International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-5, March 2020

A Two Phase Mathematical Model of Fluid Flow through Bell Shaped Stenotic Artery

Madan Lal, Yantish Dev Jha, Haris Alam Zuberi

***Abstract:** The present research paper concerns with a two phase fluid flow, consists an acentric plasma layer region free from red cells and a central core region represented by Hershel – Bulkley fluid through a bell shaped stenosed artery. Mathematical expressions for characteristics of blood flow namely core velocity (u_c), peripheral velocity (u_p), shear stress at wall (τ_w) and total volumetric fluid flow rate (Q) have been estimated and depicted graphically . The effect of shape parameter peripheral layer viscosity, on these characteristics has been depicted with graphs. It has been noticed that the fluid flow rate (Q) and shear stress at wall (τ_w) decreases as the increases of peripheral layer viscosity.*

***Keywords:** Blood flow, bell shaped stenosis, flow rate , plasma layer, shear stress at wall*.

I. INTRODUCTION

World Health Organisation report manifested that approximately 17 million people are dying yearly due to suffer of cardiovascular related diseases. Also it is predicted that 23.3 million deaths yearly will happen due to cardiovascular related diseases throughout the world by 2023 and it will be a most focusing subject of scientific research (World Health Organisation, 2013)[10]. Some reasons of the related deaths are : first is coronary heart diseases which are associated with the blood vessels and blood circulation to the heart muscles. Second is cerebrovascular diseases associated with affecting of blood vessels and blood circulation to the brain. Brain stroke and heart attack happens when circulation of blood disturbed to the brain and heart respectively. Third one is peripheral arterial diseases which affect blood circulation in arms and legs. The exact factor of construction of atherosclerosis in blood vessels is not confirmed. It is taken into account that the deposition of fatty substances and cholesterol etc. at interior wall of artery are main factors of stenosis in an artery which is major cause of heart disease like heart attack and stroke. Construction of atherosclerosis in the blood flow tubes , flow characteristics of fluid are disturbed. Several researchers prepared a number of models to explore the different characteristics of blood flow in constricted artery including Smith, Pullan, & Hunter [9] and Shukla, Parihar & Gupta[7]. A mathematical model has been prepared by Ponalagusamy [3] in which core velocity has been expressed theoretically and experimentally slip variable and variable plasma layer considering Newtonian blood. Sharma & Yadav [5] studied the influence of permeability, thickness of plasma layer, yield stress and constriction shape on fluid flow characteristics. Shukla *et al.* [6]1980 Singh , Joshi & Joshi, 2014 [8] and Joshi & Gadkari [1] taken stenosis shaped as cosine, trapezium and triangular respectively and investigated plasma layer viscosity effect on flow characteristics. Sankar & Gunakala [4] have studied the fluid flow through small composite constricted vessels using a two – layered model applying uniform transverse external magnetic field and found that flux of blood and velocity both reduced with increasing of intensity of applied magnetic field . Recently Neeraja *et al*. [2] investigated the peripheral layer viscosity effect on fluid flow characteristics, through the stenotic blood vessels considering fluid as Herschel-Bulkley fluid for cosine shaped geometry.

Present model consist two layer of fluid namely red blood cells layer in core region and plasma layer fluid as Hershel- Bulkley fluid. Mathematical expressions are established for core and peripheral velocity (u_c and u_p), flow rate (Q) and shear stress at wall (τ_w). The influence of yield stress, shape parameter of constriction and plasma layer viscosity on velocity, shear stress at wall and fluid flow rate is represented as graphically and discussed briefly.

II. MATHEMATICAL FORMULATION

Let the blood flow be steady, laminar and incompressible in z - direction. Figure (1) shows geometry of stenosis.

Mathematically geometry of stenosis can be represented as

$$R(z) = R_0\left[1 - \frac{\delta}{R_0}\exp\left(\frac{-m^2\epsilon^2 z^2}{R_0^2}\right)\right]. \quad (1)$$

where

R_0 and R(z) represent the radius of non-constricted and constricted portion of artery respectively, δ is maximum depth of stenosis with parametric constant m and $\epsilon = \frac{R_0}{L_0}$ is relative half length of stenosed part of artery.

$$\frac{R(z)}{R_0} = [1 - a\exp(-bz^2)]. \quad (2)$$

Revised Manuscript Received on March 30, 2020.

* Correspondence Author

Madan Lal*, Department of Applied Mathematics, M. J. P. Rohilkhand University, Bareilly, India. Email:madan.mjpru@gmail.com

Yantish Dev Jha*, Department of Applied Mathematics, M. J. P. Rohilkhand University, Bareilly, India. Email: yantishdev.jha@gmail.com

Haris Alam Zuberi, Department of Applied Mathematics, M. J. P. Rohilkhand University, Bareilly, India. Email: haris.mjpru@gmail.com

Where $a = \frac{\delta}{R_0}$, $b = \frac{m^2 \epsilon^2}{R_0^2}$.

The viscosity functions are given as

μ_C is central layer viscosity and μ_p peripheral layer viscosity.

$$\mu = \mu_c \qquad 0 \le r \le R_1(z)$$ (3)

$$\mu = \mu_p \quad R_1(z) \le r \le R(z).$$

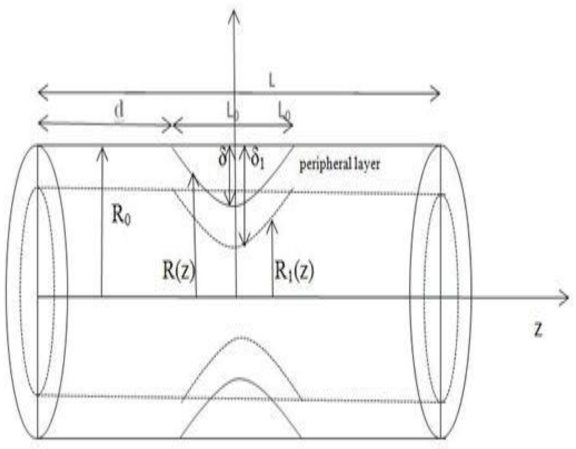

Figure 1. Bell shaped stenosed artery.

μ_C is central layer viscosity and μ_p peripheral layer viscosity.

The central layer geometry can be taken as

$$R_1(z) = R_0\left[1 - \frac{\delta_1}{R_0}\exp\left(\frac{-m^2\epsilon^2 z^2}{R_0^2}\right)\right]. \qquad (4)$$

The governing equation of blood flow in the arterial segment is given by

$$-\frac{dp}{dz} - \frac{1}{r}\frac{d(r\tau)}{dr} = 0$$

or $-\frac{dp}{dz} = \frac{1}{r}\frac{d(r\tau)}{dr}$ (5)

where r and z be the radial and axial coordinates respectively, p represent pressure and τ be the shear stress. Consider the blood as a Hershel Bulkley type fluid, for which for which the constitutive equations for shear stress and shear rate is

$$\tau = \mu^{\frac{1}{n}}\left(-\frac{du}{dr}\right)^{\frac{1}{n}} + \tau_0 \qquad \tau \ge \tau_0. \qquad (6)$$

$$\left(-\frac{du}{dr}\right) = 0, \qquad \tau < \tau_0. \qquad (7)$$

Above equations may be written as

$$-\frac{du}{dr} = \frac{1}{\mu}(\tau - \tau_0)^n \qquad \tau \ge \tau_0.$$

(8)

$$\left(-\frac{du}{dr}\right) = 0, \qquad \tau < \tau_0. \qquad (9)$$

And boundary conditions are

(1) $u_c = 0$ at $r = R_1(z)$, core velocity (u_c). (10)

(2) $u_p = 0$ at $r = R(z)$, peripheral velocity (u_p) (11)

$\tau_p = \tau_c$ at $r = R_1(z)$, $\frac{\partial u_c}{\partial r} = 0$ at r = 0.

where u_c is core velocity of fluid and u_p is peripheral velocity of fluid.

III. SOLUTION OF THE PROBLEM

On integrating equation (5)

$$\frac{dp}{dz} = -\frac{1}{r}\frac{d(r\tau)}{dr}.$$

$$\tau = -\frac{1}{2} r \frac{dp}{dz} \qquad (12)$$

$$-\frac{du}{dr} = \frac{1}{\mu}(\tau - \tau_0)^n$$

Using equation (12) in above equation

$$-\frac{du}{dr} = \frac{1}{\mu}\left(\frac{rP}{2} - \tau_0\right)^n \qquad (13)$$

On integrating and using boundary conditions (10) and (11)

$$u_c = \frac{1}{\mu_c(n+1)}\left(\frac{2}{P}\right)\left[\left(\frac{RP}{2} - \tau_0\right)^{n+1} - \left(\frac{rP}{2} - \tau_0\right)^{n+1}\right] \qquad (14)$$

$$u_p = \frac{1}{\mu_p(n+1)}\left(\frac{2}{P}\right)\left[\left(\frac{RP}{2} - \tau_0\right)^{n+1} - \left(\frac{R_1 P}{2} - \tau_0\right)^{n+1}\right] \qquad (15)$$

$$u_c = \frac{1}{\mu_c(n+1)}\tau_R R\left[(1-\alpha)^{n+1} - \left(\frac{\tau_P}{\tau_R} - \alpha\right)^{n+1}\right] \qquad (16)$$

$$u_p = \frac{1}{\mu_p(n+1)}\tau_R R\left[(1-\alpha)^{n+1} - \left(\frac{r}{R} - \alpha\right)^{n+1}\right] \qquad (17)$$

where $\alpha = \frac{\tau_0}{\tau_R}$.

Flow rate Q is given by

$$Q = \int_0^R u.\,2\pi r dr.$$

$$Q = \int_0^{R_1} u_c.\,2\pi r dr + \int_{R_1}^{R} u_p.\,2\pi r dr \qquad (18)$$

Using equation (16) and (17) we obtained

$$Q = \frac{2\pi\tau_R R}{(n+1)}\left\{\left(\frac{1}{u_c} - \frac{1}{u_p}\right)\frac{R_1^2}{2} + \frac{R^2}{2u_p} - \frac{R(1-\alpha)}{(n+2)} - \frac{(1-\alpha)^2}{(n+2)(n+3)}\right\}(1-\alpha)^{n+1}$$

(19)

Also

$$\tau_R = \frac{Q(n+1)}{2\pi R\left\{\left(\frac{1}{u_c} - \frac{1}{u_p}\right)\frac{R_1^2}{2} + \frac{R^2}{2u_p} - \frac{R(1-\alpha)}{(n+2)} - \frac{(1-\alpha)^2}{(n+2)(n+3)}\right\}(1-\alpha)^{n+1}}$$

(20)

$$\tau_N = \frac{Q(n+1)}{2\pi R_0\left\{\left(\frac{1}{u_c} - \frac{1}{u_p}\right)\frac{R_1^2}{2} + \frac{R_0^2}{2u_p} - \frac{R(1-\alpha)}{(n+2)} - \frac{(1-\alpha)^2}{(n+2)(n+3)}\right\}(1-\alpha)^{n+1}}$$

(21)

Shear stress at wall is given as

$$\tau_w = \frac{\tau_R}{\tau_N}$$

$$\tau_w = \frac{R_0}{R}\,\frac{\left\{\left(\frac{1}{u_c} - \frac{1}{u_p}\right)\frac{R_1^2}{2} + \frac{R_0^2}{2u_p} - \frac{R(1-\alpha)}{(n+2)} - \frac{(1-\alpha)^2}{(n+2)(n+3)}\right\}}{\left\{\left(\frac{1}{u_c} - \frac{1}{u_p}\right)\frac{R_1^2}{2} + \frac{R^2}{2u_p} - \frac{R(1-\alpha)}{(n+2)} - \frac{(1-\alpha)^2}{(n+2)(n+3)}\right\}} \qquad (22)$$

IV. RESULT AND DISCUSSION

In this paper a two layer model of fluid flow through a bell shaped constricted artery has been developed. Two layers exist in blood flow in artery. It is considered that when blood flows in artery there exist two layers namely peripheral plasma layer and central core layer. Core layer surrounded by peripheral layer. Both layers consist distinct viscosities.

The flow characteristics of blood flow are explained by various analytical expressions and depicted graphically as shown below. The values of parameters are taken as τ_0 = 0.05,0.10,0.15. δ = 0.10,0.15,0.20,0.25, 0.30,0.35 and μ_p = $\mu_c/10$, $\mu_c/5$, $\mu_c/13$.

Fig.2. and fig. 3. shows the variation of core velocity and peripheral velocity for different values of τ_0. It is found that core velocity of the blood and peripheral velocity of the blood both decreases upto the zero value of z and then increase. Both velocity decrease with increasing of τ_0.

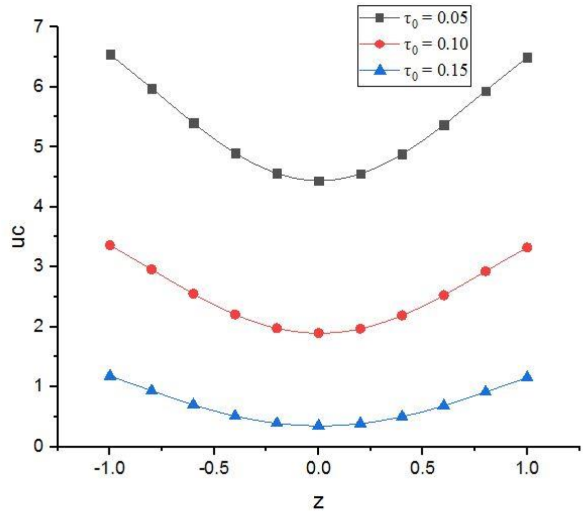

Fig. 2. Core velocity profile for τ_0 = 0.05,0.10,0.15.

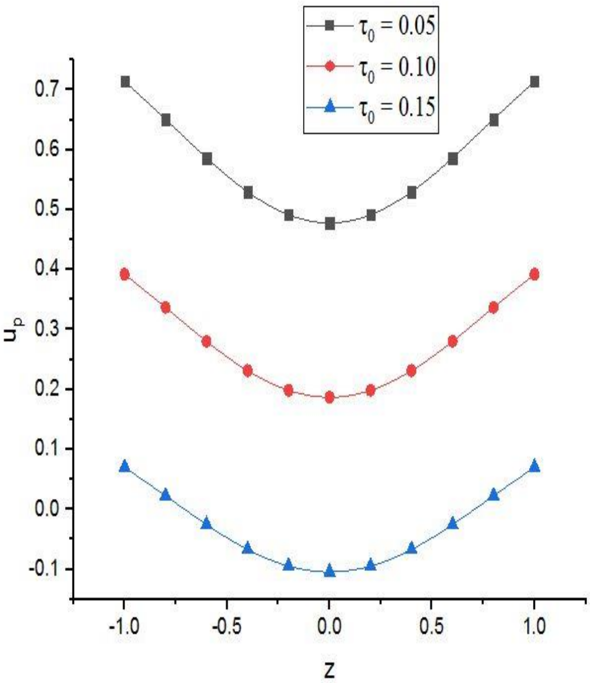

Fig. 3. Peripheral velocity profile for τ_0 = 0.05,0.10,0.15.

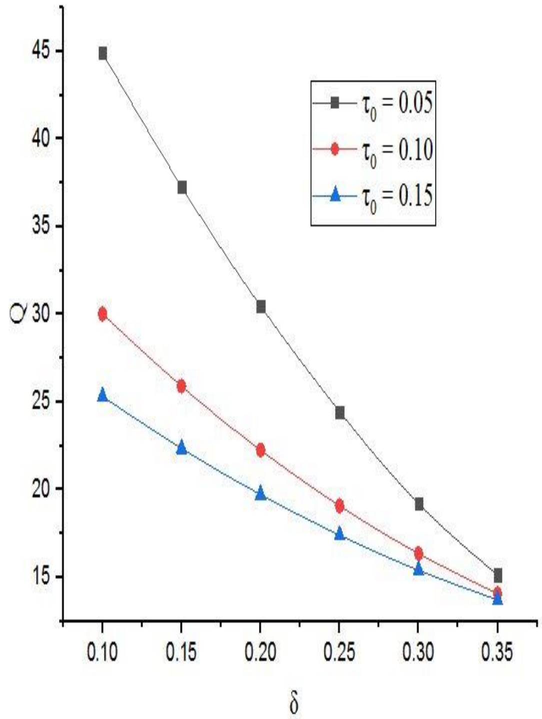

Fig. 4. Flow rate with stenosis size for τ_0 = 0.05,0.10,0.15.

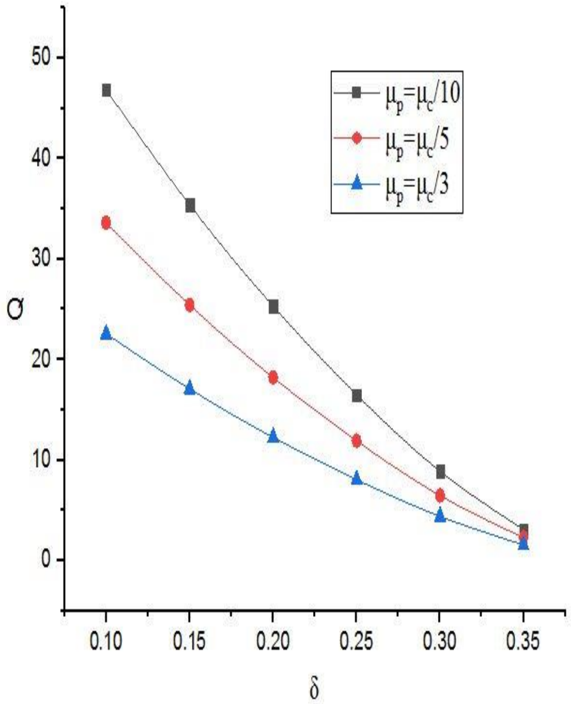

Fig. 5.Flow rate with stenosis size for μ_p = $\mu_c/10$, $\mu_c/5$, $\mu_c/13$.

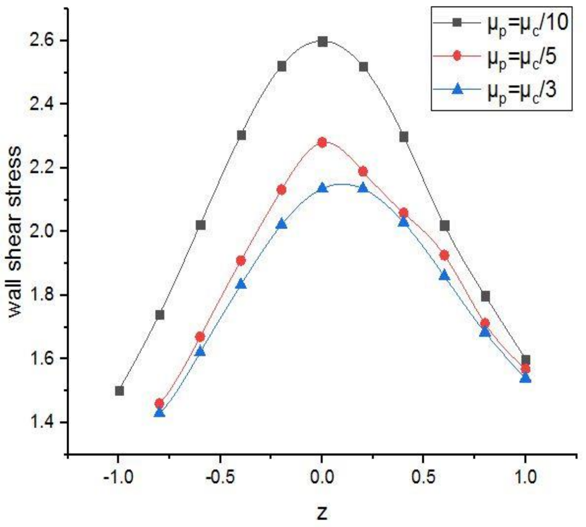

Fig. 6. Shear stress at wall for $\mu_p = \mu_c/10, \mu_c/5, \mu_c/13$.

The flow rate Q of fluid is obtained by equation (19). Fig. 4. and fig. 5. describe the changes in flow rate of fluid against height of stenosis for distinct value of yield stress (τ_0) and viscosity ratio (μ_p) respectively. It is noticed that Q decreases rapidly as shape parameter increases. This is due to decrease of cross sectional area in the stenosed arterial segment. Also the flow flux rate (Q) decaying with increasing value of yield stress (τ_0). In case of viscosity ratio same changes has seen as displayed in figure 5.

Fig. 6. explicit the effect of shear stress at wall corresponding to axial distance (z) for distinct viscosity ratios. The variation in shear stress at wall against axial distance (z) can be seen at any point in arterial segment from the graph. It is clear that shear stress (τ_w) at wall of fluid increases as the axial distance increase (from z = -1 to z = 0) and after that it reduces gradually from (from z = 0 to z = 1). The reason of this variation is intensity of the stenosis which significantly affects the shear stress at wall of blood flow. The maximum shear stress occur at the center at equal to (z = 0) of the stenosis.

V. CONCLUSION

An effort has been made to examine various aspects of fluid flow via a bell shaped stenosed artery. The fluid velocity (core velocity u_c and plasma layer velocity u_p), flow rate (Q) and shear stress at wall (τ_w) are examine graphically. From the above discussions important outcomes are summarized as

1. The core velocity and peripheral velocity decreases as the yield stress increases.
2. Flow rate Q reduced as value of yield stress τ_0 and viscosity ratio increased.
3. Shear stress at wall (τ_w) decreases when viscosity ratio increases.

Therefore the mathematical expressions and above discussion may useful to explain the flow of blood in stenosed artery.

REFERENCES

1. Joshi, P., & Garima, G., (2015). Flow of blood through stenosed artery, A Peripheral layer model. Advances in Applied Science Research, 6(2), 55-64.
2. Neeraja,G. , Dinesh, P.A. , Vidya, K. & Raju C.S.K. (2017). Peripheral layer viscosity on the stenotic blood vessels for Herschel-Bulkley fluid model", Informatics and Medicine Unlocked, 9, 161-165.
3. Ponalagusamy, R. (2007). Blood flow through an artery with mild stenosis, A twoLayered model, different shapes of stenosis and slip velocity at the wall. Journal of Applied Sciences, 7(7), 1071-1077.
4. Sankar, A.R., Gunakala,S.R., & Donna, Comissiong, D.M.G., (2013). Two layered blood Flow through a composite stenosis in the presence of a magnetic field. International Journal of Application or Innovation in Engineering & Management, vol. 6(7), 30-41.
5. Sharma, B. D. & Yadav, P. K.,(2017). A Two-Layer Mathematical Model of Blood Flow in Porous Constricted Blood Vessels. Transport in Porous Media 239–254.
6. Shukla, J. B., Parihar, R. S. and & Gupta S. P., (1980). Effect of peripheral layer viscosity on blood flow through the artery with mild stenosis. Bull. Math. Biol, 42, 797-805.
7. Shukla, J. B., Parihar, R. S., Rao, B. R. P. & Gupta, S.P. (1980). Effect of peripheral layer viscosity on peristaltic transport of a bio-fluid. J. Fluid Mech., 97, 225-235.
8. Singh, B., Joshi, P. & Joshi, B.K., (2014). Analysis of blood flow through an artery With Mild Stenosis, A Two- Layered model. Italian Journal of Pure and Applied Mathematics, 32, 35-44.
9. Smith, N. P., Pullan, A. J. & Hunter, P. J. (2002). An anatomically based model of transient coronary blood flow in the heart. SIAM J. Applied Mathematics, 62, 990-1018.
10. World Health Organisation (2013). Cardiovascular diseases (CVDs. [Online].

AUTHORS PROFILE

Dr. Madan Lal has completed M.Sc. degree in Mathematics in the year 1997 from M.J.P.Rohilkhand University, Bareilly (U.P.) India. and cleared NET (CSIR JRF) examination in December 2001. Ph.D. degree was awarded to him in 2003 (Mathematics) on the topic "Some Problems on Hydrodynamics and MHD flows" and his more than 20 research papers on "Magnetohydrodynamics" has been published in reputed National/International journals . He has participated more than 14 National /International conferences /workshops/seminars etc. Dr. Lal also chaired the session in various National and International conferences. He is also the life member of Indian Science Congress Association, Indian Mathematical Society etc. Currently he is working as Head of Applied Mathematics Department, M.J.P.Rohilkhand University Bareilly (U.P.) India.

Yantish Dev Jha has completed M.Sc. degree in Applied Mathematics in the year 2007 from M.J.P. Rohilkhand University, Bareilly (U.P.) India and cleared NET examination in December 2012. Author has joined for Ph.D. degree in Mathematics on the topic "Effect of Magnetohydrodynamics on the Blood Flow in Artery" in the year 2014 and four research papers has been published in National /International journals. He has also participated National/ International conferences/ Workshops/ Seminars. He is the life member of Indian Science Congress Association and Indian Mathematical Society

Haris Alam Zuberi has completed his M.Sc. degree in Mathematics in the year 2015 from Bareilly College affiliated to M. J. P. Rohilkhand University, Bareilly (U.P.) India. Author is a research scholar in the Department of Applied Mathematics, M. J. P. Rohilkhand University, Bareilly since November, 2018. His research topic for the award of Ph.D. degree is "Magneto-Blood-Dynamics". He has participated in many National/International Workshops, Conferences, Faculty Development Programmes. He is also the life member of Indian Science Congress Association and Indian Mathematical Society.