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Fuzzy Parameterized Hesitant Fuzzy Linguistic Term Soft Sets (FPHFLTSSs) in Multi-Criteria Decision Making

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Abstract: ***In this research, we suggest the theory of FPHFLTSSs. This theory is combination of hesitant fuzzy linguistic term soft sets (HFLTSSs) and weights of criteria in the set namely fuzzy parameterized where all the information are given in a single set. Then, we address several related concepts and the fundamental operations of this theory, namely the addition, union and intersection. In addition, we introduce the concept of scores in FPHFLTSSs based on arithmetic mean, geometry mean, and fractional of FPHFLTSSs. Next, we introduce the concept of distance between two FPHFLTSSs based on these scores which can be used in TOPSIS approach. Three algorithms are introduced to cater some problems in algorithm discovered in previous study. The first algorithm is an easy and simple step without transforming hesitant linguistic fuzzy elements into hesitant fuzzy elements. This second approach is the extended version of Liu et al. in which they suggested their positive ideal solution (PIS) distance algorithm. For the third algorithm, we suggest FPHFLTSS's arithmetic mean distances, geometry mean, and fractional distances in TOPSIS approach. Lastly, these algorithms are used for the issue of decision making in FPHFLTSSs environment to show the feasibility and efficiency of our methods.***

Keywords: ***A fuzzy parameterized, HFLTSSs, hesitant fuzzy linguistic term sets FPHFLTSSs, TOPSIS.***

I. INTRODUCTION

Decision-making challenges typically exist in different areas of society today, including supply chain management, forecasting, medicine, economics and others. The decision-making process does not always give a measured meaning to many realistic decision-making issues as priorities and human thinking are growing complex and uncertain. Therefore, it is easier to use a quantitative language type to convey findings. Zadeh's initial fuzzy sets (FSs) [1] has been a successful approximate approach to productive and quality language knowledge. However, decision makers may hesitate to choose a suitable fuzzy set in a complex decision-making setting to determine an alternative in some situations. Torra [2], [3] has suggested the idea of hesitant fuzzy sets (HFSs) to tackle these circumstances, using generalizations of FSs. Since then, several extensions have been made of this theory. The definition of hesitant fuzzy linguistic terms sets (HFLTSs) has been introduced and certain primary operations and properties of HFLTSs have been explored by Rodríguez et al. [4]. Since then, there are many applications of HFLTS in decision-making have been developed [5]–[13].

Meanwhile, About 20 years ago the soft set approach [14] was introduced and was considered an innovative way of addressing ambiguity and vagueness. The insufficiency of the parameterization tool within traditional concepts does not impact to this approach. Since the formulation of this hypothesis, several problems such as organizational research, forecasting, decision-making and medical science have been effectively solved. The hybrid concept that combines soft and other mathematical models developed very quickly and became one of the active areas in soft sets field. Maji et al. (2003) presented fuzzy soft set through joining a soft set with fuzzy sets. Several researchers have recently shown an immense interest in combining hesitant fuzzy sets and their extensions with the soft sets. The hesitant fuzzy soft sets were presented by Babitha and John [17] and Wang et.al [18]. While dual hesitant fuzzy soft sets were presented by Zhang and Shu [19] and He [20]. Hesitant intuitionistic fuzzy soft sets proposed by Nazra et al. [19] and hesitant linguistic term fuzzy soft sets were presented by Liu et al. [20]. The introduction of the fuzzy parameterized aspect is a significant milestone in the expansion of FSs and their generalizations. Cagman et al. [21], who suggested fuzzy parameterized fuzzy soft (FPFS) sets and their basic operations, first defined the fuzzy parameterized aspect. Then followed by others researchers [22]–[28].

The following are the characteristics of this work:

1. We broaden the LHFSSs definitions to the fuzzy parameters that can enhance this theory by giving weight to each parameter, namely FPHFLTSSs. We are researching certain relationships between two FPHFLTSSs and certain simple set operations for FPHFLTSSs based on the binary relationship. The properties of these operators are also discussed.
2. We define FPHFLTSS score based on arithmetic mean, geometry mean and fractional approach. Then based on these scores, we define the distance of any two FPHFLTSSs.

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3. We develop three algorithms to resolve the decision-making issue in FPHFLTSSs setting. The first algorithm is a simple and easy step without changing the linguistic hesitant fuzzy elements to hesitant fuzzy elements terms.
The second algorithm is based on the TOPSIS method, which has the shortest distance from the ideal positive ideal solution (PIS) as well as being farthest from the negative ideal solution. This approach is the extended version of Liu et al. [20] where they suggested their algorithm created on the distance to PIS. For the third algorithm, we use FPHFLTSSs arithmetic mean distances, geometry mean and fractional distances in TOPSIS approach.
4. In order to show the feasibility, superiority of our algorithm, and to compare results, an example of numerically modified from Liu et al. [20] in the "third-party evaluation" in Green Development is presented.
We structure our work as follows. Section 2 briefly reviews some basic definitions of HFLTS, HFLTSSs and fuzzy soft set and related notes. Section 3 will explain the FPHFLTSS and certain relationships, such as inclusion, equivalence and complementation, between any of the two FPHFLTSSs. We also create other basic FPHFLTSSs operations, including AND, OR, union and intersection operators. Apart from that, suitable properties are discussed for these operators. In Section 4, we suggest three scores of FPHFLTSSs and distance of any two FPHFLTSSs. Section 5 sets out three FPHFLTSS decision-making algorithms. Real-life example of the efficacy of our algorithm is given in Section 6. Eventually, Section 7 suggests some closing remarks and potential for further research.

II. PRELIMINARIES

This section offers a brief review of some related concepts to allow further discussion.
Definition 1. [2], [3] Given a fixed set X, then a hesitant fuzzy set (HFS) K on X is a term of a function that when applied to X returns a subset of $[0,1]$. The HFS can be expressed by mathematical symbol

$$K=\left(\langle x,h_E(x)\rangle \mid x\in X\right)$$

where $h_E(x)$ is a set of some value in $[0,1]$, denoting the possible membership degree of the element $x\in X$ for the set E. For convenience, call is a hesitant fuzzy element (HFE) and the set of all HFEs. The $h=h_E(x)$ is a hesitant fuzzy element (HFE) and H the set of all HFEs [29].

Definition 2. [30] Let U be an initial universe set and E be a universe set of parameters. A pair (F,E) is called a fuzzy soft set over U if $F:E\rightarrow F(U)$ where $F(U)$ is the set of all fuzzy subsets of U.

Definition 3. [31]. Assume that $x_i\in X\left(i=0,1,...,N\right)$ is fixed and $S=\{s_t \mid t=-n,...,0,1,2,...,n\}$ is linguistic term set. An HFLTS on X H_S is a mathematical form of $H_S=\left\{\langle x_i,h_s(x_i)\rangle \middle| x_i\in X\right\}$.

where $h_s(x_i)$ is a set of some values in S and can be regarded as $h_s(x_i)=\left\{s_{\phi_l} \middle| s_{\phi_l}\in S, l=1,2,...,L\right\}$, where L is a number of linguistic terms in $h_s(x_i)$. For convenience $h_s(x_i)$ and h_s are called HFLE and HFLEs.

Definition 4. [5]. Let $S=\{s_t \mid t=-n,...,0,1,2,...,n\}$ be a linguistic terms set, $\{h_s=s_{\tau l} \mid s_{\tau l}\in S, l=1,2,...,L;[-n,n]\}$ be an HFLE with L and $h_p=\{p_l \mid p_l=[0,1]; l=1,2,...,L\}$ be an HFS. Then, the transformation of the membership degree p_l and the subscript of τ_l of the linguistic term $s_{\tau l}$ to each other can be performed using functions g and g^{-1} respectively, given by

$$g:[-n,n]\rightarrow[0,1], g(\tau l)=\frac{\tau l}{2n}+\frac{1}{2}=p_l \quad (1)$$

$$g^{-1}:[0,1]\rightarrow[-n,n],\quad g(p_l)=s(2p_l-1)n=p_l \quad (2)$$

Definition 5. [20]. Let $S=\{s_t \mid t=-n,...,0,1,2,...,n\}$ be a linguistic terms set, $\{h_s^1=s_\tau^1 \mid s_\tau^1\in S, l=1,2,...,L^1;[-n,n]\}$ and $\{h_s^2=s_\tau^2 \mid s_\tau^2\in S, l=1,2,...,L^2;[-n,n]\}$ be two HFLEs with $L=L^1=L^2$ .Then, by adding the linguistic terms, the shorter one may be extended. While, g and g^{-1} are the corresponding functions of the HFEs and HFLEs transformation. Then, the distance between two HFLEs is then defined as

$$d(h_s^1,h_s^2)=\frac{\sum_{l=1}^{L}\left[\left|g(s_\tau^1)-g(s_\tau^2)\right|\right]}{L} \quad (3)$$

(assume that the linguistic terms are ascending).
Definition 6. [20]. Suppose all HFLTSs in U are set to $\vec{H}(U)$. An HFLTSS over U is $\vec{F}:A\rightarrow\vec{H}(U)$, with pair of $\left(\vec{F},A\right)$.

$\vec{F}(e)$ is the set of HFLTSS, $\left(\vec{F},A\right)$ where e-approximate elements used for any parameter $x\in A$, can be regarded as

$$\vec{F}(e)=\left\{\langle x,H_{\vec{F}(e)}(x)\rangle \middle| x\in U\right\}. \quad (4)$$

III. FUZZY PARAMETERIZED HESITANT FUZZY LINGUISTIC TERM SOFT SETS (FPHFLTSSS)

This section explains the extension of LHFSSs by combining the weightage for each parameter to LHFSSs namely fuzzy parameterized hesitant fuzzy linguistic term soft sets (FPHFLTSSs).
Definition 7. Let U be a universe set, E be the set of parameters, $S=\{s_t \mid t=-n,...,0,1,2,...,n\}$ is a linguistic term set, $\hat{f}_K$ is called fuzzy parameterized linguistic hesitant fuzzy soft sets (FPHFLTSSs) over U denoted by

$$\hat{f}_K = \left\{ \left(\frac{e_i}{\mu_K(e_i)}, h_S^K(e_i) \right) | \, e_i \in E, \mu_K(e_i) \in [0,1] \right\} \tag{5}$$

where $\mu_K(e_i)$ is a value of fuzzy parameterized set between 0 to 1, while $h_S^K(e_i)$ is a set of certain values in S and can be considered as $\{h_S^K(e_i) = s_l^K(e_i) | \; s_l^K(e_i) \in S, l = 1, 2, ..., L\}$ where L is a number of linguistic term in $h_S^K(e_i)$. For convenience, $h_S^K(e_i)$ is called linguistic hesitant fuzzy elements (LHFE).

Example 1. Let U be set of three shortlisted houses to be bought which is denoted by $U = \{r_1, r_2, r_3\}$. Let $E = \{m_1, m_2, m_3\}$ be a set of parameters where m_1 is the price, m_2 is the location and m_3 is the size. Suppose the weight for each parameter is given in fuzzy crisp sets as $K = \left\{ \frac{e_1}{0.4}, \frac{e_2}{0.5}, \frac{e_3}{0.7} \right\}$. The rating of alternatives is given in HFLTSSs as shown below

$$\vec{f}(e_1) = \left\{ \frac{r_1}{\{s_1, s_2, s_3\}}, \frac{r_2}{\{s_{-2}, s_{-1}\}}, \frac{r_3}{\{s_0, s_1, s_2\}} \right\}$$
$$\vec{f}(e_2) = \left\{ \frac{r_1}{\{s_{-1}, s_0, s_1\}}, \frac{r_2}{\{s_{-2}, s_{-1}, s_0\}}, \frac{r_3}{\{s_2, s_3\}} \right\}.$$
$$\vec{f}(e_3) = \left\{ \frac{r_1}{\{s_2, s_3\}}, \frac{r_2}{\{s_1, s_2\}}, \frac{r_3}{\{s_{-2}, s_{-1}, s_0\}} \right\}$$

Then the FPHFLTSS is given by

$$\hat{f}_K = \left\{ \left(\frac{e_1}{0.4}, \frac{r_1}{\{s_1, s_2, s_3\}}, \frac{r_2}{\{s_{-2}, s_{-1}\}}, \frac{r_3}{\{s_0, s_1, s_2\}} \right), \left(\frac{e_2}{0.5}, \frac{r_1}{\{s_{-1}, s_0, s_1\}}, \frac{r_2}{\{s_{-2}, s_{-1}, s_0\}}, \frac{r_3}{\{s_2, s_3\}} \right), \left(\frac{e_3}{0.7}, \frac{r_1}{\{s_2, s_3\}}, \frac{r_2}{\{s_1, s_2\}}, \frac{r_3}{\{s_{-2}, s_{-1}, s_0\}} \right) \right\}$$

Definition 8. Let $\hat{f}_K$ be FPHFLTSS over U. Then $\hat{f}_K$ is called an empty FPHFLTSS set if it is denoted and defined as

$$\hat{f}_\phi = \left\{ \left(\frac{e_i}{0}, \{\tilde{0}\} \right) : e_i \in E, u_j \in U \right\}$$

Definition 9. Let $\hat{f}_K$ be FPHFLTSS over U. Then $\hat{f}_K$ is called a universal FPHFLTSS set if it is denoted and defined as

$$\hat{f}_\tau = \left\{ \left(\frac{e_i}{1}, \{\tilde{1}\} \right) : e_i \in E \right\}$$

Proposition 1. Let $\hat{f}_K$, $\hat{f}_K$ and $\hat{f}_K$ be any three FPHFLTSSs. Then the following results hold:

1. $\hat{f}_K \subseteq \hat{f}_\iota$,
2. $\hat{f}_\phi \subseteq \hat{f}_\tau$,
3. $\hat{f}_K \subseteq \hat{f}_K$
4. $\hat{f}_K \subseteq \hat{f}_L$ and $\hat{f}_L \subseteq \hat{f}_M$ then $\hat{f}_K \subseteq \hat{f}_M$,
5. $\hat{f}_K = \hat{f}_L$ and $\hat{f}_L = \hat{f}_M$ then $\hat{f}_K = \hat{f}_M$,
6. $\hat{f}_K \subseteq \hat{f}_L$ and $\hat{f}_L \subseteq \hat{f}_K$ then $\hat{f}_K = \hat{f}_L$.

Proof: The proof is straightforward**.**

Definition 10. Let Ψ_K is FPHFLTSS, then the complement of $\hat{f}_K = \left\{ \left(\frac{e_i}{\mu_K(e_i)}, h_S^K(e_i) \right) | \, e_i \in E, \mu_K(e_i) \in [0,1] \right\}$ is denoted by $\hat{f}_K^c = \left\{ \left(\frac{e_i}{\mu_K(e_i)}, h_S^K(e_i) \right) | \, e_i \in E, \mu_K(e_i) \in [0,1] \right\}^c$ defined by

$$\hat{f}_K = \left\{ \left(\frac{e_i}{(1 - \mu_K(e_i))}, -h_S^K(e_i) \right) | \, e_i \in E, \mu_K(e_i) \in [0,1] \right\}.$$

Example 2. Reconsider Example 1. The complement of $\hat{f}_K$ is given as

$$\hat{f}_K^c = \left\{ \left(\frac{e_1}{0.6}, \frac{r_1}{\{s_{-3}, s_{-2}, s_{-1}\}}, \frac{r_2}{\{s_1, s_2, s_2\}}, \frac{r_3}{\{s_0, s_1, s_2\}} \right), \left(\frac{e_2}{0.5}, \frac{r_1}{\{s_1, s_0, s_{-1}\}}, \frac{r_2}{\{s_0, s_{-1}, s_{-2}\}}, \frac{r_3}{\{s_{-3}, s_{-2}, s_{-2}\}} \right), \left(\frac{e_3}{0.3}, \frac{r_1}{\{s_{-3}, s_{-2}, s_{-2}\}}, \frac{r_2}{\{s_{-2}, s_{-1}, s_{-1}\}}, \frac{r_3}{\{s_0, s_1, s_2, s_2\}} \right) \right\}$$

Proposition 2. Let Ψ_K is FPHFLTSS(U). Then the following results hold:

1. $(\hat{f}_K^c)^c = \hat{f}_K$
2. $\hat{f}_\phi^c = \hat{f}_{\tilde{1}}$

Proof. The proofs are straightforward.

Definition 11. Let $\hat{f}_K$ and $\hat{f}_L$ be FPHFLTSSs over U. The intersection of $\hat{f}_K$ and $\hat{f}_L$, denoted by $\hat{f}_K \,\tilde{\cap}\, \hat{f}_L$ is defined by

$$\hat{f}_K \,\tilde{\cap}\, \hat{f}_L = \left\{ \left(\frac{e_i}{\min(\mu_K(e_i), \mu_L(e_i))}, \min\left(h_S^K(e_i), h_S^L(e_i) \right) \right) \right. \tag{8}$$
$$\left. | \, e_i \in E : \mu_K(e_i), \mu_L(e_i) \in [0,1] \right\}$$

Example 3. Let $\hat{f}_K$ is given in Example 1 and $\hat{f}_L$ is given below

$$\hat{f}_L = \left\{ \left(\frac{e_1}{0.6}, \frac{r_1}{\{s_2, s_3, s_3\}}, \frac{r_2}{\{s_{-3}, s_{-2}, s_{-2}\}}, \frac{r_3}{\{s_2, s_3, s_3\}} \right), \left(\frac{e_2}{0.7}, \frac{r_1}{\{s_{-3}, s_{-2}, s_{-1}\}}, \frac{r_2}{\{s_1, s_2, s_3\}}, \frac{r_3}{\{s_3, s_3, s_3\}} \right), \left(\frac{e_3}{0.2}, \frac{r_1}{\{s_1, s_2, s_3\}}, \frac{r_2}{\{s_1, s_2, s_3\}}, \frac{r_3}{\{s_{-3}, s_{-2}, s_{-1}\}} \right) \right\}$$

The intersection of $\hat{f}_K$ and $\hat{f}_L$ is given as

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$$\hat{f}_K \tilde{\cap} \hat{f}_L = \left\{ \left(\frac{e_1}{0.4}, \frac{r_1}{s_1, s_2, s_3}, \frac{r_2}{\{s_{-3}, s_{-2}, s_{-2}\}}, \frac{r_3}{\{s_0, s_1, s_2\}} \right), \left(\frac{e_2}{0.5}, \frac{r_1}{\{s_{-3}, s_{-2}, s_{-1}\}}, \frac{r_2}{\{s_{-2}, s_{-1}, s_0\}}, \frac{r_3}{\{s_2, s_3, s_3\}} \right), \left(\frac{e_3}{0.2}, \frac{r_1}{\{s_1, s_2, s_3\}}, \frac{r_2}{\{s_1, s_2, s_2\}}, \frac{r_3}{\{s_{-3}, s_{-2}, s_{-1}\}} \right) \right\}$$

Proposition 3. Let $\hat{f}_K$, $\hat{f}_L$ and $\hat{f}_M$ be any three FPHFLTSSs. Then the following results hold:

1. $\hat{f}_K \tilde{\cap} \hat{f}_K = \hat{f}_K$
2. $\hat{f}_\phi \tilde{\cap} \hat{f}_K = \hat{f}_\phi$
3. $\hat{f}_K \tilde{\cap} \hat{f}_\tau = \hat{f}_K$
4. $\hat{f}_K \tilde{\cap} \hat{f}_L = \hat{f}_L \tilde{\cap} \hat{f}_K$

Proof. The proofs are straightforward.

Definition 12. Let $\hat{f}_K$ and $\hat{f}_L$ be FPHFLTSSs over U. The union of $\hat{f}_K$ and $\hat{f}_L$, denoted by $\hat{f}_K \tilde{\cup} \hat{f}_L$ is defined as

$$\hat{f}_K \tilde{\cup} \hat{f}_L = \left\{ \left(\frac{e_i}{\max(\mu_K(e_i), \mu_L(e_i))}, \max\left(h_S^K(e_i), h_S^L(e_i)\right) \right) \quad (9)$$

$$| e_i \in E : \mu_K(e_i), \mu_L(e_i) \in [0,1] \}$$

Example 4. Let $\hat{f}_K$ and $\hat{f}_L$ are given in Example 3. The union of $\hat{f}_K$ and $\hat{f}_L$ is given as

$$\hat{f}_K \tilde{\cup} \hat{f}_L = \left\{ \left(\frac{e_1}{0.6}, \frac{r_1}{\{s_2, s_3, s_3\}}, \frac{r_2}{\{s_{-2}, s_{-1}, s_{-1}\}}, \frac{r_3}{\{s_2, s_3, s_3\}} \right), \left(\frac{e_2}{0.7}, \frac{r_1}{\{s_{-1}, s_0, s_1\}}, \frac{r_2}{\{s_1, s_2, s_2\}}, \frac{r_3}{\{s_3, s_3, s_3\}} \right), \left(\frac{e_3}{0.7}, \frac{r_1}{\{s_2, s_3, s_3\}}, \frac{r_2}{\{s_1, s_2, s_3\}}, \frac{r_3}{\{s_{-2}, s_{-1}, s_0\}} \right) \right\}$$

Proposition 4. Let Ψ_K and Ψ_L be any three FPHFLTSS. Then, the findings below hold:

1. $\hat{f}_K \tilde{\cup} \hat{f}_K = \hat{f}_K$
2. $\hat{f}_\phi \tilde{\cup} \hat{f}_K = \hat{f}_K$
3. $\hat{f}_K \tilde{\cup} \hat{f}_\tau = \hat{f}_\tau$
4. $\hat{f}_K \tilde{\cup} \hat{f}_L = \hat{f}_L \tilde{\cup} \hat{f}_K$

Proof. The proofs are straightforward.

IV. THE SCORES AND DISTANCE OF FPHFLTSSs

The score of FPHFLTSSs and distance between two FPHFLTSSs play important role for this research. A lot of distance measures have been proposed for HFS and dual HFSs. First, let's take a look at the distance property as follows:

Definition 13. Let $\hat{f}_K$ and $\hat{f}_L$ be FPHFLTSSs over U. Then distance measurement between $\hat{f}_K$ and $\hat{f}_L$ is defined as $d(\hat{f}_K, \hat{f}_L)$ which satisfies the following properties:

(P1) Boundary: $0 \le d(\hat{f}_K, \hat{f}_L) \le 1$

(P2) Symmetry: $d(\hat{f}_K, \hat{f}_L) = d(\hat{f}_L, \hat{f}_K)$

(P3) Reflexivity: $d(\hat{f}_K, \hat{f}_L) = 0$ if only if $\hat{f}_K = \hat{f}_L$.

Definition 14. Let $\hat{f}_K$ be FPHFLTSSs over U where $h = \{h_1, h_2, ..., h_n\}$ be an LTHFE. The following functions can be considered as the score functions of FPHFLTSSs;

a) The arithmetic means score function of FPHFLTSS.

$$S_{AM}(\hat{f}_K(e_i)) = \left(\mu_K(e_i) \frac{1}{l} \sum_{1=j}^{l} p_{\tau l} \right). \quad (10)$$

b) The geometric-mean score function of FPHFLTSS.

$$S_{GM}(\hat{f}_K(e_i)) = \left(\mu_K(e_i) \left(\prod_{1=j}^{l} p\tau_l \right)^{\frac{1}{l}} \right) \quad (11)$$

c) and the fractional score function of FPHFLTSS.

$$S_F(\hat{f}_K(e_i)) = \left(\mu_K(e_i) \frac{\prod_{1=j}^{l} p_{\tau l}}{\prod_{1=j}^{l} p_{\tau l} + \prod_{1=i}^{l} 1 - p_{\tau l}} \right) \quad (12)$$

Example 1.

$$\hat{f}_K = \left\{ \left(\frac{e_1}{0.4}, \frac{r_1}{\{s_1, s_2, s_3\}}, \frac{r_2}{\{s_{-2}, s_{-1}, s_{-1}\}}, \frac{r_3}{\{s_0, s_1, s_2\}} \right), \left(\frac{e_2}{0.5}, \frac{r_1}{\{s_{-1}, s_0, s_1\}}, \frac{r_2}{\{s_{-2}, s_{-1}, s_0\}}, \frac{r_3}{\{s_2, s_3, s_3\}} \right), \left(\frac{e_3}{0.7}, \frac{r_1}{\{s_2, s_3, s_3\}}, \frac{r_2}{\{s_1, s_2, s_2\}}, \frac{r_3}{\{s_{-2}, s_{-1}, s_0\}} \right) \right\}$$

$$S_{AM}(\hat{f}_K(e_1)) = \frac{r_1}{0.4\left((0.667+0.833+1)/3\right)}, \frac{r_2}{0.4\left((0.167+0.333+0.333)/3\right)}, \frac{r_3}{\left(0.4(0.5+0.667+0.8333)/3\right)} = \frac{r_1}{0.333}, \frac{r_2}{0.111}, \frac{r_3}{0.267}$$

$$S_{GM}(\hat{f}_K(e_1)) = \frac{r_1}{0.4\left((0.667\times0.833\times1)^{1/3}\right)}, \frac{r_2}{0.4\left((0.167\times0.333\times0.333)^{1/3}\right)}, \frac{r_3}{\left(0.4(0.5\times0.667\times0.8333)^{1/3}\right)} = \frac{r_1}{0.329}, \frac{r_2}{0.106}, \frac{r_3}{0.261}$$

$$S_{AM}(\hat{f}_K(e_1)) = \frac{r_1}{0.4\left(\frac{(0.667\times 0.833\times 1)}{(0.667\times 0.833\times 1)+((1-0.667)\times(1-0.833)\times(1-1))}\right)}, \frac{r_2}{0.4\left(\frac{(0.167\times 0.333\times 0.333)}{(0.167\times 0.333\times 0.333)+((1-0.167)\times(1-0.333)\times(1-0.333))}\right)}, \frac{r_3}{0.4\left((0.5+0.667+0.8333)+((1-0.5)\times(1-0.667)\times(1-0.8333))\right)}$$

$$= \frac{r_1}{0.400}, \frac{r_2}{0.019}, \frac{r_3}{0.364}$$

Definition 15. Let $\hat{f}_K$ and $\hat{f}_L$ be FPHFLTSSs over U. The distance measurement between $\hat{f}_K$ and $\hat{f}_L$ is given by these mathematical formulae

a) The FPHFLTSSs arithmetic mean distances

$$d_{AM}(\hat{f}_K, \hat{f}_L) = \left(\sum \left(S_{AM}(\hat{f}_K) - S_{AM}(\hat{f}_L)\right)^{\lambda}\right)^{\frac{1}{\lambda}}, \quad (13)$$

b) The FPHFLTSSs geometry mean distances

$$d_{GM}(\hat{f}_K, \hat{f}_L) = \left(\sum \left(S_{GM}(\hat{f}_K) - S_{GM}(\hat{f}_L)\right)^{\lambda}\right)^{\frac{1}{\lambda}} \quad (14)$$

c) The FPHFLTSS fractional distances

$$d_{F}(\hat{f}_K, \hat{f}_L) = \left(\sum \left(S_{F}(\hat{f}_K) - S_{F}(\hat{f}_L)\right)^{\lambda}\right)^{\frac{1}{\lambda}} \quad (15)$$

when $\lambda = 1$ it is also known as Hamming distance of FPHFLTSSs and when $\lambda = 2$ it is also known as Euclidean distance of FPHFLTSSs.

V. THE ALGORITHMS OF FPHFLTSSs

In this section, we provide three algorithms to solve the problems in FPHFLTSSs environments.

Algorithm 5.1

For the first algorithm, we give a simpler way to cater problem given by Liu et al. [20]. In their algorithm 4.1, they gave adjustable method to solve the example given. They choose the decision rule based on the top level. However, we found that this algorithm has a flaw where choices values obtained to be equal is high. As shown in their example, we can see that the choices values for U2 and U3 are the same. Next, the rankings for u2 and u3 are equal although the ratings according to the criteria are different. This becomes an issue if decision makers need to choose two of the best alternatives.

The first algorithm is shown as follows:

The algorithm of FPHFLTSSs

Step 1. *Built up the matrix of decision R.*

Step 2. *Transform to matrix of decision T as in* τ_l.

Step 3. *Find the scores of alternatives,* $m(A_i) = \sum \mu_A.\tau l_i$.

Step 4. *Rank each alternative according to the downward values.*

Algorithm 5.2.

The second proposed algorithm is the well-known method of TOPSIS in FPHFLTSS environment. The alternative should be the shortest distance from the positive ideal solution (PIS) as well as the most distant from the negative ideal solution (NIS) according to the TOPSIS principle. Liu [20] presented an algorithm based on TOPSIS approach on the basis of distance from the PIS. However, the ranking of alternatives that they offer is not the same when we apply in the TOPSIS concepts. We will show this in the numerical calculation. The second algorithm is displayed as:

The algorithm of FPHFLTSSs in TOPSIS

Step 1. *Build the matrix of decision R.*

Step 2. *Determine the PIS, and the NIS.*

Step 3. *Find the distance from the PIS and NIS.*

Step 4. *Find the relative value of closeness for every alternative* $RC(A_i) = \frac{d_i^-}{d_i^- + d_i^+}$.

Step 5. *Rank the alternatives by the value of RC. The best choice is the one with the largest RC.*

Algorithm 5.3.

The third proposed algorithm is also TOPSIS in FPHFLTSS environment. Usually, the length of the linguistic hesitant fuzzy element is different in practical application. The method suggested in [32] adds some linguistic hesitant fuzzy element to a shortening linguistic hesitant fuzzy element that is equal to another. Some researchers may agree that the original data structure is destroyed and the information is changed. Therefore, we provide the algorithm without adding any linguistic hesitant fuzzy element to the shorter one to ensure that it has the same length with other elements. In this algorithm, we used the FPHFLTSSs arithmetic mean distances, the FPHFLTSSs geometry mean distances and the FPHFLTSS fractional distances as mentioned in section 4.

The algorithm scores of FPHFLTSSs in TOPSIS

Step 1. *Build the matrix of decision R.*

Step 2. *Transform the matrix of the decision G.*

Step 3. *Set up the score matrix of FPHFLTSSs.*

Step 4. *Identify the PIS and NIS.*

Step 5. *Calculate a distance from PIS and NIS.*

Step 6. *Find the value of relative closeness for each alternative,* $RC(A_i) = \frac{d_i^-}{d_i^- + d_i^+}$.

Step 7. *The alternatives are ranked by RC value. The best choice is the one with the largest RC.*

VI. NUMERICAL EXAMPLES

Based on Liu et al. [20], China is looking at a new path towards sustainable green model of development.

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The green development model is very different from the traditional economic growth style. It encompasses the social, economic and environment structures and sets the path for Chinese development.

Evaluation processes are coordinated with numerous third-party organizations involved, such as the Environmental Protection Agency, the Committee for National Development and Reform and the local public opinion committee, with a view to providing a comprehensive and unbiased evaluation.

Now we examine four towns denoted by $T=\{T_1,T_2,T_3,T_4\}$ as alternatives. $C=\{c_1,c_2,c_3,c_4\}$ is a set of parameters whose parameters are "air quality," "drinking water quality," "environmental boosting" and "domestic waste processing" respectively.

Assume the linguistic term be defined as:

$$\bar{S}=\{s_{-3}=very\,bad, s_{-2}=bad, s_{-1}=somewhat\,bad,$$
$$s_0=fair, s_1=somewhat\,good, s_2=good, s_3=very\,good\}$$

However, we suggest that the weights of criteria are given directly by decision makers and not generated from the rating given by third-party stakeholder organizations. Suppose the weight of criteria is similar with the one given in their example. Then, the FPLHFSSs are given as below,

$$\Psi_A=\left\{\left(\frac{c_1}{0.28571},\frac{T_1}{\{s_{-0},s_1\}},\frac{T_2}{\{s_0,s_1\}},\frac{T_3}{\{s_0,s_1,s_2\}},\frac{T_4}{\{s_{-2},s_{-1}\}}\right),\right.$$
$$\left(\frac{c_2}{0.25},\frac{T_1}{\{s_0,s_1,s_2\}},\frac{T_2}{\{s_0\}},\frac{T_3}{\{s_{-1},s_0\}},\frac{T_4}{\{s_1,s_2\}}\right),$$
$$\left(\frac{c_3}{0.34524},\frac{T_1}{\{s_{-2},s_{-1}\}},\frac{T_2}{\{s_1,s_2\}},\frac{T_3}{\{s_{-1},s_0,s_1\}},\frac{T_4}{\{s_0,s_1\}}\right),$$
$$\left.\left(\frac{c_4}{0.11905},\frac{T_1}{\{s_{-1},s_0\}},\frac{T_2}{\{s_{-1},s_0,s_1\}},\frac{T_3}{\{s_0,s_1\}},\frac{T_4}{\{s_{-1},s_0\}}\right)\right\}$$

Algorithm 5.1.

Step 1. *The decision matrix of FPHFLTSSs, R is obtained as in Table 1.*

Table 1. Decision matrix of FPHFLTSSs

	$c_1(0.28571)$	$c_2(0.25)$	$c_3(0.34524)$	$c_4(0.11905)$
T_1	s_0, s_1	s_0, s_1, s_2	s_{-2}, s_{-1}	s_{-1}, s_0
T_2	s_{-1}, s_0	s_0	s_1, s_2	s_{-1}, s_0, s_1
T_3	s_0, s_1, s_2	s_{-1}, s_0	s_{-1}, s_0, s_1	s_0, s_1
T_4	s_{-2}, s_{-1}	s_1, s_2	s_0, s_1	s_{-1}, s_0

Step 2. *Transform to decision matrix T as we can see in Table 2.*

Table 2. The score decision matrix of FPHFLTSSs

	$c_1(0.28571)$	$c_2(0.25)$	$c_3(0.34524)$	$c_4(0.11905)$
T_1	0.5	1	-1.5	-0.5
T_2	-0.5	0	1.5	0
T_3	1	-0.5	0	0.5
T_4	1.5	1.5	0.5	0.5

Step 3. *Calculate the score,* $m(A_i)$. From here we can see that the result is $m(T_1)=-0.18453$, $m(T_2)=0.3750$ $m(T_3)=0.2202$ and $m(T_4)=0.1786$.

Step 4. *Rank the alternatives.*

$T_2 \succ T_3 \succ T_4 \succ T_1$. The best alternative is the city T_2.

Algorithm 5.2.

For the second algorithm, we give the TOPSIS approach for problem solving given by Liu et al.

Step 1. For this step, see Algorithm 5.1.

Step 2. Determine the PIS and NISs as seen in Table 3.

Table 3. PISs and NISs

	c_1	c_2	c_3	c_4
PIS	s_0, s_1, s_2	s_1, s_2, s_2	s_1, s_2, s_2	s_0, s_1, s_1
NIS	s_{-2}, s_{-1}, s_{-1}	s_{-1}, s_0, s_0	s_{-1}, s_0, s_1	s_{-1}, s_0, s_0

Step 3. The distance from the positive and NISs are given in Table 4.

Table 4. The distance from PISs and NISs

	d^+	d^-
T_1	0.30592	0.20998
T_2	0.18673	0.30885
T_3	0.23327	0.25212
T_4	0.23687	0.25717

Note : The shaded area is the result using algorithm by Liu et al. [20].

Step 4. *The relative closeness for* $\lambda=0.1,1,2,4,6,8,10$ *to the ideal solution is obtained as shown in Table 5.*

Table 5. The relative closeness

	0.1	**1**	**2**	**4**	**6**	**8**	**10**
T_1	0.00320	0.38974	0.40702	0.39784	0.39538	0.39534	0.39583
T_2	0.98201	0.62222	0.62320	0.63946	0.64516	0.64707	0.64760
T_3	0.89878	0.53675	0.51941	0.52331	0.53005	0.53439	0.53689
T_4	0.12464	0.51282	0.52055	0.50400	0.49163	0.48435	0.47981

Step 5. *As a result, we have the ranking for each alternative, where* $T_2 \succ T_3 \succ T_4 \succ T_1$ *except when* $\lambda = 2$. *It is bit different from the ranking obtained by Liu's.*

Algorithm 5.3.

Step 1. *Build the matrix of decision R as in Algorithm 5.1.*
Step 2. *Transform the decision matrix G as in Table 5.*

Table 5. Decision matrix G

	$c_1(0.28571)$	$c_2(0.25)$	$c_3(0.34524)$	$c_4(0.11905)$
T_1	0.5,0.6667	0.5,0.6667,0.8333	0.1667,0.3333	0.3333,0.5
T_2	0.3333,0.5	0.5	0.6667,0.8333	0.3333,0.5,0.6667
T_3	0.5,0.6667,0.8333	0.3333, 0.5	0.3333,0.5,0.6667	0.5,0.6667
T_4	0.1667,0.3333	0.6667,0.8333	0.5,0.6667	0.3333,0.5

Step 3. *Set up the score matrix of FPHFLTSSs. Tables 6, 7 and 8 are arithmetic mean, geometric mean and fractional score matrix of FPHFLTSSs respectively.*

Table 5. Arithmetic mean score matrix of FPHFLTSSs

	c_1	c_2	c_3	c_4
T_1	0.1667	0.1667	0.0959	0.0496
T_2	0.1190	0.1250	0.2589	0.0595
T_3	0.1905	0.1042	0.1726	0.0694
T_4	0.0952	0.1875	0.2014	0.0496

Table 6. Geometric mean score matrix of FPHFLTSSs

	c_1	c_2	c_3	c_4
T_1	0.1650	0.1631	0.0913	0.0486
T_2	0.1166	0.1250	0.2573	0.0572
T_3	0.1864	0.1021	0.1660	0.0687
T_4	0.0865	0.1863	0.1993	0.0486

Table 7. Fractional score matrix of FPHFLTSSs

	c_1	c_2	c_3	c_4
T_1	0.1614	0.1571	0.0884	0.0482
T_2	0.1142	0.1250	0.2505	0.0562
T_3	0.1786	0.1002	0.1579	0.0681
T_4	0.0833	0.1827	0.1941	0.0482

Steps 4 - 6. *As in Algorithm 5.2.*
Step 7. *Ranking the alternatives by RC value. The best choice is the one with the largest RC. Table 9 shows the ranking of alternatives according to three different approaches.*

Table 8. Ranking of alternatives

λ	Arithmetic mean score	Geometry mean score	Fractional score	Liu et al Algorithm 4.2
1	$T_2 \succ T_3 \succ T_4 \succ T_1$	$T_2 \succ T_3 \succ T_4 \succ T_1$	$T_2 \succ T_3 \succ T_4 \succ T_1$	$T_2 \succ T_3 \succ T_4 \succ T_1$
2	$T_2 \succ T_3 \succ T_4 \succ T_1$	$T_2 \succ T_3 \succ T_4 \succ T_1$	$T_2 \succ T_3 \succ T_4 \succ T_1$	$T_2 \succ T_3 \succ T_4 \succ T_1$
4	$T_2 \succ T_3 \succ T_4 \succ T_1$	$T_2 \succ T_3 \succ T_4 \succ T_1$	$T_2 \succ T_3 \succ T_4 \succ T_1$	$T_2 \succ T_3 \succ T_4 \succ T_1$
5	$T_2 \succ T_3 \succ T_4 \succ T_1$	$T_2 \succ T_3 \succ T_4 \succ T_1$	$T_2 \succ T_3 \succ T_4 \succ T_1$	$T_2 \succ T_3 \succ T_4 \succ T_1$

From this table, we can see that all the rankings obtained by arithmetic mean distance, geometry mean distance, fractional distance in TOPSIS and algortihms 4.2 given by Liu et.al gave the same results for values of $\lambda = 1, 2, 3, 4, 5.$.
This shows that our proposed algorithm can be used to solve decision-making problems.

VII. CONCLUSION

The idea of FPHFLTSSs has been described in this paper through the combination of HFLTSSs with the fuzzy parameterized concept. Our central work is outlined and summarized below.

1. We described some related concepts and the fundamental operations of FPHFLTSSs such as complement, union and intersection.
2. We suggested the FPHFLTSS score based on arithmetic mean, geometry mean and fractional approach. Then we established the distance of any two FPHFLTSSs, based on those scores.
3. Within the FPHFLTSS environment, we developed three algorithms to solve the decision-making problem. The first algorithm is simple and easy step without shifting the hesitant linguistic elements to hesitant fuzzy elements. The second is the TOPSIS-based algorithm the shortest distance from the PIS and the longest distance from the NIS. This approach is an expansion of Liu et al.

variant, with an algorithm created on the distance from the PIS. In practical application the length of the linguistic hesitant fuzzy elements is usually different. Some approach suggested to add a hesitant linguistic element to a shortening linguistic hesitant fuzzy element equivalent to another. Many researchers may consent to the destruction and alteration of the original data structure. Therefore, without any fluidity in language, we provide the third algorithm based on arithmetic mean, geometry mean and fractional distance in TOPSIS.

4. In order to explain how our algorithms can be utilized, an empirical example restructured by Liu et al. is presented. It shows that all three algorithms proposed can be applied to solve MCDM problem in FPHFLTSS environment.

In upcoming study, we will look at aggregates of knowledge for FPHFLTSS,

Particularly circumstances of language weight of arguments, in more general contexts and build more MCDM decision-making approaches with information for FPHFLTSSs.

REFERENCES

1. L. . Zadeh, "Fuzzy Sets," *Inf. Control*, vol. 8, pp. 338–353, 1965, doi: https://doi.org/10.1016/S0019-9958(65)90241-X.
2. V. Torra and Y. Narukawa, "On hesitant fuzzy sets and decision," *IEEE Int. Conf. Fuzzy Syst.*, pp. 1378–1382, 2009, doi: 10.1109/FUZZY.2009.5276884.
3. V. Torra, "Hesitant Fuzzy Sets," *Int. J. Intell. Syst.*, vol. 25, pp. 529–539, 2010, doi: 10.1002/int.
4. R. M. Rodriguez, L. Martinez, and F. Herrera, "Hesitant fuzzy linguistic term sets for decision making," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 1, pp. 109–119, 2012, doi: 10.1109/TFUZZ.2011.2170076.
5. X. Gou, Z. Xu, and H. Liao, "Multiple criteria decision making based on Bonferroni means with hesitant fuzzy linguistic information," *Soft Comput.*, vol. 21, no. 21, pp. 6515–6529, 2017, doi: 10.1007/s00500-016-2211-1.
6. C. Wei, N. Zhao, and X. Tang, "Operators and comparisons of hesitant fuzzy linguistic term sets," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 3, pp. 575–585, 2014, doi: 10.1109/TFUZZ.2013.2269144.
7. W. Yu, Z. Zhang, Q. Zhong, and L. Sun, "Extended TODIM for multi-criteria group decision making based on unbalanced hesitant fuzzy linguistic term sets," *Comput. Ind. Eng.*, vol. 114, no. 2, pp. 316–328, 2017, doi: 10.1016/j.cie.2017.10.029.
8. S. M. Chen and J. A. Hong, "Multicriteria linguistic decision making based on hesitant fuzzy linguistic term sets and the aggregation of fuzzy sets," *Inf. Sci. (Ny).*, vol. 286, no. June, pp. 63–74, 2014, doi: 10.1016/j.ins.2014.06.020.
9. M. Tang, H. Liao, Z. Li, and Z. Xu, "Nature Disaster Risk Evaluation with a Group Decision Making Method Based on Incomplete Hesitant Fuzzy Linguistic Preference Relations," *Int. J. Environ. Res. Public Health*, vol. 15, no. 4, p. 751, 2018, doi: 10.3390/ijerph15040751.
10. D. K. Joshi, I. Beg, and S. Kumar, "Hesitant probabilistic fuzzy linguistic sets with applications in multi-criteria group decision making problems," *Mathematics*, vol. 6, no. 4, 2018, doi: 10.3390/math6040047.
11. K. Gong and C. Chen, "Multiple-attribute decision making based on equivalence consistency under probabilistic linguistic dual hesitant fuzzy environment," *Eng. Appl. Artif. Intell.*, vol. 85, no. October 2018, pp. 393–401, 2019, doi: 10.1016/j.engappai.2019.05.008.
12. Merve Güler, E. Mukul, and Gülçin Büyüközkan, "Analysis of e-Government Strategies with Hesitant Fuzzy Linguistic Multi-Criteria Decision Making Techniques Merve," *Intell. Fuzzy Tech. Big Data Anal. Decis. Mak.*, vol. 1029, pp. 1068–1075, 2019, doi: 10.1007/978-3-030-23756-1.
13. H. Liu and L. Jiang, "Optimizing consistency and consensus improvement process for hesitant fuzzy linguistic preference relations and the application in group decision making," *Inf. Fusion*, vol. 56, pp. 114–127, 2020, doi: 10.1016/j.inffus.2019.10.002.
14. D. Molodtsov, "Soft set theory—First results," *Comput. Math. with Appl.*, vol. 37, no. 4–5, pp. 19–31, 1999, doi: 10.1016/s0898-1221(99)00056-5.
15. F. Wang, X. Li, and X. Chen, "Hesitant Fuzzy Soft Set and Its Applications in Multicriteria Decision Making," *J. Appl. Math.*, vol. 2014, pp. 1–10, 2014, doi: 10.1155/2014/643785.
16. K. V Babitha and S. J. John, "Hesitant fuzzy soft sets," *J. New Results Sci.*, vol. 3, pp. 98–107, 2013.
17. H. Zhang and L. Shu, "dual hesitant fuzzy soft set and its properties," *Adv. Intell. Syst. Comput.*, vol. 367, pp. 171–182, 2016, doi: 10.1007/978-3-319-19105-8.
18. Yan-ping He, "An Approach to Dual Hesitant Fuzzy Soft Set Based on Decision Making," *Adv. Intell. Syst. Comput.*, vol. 367, 2016, doi: 10.1007/978-3-319-19105-8.
19. A. Nazra, Syafruddin, R. Lestari, and G. C. Wicaksono, "Hesitant intuitionistic fuzzy soft sets," *J. Phys. Conf. Ser.*, vol. 890, no. 1, 2017, doi: 10.1088/1742-6596/890/1/012118.
20. X. Liu, H. Chen, and L. Zhou, "Hesitant Fuzzy Linguistic Term Soft Sets and Their Applications in Decision Making," *Int. J. Fuzzy Syst.*, 2018, doi: 10.1007/s40815-018-0506-3.
21. N. Çağman, F. Çıtak, and S. Enginoğlu, "Fuzzy Parameterized Fuzzy Soft Set Theory and Its Applications," *Turkish J. Fuzzy Syst.*, vol. 1, no. 1, pp. 21–35, 2010.
22. S. Alkhazaleh, A. R. Salleh, and N. Hassan, "Fuzzy Parameterized Interval-Valued," *Appl. Math. Sci.*, vol. 5, no. 67, pp. 3335–3346, 2011.
23. M. Bashir and A. R. Salleh, "Fuzzy Parameterized Soft Expert Set," *Abstr. Appl. Anal.*, vol. 2012, pp. 1–15, 2012, doi: 10.1155/2012/258361.
24. B. K. Tripathy and A. Panigrahi, "Interval-valued intuitionistic fuzzy parameterized soft set theory and its application in decision-making," *Proc. 10th Int. Conf. Intell. Syst. Control. ISCO 2016*, no. Iv, 2016, doi: 10.1109/ISCO.2016.7726952.
25. S. Ganeshsree and S. Abdul Razak, "Fuzzy Parameterized Intuitionistic Fuzzy Soft Expert Set Theory and its Application in Decision Making," *Int. J. Soft Comput.*, vol. 11, no. 2, pp. 52–63, 2016, doi: 10.1504/ijads.2016.10000983.
26. Y. Al-Qudah, M. Hassan, and N. Hassan, "Fuzzy Parameterized Complex Multi-Fuzzy Soft Expert Set Theory and Its Application in Decision-Making," *Symmetry (Basel).*, vol. 11, no. 3, p. 358, 2019, doi: 10.3390/sym11030358.
27. A. Al-Quran, N. Hassan, and S. Alkhazaleh, "Fuzzy Parameterized Complex Neutrosophic Soft Expert Set for Decision under Uncertainty," *Symmetry (Basel).*, vol. 11, no. 3, p. 382, 2019, doi: 10.3390/sym11030382.
28. Z. Rodzi and A. G. Ahmad, "Fuzzy Parameterized Dual Hesitant Fuzzy Soft Sets and Its Application in TOPSIS," *Math. Stat. 8*, vol. 8, no. 1, pp. 32–41, 2020, doi: 10.13189/ms.2020.080104.
29. M. Xia and Z. Xu, "Hesitant fuzzy information aggregation in decision making," *Int. J. Approx. Reason.*, vol. 52, no. 3, pp. 395–407, 2011, doi: 10.1016/j.ijar.2010.09.002.
30. A. Kharal and B. Ahmad, "On fuzzy soft sets," *Adv. Fuzzy Syst.*, vol. 2009, 2009, doi: 10.1155/2009/586507.
31. H. Liao, Z. Xu, X. J. Zeng, and J. M. Merigó, "Qualitative decision making with correlation coefficients of hesitant fuzzy linguistic term sets," *Knowledge-Based Syst.*, vol. 76, pp. 127–138, 2015, doi: 10.1016/j.knosys.2014.12.009.
32. H. Liao, D. Wu, Y. Huang, P. Ren, Z. Xu, and M. Verma, "Green logistic provider selection with a hesitant fuzzy linguistic thermodynamic method integrating cumulative prospect theory and PROMETHEE," *Sustain.*, vol. 10, pp. 1–16, 2018, doi: 10.3390/su10041291.

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