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Inverse of K Regular Super Fuzzy Matrix

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Abstract: ***Fuzzy matrix (FM) is a rich topic in modeling uncertain situation occurred. Every FM can be visualized as a three dimensional figure, but this representation is not possible for classical matrix without any proper scaling. To overcome this problem we need a certain special classical fuzzy matrix. In this paper, the concept of inverse of k-regular fuzzy matrix is introduced and derived some basic properties of an inverse of k-regular fuzzy matrix. This leads to the characterization of a matrix for which the regularity indicator is equal. Further the connection between regular, k-regular and consistency of powers of fuzzy matrices are discussed.***

Keywords**: *inverse, k-regular, super fuzzy matrix, Fuzzy matrix, regular.***

I. INTRODUCTION

A Boolean matrix is a matrix with factors, every has the value 0 or 1. A fuzzy matrix is a matrix with essentials having values in [0,1]. The idea of sections of a fuzzy matrix was delivered by way of kim and Roush. Dubois and Prade, Horst, Kandasamy et al., Kaufmann, M.M.Gupta, kim J.B, Ragab and Emam and Thomason gave the superior improvement of fuzzy matrices. Later, Zheng and Wang discussed the m x n widespread fuzzy linear machine and the inconsistent fuzzy linear system by using modified regular inverses of the coefficient matrix. In 2008, Abbasbandy et al., investigated the minimum answer of the overall twin fuzzy linear device by using matrix modified everyday inverse theory. In this paper, we propose a new technique to the inverse of terrific fuzzy matrix is modified everyday idea to be applied.

In section 2 preliminaries and basic concepts of super fuzzy matrix with an example. Review some fundamental results and theorems of inverses of k regular super fuzzy matrices in section 3 and we draw the conclusion in Section 4.

II. EXISTING MODELS FUZZY TECHNIQUES

In this section discuss the simple definitions and notations of super fuzzy matrix. Here, we are involved with super fuzzy matrices (SFM) with support [0,1], underneath maxmin (min max) operations and the same old ordering of actual numbers. Let $(SF)_{mxn}$ be the set of all super fuzzy matrices of order m x n and $(SF)_n$ be the set of all super fuzzy matrices of order n x n. R(A) or C(A) is the space generated by the row (or) column.

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2.1 Corners & Edge detection Approach

Definition 2.1: Fuzzy Matrix

A *fuzzy matrix* is a matrix which has its elements from [0, 1]. We have a tendency to as just in case of matrix have a rectangular fuzzy matrix, fuzzy square matrix, fuzzy row matrix and fuzzy column matrix.

Definition 2.2: super fuzzy matrix

Let us consider a fuzzy matrix $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$

where A_{11}, A_{12}, A_{13}, A_{21}, A_{22} and A_{23} be fuzzy sub-matrices and number of columns in the fuzzy sub-matrices A_{11} and A_{21} are equal. Similarly the columns in fuzzy sub-matrices of the A_{12} and A_{22} are equal and columns of fuzzy matrices A_{13} and A_{23} are equal. This is evident from the second index of the fuzzy sub-matrices. One can also see, the number of rows in fuzzy sub-matrices A_{11},A_{12} and A_{13} are equal.

Similarly for fuzzy sub-matrices of A_{21}, A_{22} and A_{23} the numbers of rows are equal. Thus a general ***super fuzzy matrix***.

$$A = \begin{bmatrix} A_{11} & A_{12} \cdots & A_{1n} \\ A_{21} & A_{22} \dots & A_{2n} \\ & & \vdots \\ A_{m1} & A_{m2} \dots & A_{mn} \end{bmatrix}$$

where Aij's are fuzzy sub-matrices; i = 1, 2, …, m and j =1, 2, …, n.

Definition 2.3: Modified regular Inverse of Matrix

Modified regular Inverse of Matrix A modified regular inverse of a non-singular matrix is giving the specific answer of a positive set of equations. This modified regular inverse exists for any (possibly square) matrix by any means with complex elements. It is used right here, for solving linear matrix equations, and among other programs for locating an expression for the main idempotent elements of a matrix.

III. METHODOLOGY

In this section, the k regular super fuzzy matrix is investigated.

3.1 Inverse of k-regular Super Fuzzy Matrix

Definition 3.1

A matrix $A\in(SF)_n$, is said to be a right inverse of k regular if exists a matrix $X\in(SF)_n$ such that $A^kXA=A^k$ for some positive integer k. So X is called the inverse of K regular super fuzzy matrix A.

$A_\mu=\{1^k\}=\{X/A^kXA=A^k\}$

Definition 3.2

A matrix $A\in(SF)_n$, is said to be a left inverse of k regular if exists a matrix $Y\in(SF)_n$ such that $A^kYA=A^k$ for some positive integer k. So Y is called the inverse of K regular super fuzzy matrix A.

$A_\vartheta=\{1^k\}=\{Y/AYA^k=A^k\}$

Where, $A\{1^k\}=A_\mu\{1^k \cup A_\vartheta\{1^k\}$

Definition 3.3

Let $A = \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ & & \vdots \\ a_{m1} & a_{m2} \cdots & a_{mn} \end{bmatrix}$ be a Fuzzy matrix that is each a_{ij} satisfies $0 \le a_{ij} \le 1$, then the k regular super fuzzy matrix which we denoted as A^k.

Here the strength of each element is considered exactly. It is the name of the G-inverse.

$$A^k = \begin{bmatrix} 1-ka_{11} & 1-ka_{12} \cdots & 1-ka_{1n} \\ 1-ka_{21} & 1-ka_{22} \cdots & 1-ka_{2n} \\ & & \vdots \\ 1-ka_{m1} & 1-ka_{m2} \cdots & 1-ka_{mn} \end{bmatrix}$$

Here k is the smallest positive integer (Based on Probability). The multiplication of two fuzzy matrixes need not be fuzzy. It is replacing every positive entry is consider as 1 and a negative entry considers as 0 or negative tends to zero.

Properties 3.4

$(A^k)^k = A$

$AxA^k = A^k$,

$(A+B)^k = A^k + B^k$

$(\lambda A)^k = \lambda A^k$

$(BA)^k = A^k B^k$

$AA^k=0$ implies $A=0$

Conditions 3.5

$AXA=A$

$XAX=X$

$(AX)^k = AX$

$(XA)^k = XA$

$XX^kA^k = X$

$XAA^k=A^k$

$BA^kAA^k=A^k$

$X=XX^kA^k=XX^kA^kAY=XAY=XAA^kY^kY=A^kY^kY=Y$.

IV. RESULT

The inverse of k super fuzzy matrix satisfied the all the necessary conditions of convergence.

Lemma 4.1

For $A,B \in (SF)_{mxn}$, $R(B) \sqsubseteq R(A) \Leftrightarrow B=XA$ for some $X \in (SF)_m$, $C(B) \sqsubseteq C(A) \Leftrightarrow B=AY$ for some $Y \in (SF)_n$.

Theorem 4.2

Let A be an inverse of k regular super fuzzy matrix whose non zero rows form a standard basis. If for some super fuzzy matrix S, A satisfying the matrix equation ASA=A under the max min principles, then A is k regular.

Proof:

Here non zero rows of an inverse of k regular super fuzzy matrix A form a standard basis. Let SA=X, the rows of X are rearrangements of rows of A.

Then X is an idempotent of k regular super fuzzy, that is $X^2=X$, having same row space as A with non zero rows of Z form a standard basis also.

Since the standard basis is unique, therefore A=GX for all S. Then

$$AS^TA=SXS^TSX=SXX=SX=A^k$$

$$\Rightarrow ASA=A^k.$$

So, A is k *regular*.

Theorem 4.3

Let $A,B \in (SF)_{nxm}$ be two inverse of k regular super fuzzy matrix. If A is k regular, then we prove that

(i). $S(B) \sqsubseteq S(A)$ iff $B=BA^{I}A$ for each $A^{I} \in A(1^k)$.

(ii). $R(B) \sqsubseteq R(A)$ iff $B=A^{I}AB$ for each $A^{I} \in A(1^k)$.

Proof:

(i). let $S(B) \sqsubseteq S(A)$, then each row of B is a linear combination of the A(row). Hence $B_i^* = \sum x_{ij} A^*$.

Where $x_{ij} \epsilon F$

$\Rightarrow B=XA^k$

$\Rightarrow B=XAA^{I}A$ (since $AA^{I}A=A^k$)

$\Rightarrow B=BA^{I}A^k$

(ii). Let $R(B) \sqsubseteq R(A^k)$ (since $B=AY$ ($Y \in F_n$))

$\Rightarrow B=AA^{I}A$

$\Rightarrow B=AA^{I}AY$.

$\Rightarrow B=A^kA^{I}B$.

Theorem 4.4

Let $A \in (SF)_{nxm}$ be a regular inverse of k regular super fuzzy matrix and k regular be a g-inverse of A. Then G be the inverse of k regular super fuzzy matrix,

(i) $G^T \in A^T\{1^k\}$.

(ii) If Q and R are the inverse of k regular super fuzzy matrix, then $R^TGQ^T \in QAR\{1^k\}$.

(iii) AG and GA are idempotent.

Proof:

Let G be a inverse of k regular super fuzzy matrix A.

Then AGA= A holds. Taking transpose on both sides, we get $A^TG^T A^T = A^T$.

This implies $G^T \in A^T\{1^k\}$.

(ii) Since Q and R are the inverse of k regular super fuzzy matrix, Q and R are invertible and

$Q^{-1} = Q^T$, $R^{-1} = R^T$.

Now,

$QAR(R^TGQ^T)QAR=QA(RR^T)G(Q^TQ)AR$

$= QAGAR$ (as $RR^T=I$, $QQ^T=I$)

$= QAR$. (as $AGA=A$).

This implies $R^TGQ^T \in QAR\{1^k\}$

(iii) Again, $(AG)(AG)=(AGA)G$

$= AG$ (as $AGA=A$).

Also $(GA)(GA)=(GAG)A$

$= GA$ (as $GAG=G$).

Thus AG and GA are *idempotent*.

Theorem 4.5

Let A be an k regular super fuzzy matrix $Y,Z \in A\{1^k\}$ and $P=QAR$. Then $X \in A\{1,2\}$, that is, P is a semi inverse of A .

Proof:

Since $Q,R \in A\{1^k\} \Rightarrow AQA=A$ and $ARA=A$.

As $P=QAR$ so, $AXA=A(QAR)A$

$= (AQA)RA$

$= ARA$

$= A$ is k regular

Also, $PAP=(QAR)A(QAR)$

$= Q(ARA)(QAR)$

$= Q(AQA)R$

$= QAR$

$= P$.

So P is a *semi-inverse* of the inverse of k regular super fuzzy matrix A.

Theorem 4.6

If n, $A \in (SF)_{nxm}$ be the symmetric and the idempotent inverse of k regular super fuzzy matrix A.

Proof :

Here $A^T=A$ and $A^2=A$.

For $Q=I_n$, $QA=A$.

Then $AQA=AA=A^2=A$.

That is, $A \in A\{1\}$.

Now $(XA)^T=A^T X^T=AX^T=AA^T$ (Taking X=A, as A itself a inverse.)
$= AA=XA$.
Hence AÎ A{1,4}.

Theorem 4.7

Let m,n $A\in(SF)_{nxm}$ be an inverse of k regular super fuzzy matrix and $X\in A\{1^k\}$, then $X\in A\{2\}$ iff G(AX)=G(X).

Proof:

$X\in A\{2\}$ implies XAX = A.
That is, $A\in X\{1^k\}$.
Hence, G(X)=G(AX) (since AX is idempotent).
Conversely, let G(AX)=G(A), then for a pair of matrices A and X, if the product AX is
defined.
so, $G(AX)\in G(X)$.
That is, X=YAX , for some m $Y\in F_m$.
So X (AX)=(YAX)AX
$\Rightarrow$XAX =Y(AXA)X=YAX=X.
Hence $X\in A\{2\}$.

Theorem 4.8

If n $A\in(SF)_n$ be symmetric and idempotent SF then A is k regular super fuzzy matrix and itself a least square inverse.

Proof:

Since A is symmetric, A^T=A and A is idempotent, A^2=A.
Now QA=A if n $P=I_n$.
Then $AQA=AA=A^2$ =A.
That is, $A\in A\{1\}$.
Now $(AX)^T=X^TA^T=X^TA=A^TA$ (Taking X = A, as A itself a inverse.)
= AA=AX.
This implies, $A\in A\{1,3\}$.

Theorem 4.9

Let $A\in(SF)_n$ and k be a positive integer. The following statements are true.
(i). A is k regular in super fuzzy matrix
(ii). γA is k regular for $\gamma\neq0\in F$

Proof:

Let A is a super fuzzy matrix.
Then $A=A=(A_\mu,A_\vartheta)=((a_{ij\mu}),(a_{ij\vartheta}))$.
Then $\gamma A=\gamma\,(A_\mu,A_\vartheta)=(min(\gamma,a_{ij\mu}),max(\gamma,a_{ij\vartheta}))=A\gamma$ and $\gamma\cdot\gamma=\gamma$.
A is right k-regular $\Rightarrow A^kXA=A^k$
$\Rightarrow (\gamma A)^k X\gamma A=(\gamma A)^k$
$\Rightarrow$ λA is right k-regular.
If λA is right k-regular, then for λ=1, A is right k-regular.
Similarly, the result can be proved for left k-regular.
Thus (i) ⇔ (ii) hold.
A is right k-regular $\Rightarrow A^kXA = A^k$, X is a right k- inverse of A.

Theorem 4.10

Let $A\in(SF)_n$ and k be a positive integer, then $X\in A_\mu\{1^k\}\leftrightarrow X^T\in A^T_\vartheta\{1^k\}$.

Proof:

$X\in A_\mu\{1^k\}\leftrightarrow A^k X A=A^k$
$(A^k X A)^T=(A^k)^T$
$A^T X^T (A^T)^K=(A^T)^K$
$X^T=A^T_\vartheta\{1^k\}$

Theorem 4.11

Let $A\in(SF)_n$ X is $\{1_\mu{}^k,3^k\}$ inverse of A and G^- is a $\{1_\vartheta{}^k,3^k\}$inverse of then $A^kX=A^kG^-$.

Proof:

Since X is a $\{1_\mu{}^k,3^k\}$inverse of A

We know that, $A^k XA=A^k$ and $(A^k X)^T=(A^k)X$.
Also, we know G^- is a $\{1_\vartheta{}^k,3^k\}$inverse of A.
We know that $AG^-A^k=A^k$ and $(AG^-)^T = AG^-$.
$A^kG^-=(A^kXA)G^-=(A^kX)^T(AG^-)=(A^kX)^T(AG^-)^T=X^T(A^T)^k(G^-)^TA^T=X^T(AG^-A^k)^T=$
$X^T(A^k)^T=(A^kX)^T=A^kX$.
Hence the Theorem.

Theorem 4.12

Let $A\in(SF)_n$ X is $\{1_\mu{}^k,4^k\}$ inverse of A and G^- is a $\{1_\vartheta{}^k,4^k\}$inverse of then $A^k X=A^kG^-$.

Proof:

The Proof is similar to Theorem 3.9 and hence omitted.

Theorem 4.13

For $A,B\in(SF)_n$, with R(A)=R(B) and $R(A^k)=R(B^k)$ then, A is right k-regular ⇔ B is right k-regular.

Proof:

Let A be a right k-regular matrix, satisfying $R(B^k)\subseteq R(A^k)$ and $R(A)\subseteq R(B)$. Since $R(B^k)\subseteq R(A^k)$ and $B^k=B^kXA$ for each k is inverse X of A.
Since $R(A)\subseteq R(B)$ and A=YB for some $Y\in(SF)_n$.
Substituting for A in $B^k=B^kXA$, we get $B^k=B^kXA=B^kXYB=B^kZB$
where XY=Z.
Hence B is right k-regular.
Conversely, if B is a right k-regular matrix satisfying $R(A^k)\subseteq R(B^k)$ and $R(B)\subseteq R(A)$, then A is right k-regular can be proved in the same manner.
Hence the theorem.

Theorem 4.14

For $A,B\in(SF)_n$, with C(A)=C(B) and $C(A^k)=C(B^k)$ then, A are left k-regular ⇔B is left k-regular.

Proof:

This is similar to theorem 4.13. So it's omitted.

Theorem 4.15

(i). If $A,B\in(SF)_{mn}$ with R(A)=R(B) (or) C(A)=C(B), then A is regular ⇔B is regular.
(ii). For $A\in(SF)_n$ with $R(A)=R(A^TA)$ and $R(A^k)=R((A^TA)^k)$ then, A is right k- regular ⇔A^T, A is right k-regular.
(iii). For $A\in(SF)_n$ with $C(A)=C(A^TA)$ and $C(A^k)=C((A^TA)^k)$ then, A is left k-regular ⇔ A^T, A is left k-regular.

Theorem 4.16

For $A\in(SF)_n$ if $R(A)=R(A^k)$ then the following statements are equivalent:
A is regular
A is right k-regular
A^k is regular
A^k is right k-regular

Proof:

If A is regular, then AXA = A for some X in IF_n, for $k\geq1$, Pre-multiplying by A^{k-1} on both sides, we get $A^kXA=A^k$.
Therefore A is right k-regular for all $k\geq1$.
Thus (i) ⇒ (ii).

Since $R(A)\subseteq R(A^k)$, by Lemma (4.1) $A=YA^k$ for some $Y\in(SF)_n$. (4.1)

If A is right k-regular, then $A^kXA = A^k$ for some $X\in(SF)_n$. (4.2)

Premultiplying by Y on both sides in (4.2) and using (4.1), we get

$YA^kXA=YA^k\Rightarrow AXA=A$. Therefore A is regular.

Thus (ii) $\Rightarrow$ (i).

If A^k is regular then $A^kZA^k=A^k$ for some Z in $(SF)_n$. (4.3)

This can write as $A^kWA=A^k$ where $W=ZA^{k-1}$,

Hence A is right k-regular and W is a right k-g inverse of A .

Thus (iii) $\Rightarrow$ (ii).

By using (4.1) and (4.2), $A^kXYA^k = A^k \Rightarrow A^kVA^k = A^k$ where V = XY.

Therefore A^k is regular. Thus (ii) $\Rightarrow$ (iii). Hence (i) $\Leftrightarrow$ (ii) $\Leftrightarrow$ (iii).

Next, let us prove that (iii) $\Leftrightarrow$ (iv).

Pre-multiplying by $(A^k)^{k-1}$ on both sides in (3.3), we get $(A^k)^kZA^k=(A^k)^k$. Thus A^k is right k-regular. Hence (iii) $\Rightarrow$ (iv).

If A^k is right k-regular then $(A^k)^kUA^k=(A^k)^k$ for some $U\in(SF)_n$. (4.4)

By using (4.1) and (4.4), we get $A^kUA^k = A^k$.

Thus (iv) $\Rightarrow$ (iii).

Hence the Theorem.

Theorem 4.17

For $A\in(SF)_n$ if $C(A)=C(A^k)$ then the following statements are equivalent:

A is regular.

A is left k-regular.

A^k is regular.

A^k is left k-regular.

Proof:

This can be Proved as that of Theorem (4.16) and hence omitted.

Remarks 4.18

(i). A matrix $A\in(SF)_n$, is said to have a $\{3^k\}$, inverse if there exists a matrix $X\in(SF)n$ such that $(A^kX)^T=A^kX$, for some positive integer k. So, X is called the $\{3^k\}$, inverse of A.

Let A, $\{3^k\}=X/(A^kX)^T=A^kX$.

(ii). A matrix $A\in(SF)_n$, is said to have a $\{4^k\}$, inverse if there exists a matrix $X\in(SF)_n$ such that $(A^kX)^T= A^kX$, for some positive integer k. So, X is called the $\{4^k\}$, inverse of A.

Let A,$\{4^k\}=X/(A^kX)^T=A^kX$.

(iii). A matrix $A\in(SF)_n$, is said to have a $\{n^k\}$, inverse if there exists a matrix $X\in(SF)_n$ such that $(A^kX)^T=A^kX$, for some positive integer k. So, X is called the $\{n^k\}$, inverse of A.

Let A, $\{n^k\}=X/(A^kX)^T=A^kX$.

Theorem 4.19

Let $A\in(SF)_n$ and X = Y AZ,

(i). if Y, $Z\in A_r\{1^k\}$ then $X\in A_r\{1^k\}$.

(ii). if Y, $Z\in AA\{1^k\}$then $X\in AA\{1^k\}$.

(iii). if $Y\in A_r\{1^k\}$ and $Z\in A\{3^k\}$then $X\in A\{3^k\}$.

(iv). if $Z\in AA\{1^k\}$ and $Y\in A\{4^k\}$ then $X\in A\{4^k\}$.

Proof:

Since if Y, $Z\in A_r\{1^k\}$,

Wkt,$A^kYA=A^k$ and $A^kZA=A^k$. $A^kXA=A^k(YAZ)A=(A^kYA)ZA=A^kZA=A^k$.

Hence $X\in A_r\{1^k\}$

Since $Y,Z\in AA\{1^k\}$.

$\Rightarrow AYA^k=A^k$ and $AZA^k=A^k$. $AXA^k = A(YAZ)A^k=AY(AZA^k)ZA=AYA^k=A^k$.

Hence $X\in AA\{1^k\}$,

Since $Y\in Ar\{1^k\}$,

$A^kYA=A^k$.

Since $Z\in A\{3^k\}$.

$(A^kZ)^T=A^kZ.(A^kZ)^T = (A^kY AZ)^T = (A^kZ)^T = A^kZ = A^kY AZ = A^kX$.

Hence $X\in A\{3^k\}$.

Since $Z\in AA\{1^k\}$,

$AZA^k = A^k$. Since $Y\in A\{3^k\}$.

$(YA^k)^T=YA^k.(XA^k)^T=(YAZA^k)^T=(YA^k)^T=YA^k=YAZA^k=XA^k$.

Hence $X\in A\{4^k\}$.

Theorem 4.20

For $A\in(SF)_n$ and for any $G^-\in(SF)_n$, if $A^kX=A^kG^-$, where X is a $\{1^K,3^K\}$ inverse of A then, G^- is a $\{1^K,3^K\}$ inverse of A.

Proof:

Since X is a $\{1^K,3^K\}$

$\Rightarrow A^kXA=A^k$ and $(A^kX)^T=A^kX$.

The Post is multiplied by A on both sides of $A^kX=A^kG^-$, $A^kG^-A=A^kXA=A^k$.

$(A^kG^-)^T=(A^kX)^T=A^kX=A^kG^-$.

Hence G^- is a $\{1^K,3^K\}$inverse of A.

Theorem 4.21

For $A\in(SF)_n$ and for any $G^-\in(SF)_n$, if $A^kX=A^kG^-$, where X is a $\{1^K,4^K\}$ inverse of A then, G^- is a $\{1^K,4^K\}$ inverse of A.

Proof:

This can be Proved as that of Theorem (4.20) and hence omitted.

Theorem 4.22

For $A\in(SF)_n$, X is a $\{1^K,3^K\}$ inverse of A and G^- is a $\{1^K,3^K\}$ inverse of A then, $A^kX=A^kG^-$.

Proof:

Since X is a $\{1^K,3^K\}$ inverse of A

$\Rightarrow A^kXA=A^k$ and $(A^kX)^T=A^kX$.

G^- is a $\{1^K,3^K\}$ inverse of A,

$\Rightarrow AG^-A^k=A^k$ and $(AG^-)^T=AG^-$.

$A^kG^-=(A^kXA)G^-= (A^kX)^T(AG^-)=(A^kX)^T(AG^-)^T=X^T(A^T)^k(G^-)^TA^T$

$= X^T(AG^-A^k)^T=X^T(A^k)^T=(A^kX)^T=A^kX$.

Hence the Theorem.

Theorem 4.23

For $A\in(SF)_n$, if A^TA is a right k-regular super fuzzy matrix and $R(A^k)\subseteq R(A^TA)^k$ then A has a $\{1^K,3^K\}$ inverse. In particular for k=1, $U=(A^TA)^-A^T$ is a {1, 3} inverse of A.

Proof:

Since A^TA is right k-regular fuzzy matrix, $(A^TA)^k(A^TA)^-(A^TA) = (A^TA)^k$ for some right k inverse $(A^TA)^-$ of A^TA.

Since $R(A^k)\subseteq R((A^TA)^k)$,

$A^k=X(A^TA)^k$ for some $X\in(SF)_n$ and take $U=(A^TA)^-A^T$

$A^kUA=(A^k)(UA)=(X(A^TA)^k)((A^TA)^-A^TA)$

$=X((A^TA)^k(A^TA)^-(A^TA))$

$= X(A^TA)^k=(A^k)$.

Take $V=(A^TA)^-(A^k)^T$.

$A^kV=(A^k)V$

$=(X(A^TA)^k)((A^TA)^-(A^k)^T)$

$= X(A^TA)^k(A^TA)^-(A^TA)^kA^T$

$=X(A^TA)^k(A^TA)^-(A^TA)(A^TA)^{k-1}X^T$

$= X(A^TA)^k(A^TA)^{k-1}X^T$

$=X(A^TA)^{2k-1}X^T=(X(A^TA)^{2k-1}X^T)^T = (A^kV)^T$.

Hence A has a $\{1^K,3^K\}$ inverse.

In particular for k=1, $Y= (A^TA)^-A^T$ is a {1, 3} inverse of A.

Theorem 4.24

Let $A\in(SF)_n$ be a right k-regular super fuzzy matrix and $R(A^TA)^k\subseteq R(A^k)$ then A^TA has a $\{3^K\}$ inverse.

Proof:

Since A is right k-regular matrix, $A^kXA=A^k$ for some right k inverse $X\in(SF)_n$ of A.

Since $R((A^TA)^k)\subseteq R(A^k)$, $(A^TA)^k=ZA^k$ for some $Z\in(SF)_n$ and take $Y=XA$.

$(A^TA)^kY=(ZA^k)(XA)=Z(A^kXA)=ZA^k=(A^TA)^k= ((A^TA)^k)^T$

$=((A^TA)^kY)^T$

Hence A^TA has a $\{3^K\}$ inverse.

Theorem 4.25

Let $A\in(SF)_n$ be a left k-regular super fuzzy matrix and $C(AA^T)^k\subseteq C(A^k)$ then AA^T has a $\{4^K\}$ inverse.

Proof:

This can be Proved as that of Theorem (4.24) and hence omitted.

The above all the theorem is satisfied and give performed better for the other related fuzzy matrices. Some of the methods are satisfied some conditions (regular & inverse) but our proposed k regular super fuzzy matrix satisfied the all the conditions (regular & inverse).

V. CONCLUSIONS

In this paper newly proposed and derived various theorems based on the inverse of k regular super fuzzy matrix in diverse aspects. The original machine with matrix coefficient A is replaced with the two of n × m crisp matrix equation systems. So it is a conquer FM trouble. To further verify the relation between every regular, k-regular and regularity of powers of fuzzy matrices. Our proposed method satisfies the all the situations of inverse of k regular super fuzzy matrix. An appropriate theorem is likewise provided. In the next paper we strive to show some associated properties and application in computer vision.

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