1 \leq i \leq n\}$ be the edge set of the graph L_n.

Then L_n has $2n$ vertices and $3n-2$ edges

Case (i) when $n \equiv 1(mod\ 2)$

Define a $1-1$ function $f : U \to \mathbb{N}_0$ such that

$$f(u_i) = \begin{cases} 2^i & , i \equiv 1(mod\ 2)\ and\ 1 \leq i \leq n \\ 2^i - 1, & i \equiv 0(mod\ 2)\ and\ 1 \leq i \leq n \end{cases}$$

$$f(v_j) = \begin{cases} 2^{n+j} & , j \equiv 1(mod\ 2)\ and\ 1 \leq j \leq n \\ 2^{n+j} - 1, & j \equiv 0(mod\ 2)\ and\ 1 \leq j \leq n \end{cases}$$

and $f^* : E \to \{0,1\}$ as in the definition 2.1.

Through these functions, $\frac{3(n-1)}{2} + 1$ edges receive label 1 and $\frac{3(n-1)}{2}$ edges receive the label 0.

That is, $e_{f^*}(1) = \frac{3(n-1)}{2} + 1$ and $e_{f^*}(0) = \frac{3(n-1)}{2}$.

Therefore $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ is satisfied.

Case (ii) when $n \equiv 0(mod\ 2)$

Define a $1-1$ function $f : U \to \mathbb{N}_0$ such that

$$f(u_i) = \begin{cases} 2^i & , i \equiv 1(mod\ 2)\ and\ 1 \leq i \leq n, i = n \\ 2^i - 1, & i \equiv 0(mod\ 2)\ and\ 1 < i < n \end{cases}$$

$$f(v_j) = \begin{cases} 2^{n+j} & , j \equiv 1(mod\ 2)\ and\ 1 \leq j \leq n \\ 2^{n+j} - 1, & j \equiv 0(mod\ 2)\ and\ 1 \leq j \leq n \end{cases}$$

and $f^* : E \to \{0,1\}$ defined as in the definition 2.1.

Through these functions, $\frac{3n}{2} - 1$ edges receive the label 1 and $\frac{3n}{2} - 1$ edges receive the label 0

That is, $e_{f^*}(1) = \frac{3n}{2} - 1$ and $e_{f^*}(0) = \frac{3n}{2} - 1$. Therefore $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ is satisfied.

In all cases, the condition $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ is satisfied and therefore the ladder graph L_n is a NP-cordial graph.

The following examples illustrate the NP-cordial labeling of the ladder graph.

Example 3.1

When $n = 6$ the NP-cordial labeling of L_6 is given in Fig. 2.

Example 3.2

When $n = 5$ the NP-cordial labeling of L_5 is given in Fig. 3.

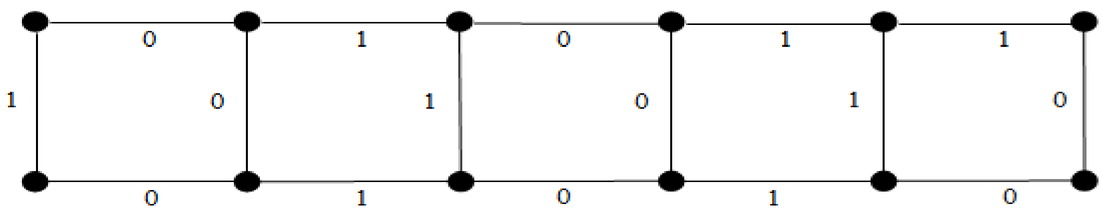

Fig. 2 NP-cordial labeling of L_6

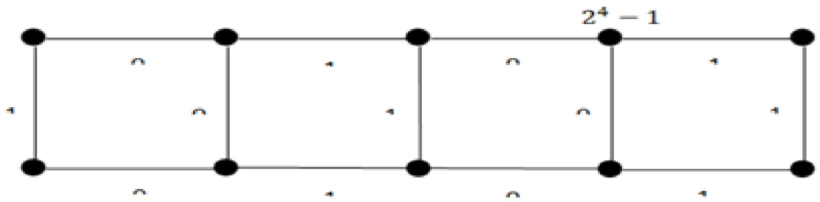

Fig. 3 NP-cordial labeling of L_5

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Theorem 3.2

The wheel graph W_n admits a NP-cordial labeling when $n \geq 3$.

Proof

Let $V = \{v_0, v_1, v_2, \ldots, v_n\}$ be the vertex set and $E = \{v_i v_{i+1} \mid 1 \leq i < n\} \cup \{v_n v_1\} \cup \{v_0 v_i \mid 1 \leq i \leq n\}$ be the edge set of W_n.

Then W_n has $n+1$ vertices and $2n$ edges.

Define a $1-1$ mapping $f: U \to \mathbb{N}_0$ as

$$f(v_0) = 2^{n+1} - 1$$

$$f(v_i) = 2^i, \ 1 \leq i \leq n$$

and $f^*: E \to \{0,1\}$ as in the definition 2.1.

These functions provide the label 0 to 'n' edges and the label 1 to 'n' edges.

That is, $e_{f^*}(1) = n$ and $e_{f^*}(0) = n$. Therefore $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ is satisfied.

Hence the wheel graph W_n admits a NP-cordial labeling.

The following example illustrates the NP-cordial labeling of the wheel graph W_n.

Example 3.3

The NP-cordial labeling of W_5 is given in Fig. 4.

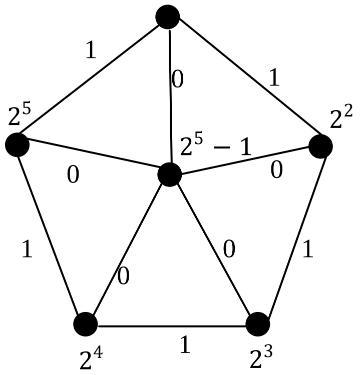

Fig. 4 NP-cordial labeling of W_5

Theorem 3.3

The fan graph F_n admits a NP-cordial labeling when $n \geq 2$.

Proof

Let $V = \{v_0, v_1, v_2, \ldots, v_n\}$ be the vertex set and $E = \{v_i v_{i+1} \mid 1 \leq i < n\} \cup \{v_0 v_i \mid 1 \leq i \leq n\}$ be the edge set of F_n.

Then F_n has $n+1$ vertices and $2n-1$ edges.

Define a $1-1$ mapping $f: U \to \mathbb{N}_0$ as

$$f(v_0) = 2$$

$$f(v_i) = \begin{cases} 2^{i+1} & , i \equiv 1 \ (mod\ 2) \\ 2^{i+1} - 1, & i \equiv 0 (mod\ 2) \end{cases}$$

and $f^*: E \to \{0,1\}$ as in the definition 2.1.

Case (i) when $n \equiv 1 \ (mod\ 2)$

Through these functions, $n-1$ edges receive the label 0 and n edges receive the label 1.

That is, $e_{f^*}(1) = n$ and $e_{f^*}(0) = n-1$. Therefore $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ is satisfied.

Case (ii) when $n \equiv 0 \ (mod\ 2)$

Through these functions, n edges receive the label 0 and $n-1$ edges receive the label 1.

That is $e_{f^*}(1) = n-1$ and $e_{f^*}(0) = n$. Therefore $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ is satisfied.

Hence in both cases the fan graph F_n admits a NP-cordial labeling.

The following example illustrates the NP-cordial labeling of the fan graph F_n.

Example 3.4

The NP-cordial labeling of the fan graph F_5 is given in the Fig. 5.

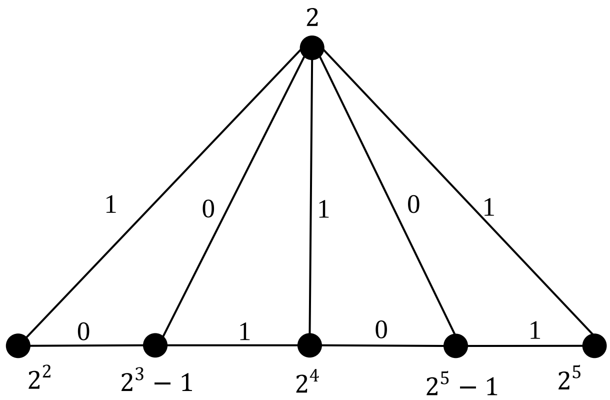

Fig. 5 NP-cordial labeling of F_5

IV. NP-CORDIAL LABELING OF TREE RELATED GRAPHS

In this section, the existence of NP-cordial labeling of some tree related graphs such as bistar graph, double star graph and generalized star graph are proved.

Theorem 4,1

The double star $S_{2,n}$ admits a NP-cordial labeling

Proof

Let $V = \{v_0\} \cup \{u_i \mid 1 \leq i \leq n\} \cup \{v_i \mid \leq i \leq n\}$ be the vertex set and $E = \{v_0 u_i \mid 1 \leq i \leq n\} \cup \{u_i v_i \mid 1 \leq i \leq n\}$ be edge set of $S_{2,n}$. Then the double star $S_{2,n}$ has $2n+1$ vertices and $2n$ edges.

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Define a $1-1$ mapping $f:U\to\mathbb{N}_0$ as

$f(v_0)=1$

$f(u_i)=2^{i+1},\quad 1\le i\le n$

$f(v_i)=3^{i+1}-1$

and $f^*:E\to\{0,1\}$ as in the definition 2.1.

Through these functions, n edges of $S_{2,n}$ receive the label 1 and n edges receive the label 0.

This means that, $e_{f^*}(0)=n$ and $e_{f^*}(1)=n$. Therefore $\left|e_{f^*}(0)-e_{f^*}(1)\right|\le 1$ is satisfied.

Hence the double star $S_{2,n}$ admits a NP-cordial labeling.

The following example illustrates the NP-cordial labeling of the double star $S_{2,n}$.

Example 4.1

The NP-cordial labeling of the graph $S_{2,5}$ is given in the Fig. 6.

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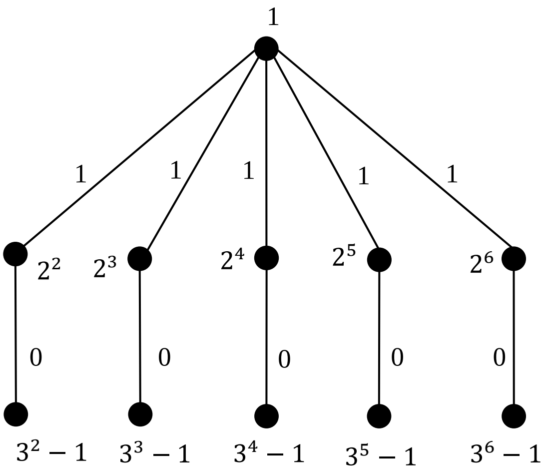

Fig. 6 NP-cordial labeling of $S_{2,5}$

Theorem 4.2

The bistar graph $B_{n,m}$ admits a NP-cordial labeling.

Proof

Let $B_{n,m}$ be the bistar graph with vertex set $V=\{u_i\,|\,0\le i\le n\}\cup\{v_i\,|\,0\le i\le m\}$ and edge set $E=\{u_0v_0\}\cup\{u_0v_j\,|1\le j\le m\}\cup\{v_0u_i|1\le i\le n\}$.

Then the bistar graph has $(m+n+2)$ vertices and $(m+n+1)$ edges.

Define a $1-1$ mapping $f:U\to\mathbb{N}_0$ such that

$f(u_0)=1,\quad f(v_0)=2$

$$f(u_i)=\begin{cases}3^i & ,\ i\equiv 1(mod\,2)\\ 3^i-1, & i\equiv 0(mod\,2)\end{cases}$$

$$f(v_i)=\begin{cases}2^{i+1} & ,\ i\equiv 1(mod\,2)\\ 2^{i+1}-1, & i\equiv 0(mod\,2)\end{cases}$$

and $f^*:E\to\{0,1\}$ as in the definition 2.1.

Case (1) when $n\equiv 0(mod\,2)\,and\;m\equiv 0(mod\,2)$

In this case, $B_{n,m}$ has odd number of edges. The functions defined above will enable the graph such that $\left(\frac{n}{2}+\frac{m}{2}\right)$ number of edges receive the label 1 and $\left(\frac{n}{2}+\frac{m}{2}+1\right)$ number of edges receive the label 0.

This means that, $e_{f^*}(0)=\left(\frac{n}{2}+\frac{m}{2}\right)$ and $e_{f^*}(1)=\left(\frac{n}{2}+\frac{m}{2}+1\right)$. Therefore $\left|e_{f^*}(0)-e_{f^*}(1)\right|\le 1$ is satisfied.

Case (2) when $n\equiv 0(mod\,2)\,and\;m\equiv 1(mod\,2)$

In this case, $B_{n,m}$ has even number of edges. Through these labeling functions $\left(\frac{n}{2}+\frac{m+1}{2}\right)$ number of edges receive the label 1 and $\left(\frac{n}{2}+\frac{m+1}{2}\right)$ number of edges receive the label 0.

This means that, $e_{f^*}(0)=\left(\frac{n}{2}+\frac{m+1}{2}\right)$ and $e_{f^*}(1)=\left(\frac{n}{2}+\frac{m+1}{2}\right)$. Therefore $\left|e_{f^*}(0)-e_{f^*}(1)\right|\le 1$ is satisfied.

Case (3) when $n\equiv 1(mod\,2)\,and\;m\equiv 0(mod\,2)$

Here $B_{n,m}$ has even number of edges. Through these functions $\left(\frac{n+1}{2}+\frac{m}{2}\right)$ number of edges receive the label 1 and $\left(\frac{n+1}{2}+\frac{m}{2}\right)$ number of edges receive the label 0.

This means that, $e_{f^*}(0)=\left(\frac{n+1}{2}+\frac{m}{2}\right)$ and $e_{f^*}(1)=\left(\frac{n+1}{2}+\frac{m}{2}\right)$. Therefore $\left|e_{f^*}(0)-e_{f^*}(1)\right|\le 1$ is satisfied.

Case (4) when $n\equiv 1(mod\,2)\,and\;m\equiv 1(mod\,2)$

In this case, $B_{n,m}$ has odd number of edges. Through these labeling functions $\left(\frac{n+1}{2}+\frac{m+1}{2}\right)$ number of edges

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Receive the label 1 and $\left(\frac{n-1}{2}+\frac{m+1}{2}\right)$ number of edges receive the label 0.

This means that, $e_{f^*}(0)=\left(\frac{n+1}{2}+\frac{m+1}{2}\right)$ and $e_{f^*}(1)=\left(\frac{n-1}{2}+\frac{m+1}{2}\right)$. Therefore $\left|e_{f^*}(0)-e_{f^*}(1)\right|\le 1$ is satisfied.

In all cases the condition $\left|e_{f^*}(0)-e_{f^*}(1)\right|\le 1$ is satisfied. And hence the bistar graph admits a NP-cordial labeling.

The following example illustrates the NP-cordial labeling of the bistar graph.

Example 4.2

The NP-cordial labeling of $B_{4,3}$ is given in the Fig. 7

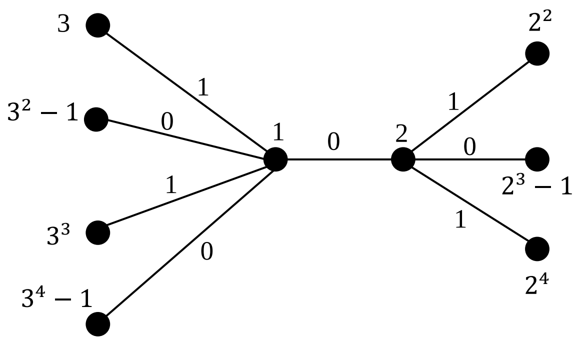

Fig. 7 NP-cordial labeling of $B_{4,3}$

Theorem 4.3

The generalized star graph $S_{m,n}$ admits a NP-cordial labeling.

Proof

Let $S_{m,n}$ be the generalized star graph with vertex set $V=\{v_0\}\cup\{v_{ij}|1\le i\le n,1\le j\le n\}$ and the edge set $E=\{v_0v_{1j}|1\le j\le n\}\cup\{v_{ij}v_{(i+1)j}|1\le i\le m-1,1\le j\le n\}$.

Then the star graph $S_{m,n}$ has $(mn+1)$ vertices and mn edges.

Case (1) when $m\equiv 0(mod\ 2)$

Define a $1-1$ mapping $f:V\to\mathbb{N}_0$ such that

$$f(v_0)=1$$

$$f(v_{ij})=\begin{cases}p_j^{\,i+1} & ,\ j\equiv 0(mod 2) and\ j\le n,\ 1\le i\le m\\ p_j^{\,i+1}-1, & j\equiv 1(mod 2) and\ j\le n,\ 1\le i\le m\end{cases}$$

and $f^*:E\to\{0,1\}$ as in the definition 2.1.

From these labeling functions $mn/2$ edges receive the value 1 and $mn/2$ edges receive the value 0.

This means that, $e_{f^*}(0)=mn/2$ and $e_{f^*}(1)=mn/2$.

Therefore $\left|e_{f^*}(0)-e_{f^*}(1)\right|\le 1$ is satisfied.

Case (2) when $m\equiv 1(mod\ 2)$

Define a $1-1$ mapping $f:V\to\mathbb{N}_0$ such that

$$f(v_0)=1$$

$$f(v_{ij})=\begin{cases}p_j^{\,i+1} & ,\ j\equiv 1(mod 2) and\ j< n,\ 1\le i\le m\\ p_j^{\,i+1}-1, & j\equiv 0(mod 2) and\ j< n,\ 1\le i\le m\end{cases}$$

$$f(v_{in})=\begin{cases}p_n^{\,i+1} & ,\ i\equiv 1(mod 2)\\ p_n^{\,i+1}-1, & i\equiv 0(mod 2)\end{cases}$$

and $f^*:E\to\{0,1\}$ as in the definition 2.1.

Subcase (2(i)) when $n\equiv 0(mod\ 2)$

In this case we have even number of edges in the graph.

From these labeling functions $mn/2$ edges receive the label 1 and $mn/2$ edges receive the label 0.

This means that, $e_{f^*}(0)=mn/2$ and $e_{f^*}(1)=mn/2$.

Therefore the condition $\left|e_{f^*}(0)-e_{f^*}(1)\right|\le 1$ is satisfied.

Subcase (2(i)) when $n\equiv 1(mod\ 2)$

In this case, we have odd number of edges in the graph.

From these labeling functions, $\frac{mn+1}{2}$ edges receive the label 1 and $\frac{mn-1}{2}$ edges receive the label 0.

This means that, $e_{f^*}(0)=\frac{mn-1}{2}$ and $e_{f^*}(1)=\frac{mn+1}{2}$.

Therefore $\left|e_{f^*}(0)-e_{f^*}(1)\right|\le 1$ is satisfied.

Hence in all cases the cordiality condition $\left|e_{f^*}(0)-e_{f^*}(1)\right|\le 1$ is satisfied.

Hence the proof.

The following example illustrates the NP-cordial labeling of the star graph.

Example 4.3

When $m=3,n=5$ the NP-cordial labeling of $S_{3,5}$ is given in Fig. 8.

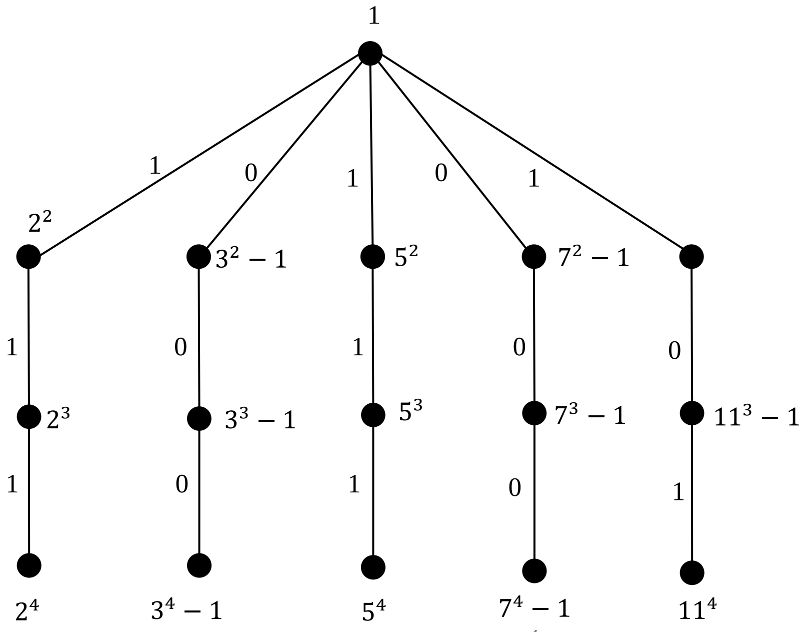

Fig. 8 the NP-cordial labeling of $S_{3,5}$

V. CONCLUSION

In this paper we have computed the NP-cordial labeling of ladder graph, wheel graph, fan graph, double star graph, bistar graph and generalized star graph. Computing NP-cordial labeling of other family of graphs is an interesting and potential area of research in graph theory and future scope for the readers.

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