International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-5, March 2020

Complete Cototal Domination Number of Middle Graphs

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***Abstract:** A total dominating set D is said to be a complete cototal dominating set if the $\langle V-D\rangle$ has no isolated nodes. The complete cototal domination number $\gamma_{cc}(G)$ is the minimum cardinality of a complete cototal dominating set of G. Our aim is to determine the Complete Cototal Domination Number of Middle Graphs and its bounds.*

***Keywords:** Domination number, Total domination number, Cototal domination number, Complete cototal domination number, Middle graph.MSC 2010 subject classification : 05C69*

I. INTRODUCTION

Domination theory in graph was established by Claude Berge around 1960's with the problem of placing minimum number of queens on a $n \times n$ chess board to dominate every square by at least one queen. After that Oystein Ore established the concept dominating set and domination number [5]. A set S of nodes of G is a dominating set of G if each node of G is dominated by some node in . The total domination in graphs which was presented by Cockayne, Dawes and Hedetniemi [2,3]. A subset D of V is called a dominating set of G if every node not in D is adjacent to some node in . A total dominating set for a graph G is a dominating set M for G with the properly that every node in M has a neighbor in . Note that total dominating sets are not defined for graphs with isolated nodes. The concept of cototal dominating set was presented by Kulli, Janakiram and Iyer [4]. This concept motivate us to do research under this topic. Throughout this paper we considered a simple connected graph the total number of nodes and edges are denoted by p and q respectively.

II. DEFINITIONS

Definition: 2.1

A total dominating set D is said to be a complete cototal dominating set (γ_{cctd}) if the $\langle V-D\rangle$ has no isolated nodes. The complete cototal domination number $\gamma_{cc}(G)$ is the minimum cardinality of a complete cototal dominating set of G [1].

Definition 2.2.

Let G be a connected graph. A subdivision graph $S(G)$ is said to be a middle graph $M(G)$ if the middle nodes lies on adjacent edges of G should be adjacent.

Revised Manuscript Received on March 30, 2020.
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III. MAIN RESULTS

Theorem: 3.1 For a Path graph P_n, $\gamma_{cc}(M(P_n)) = \begin{cases} 3 & \text{if } n = 2 \\ 2n-1 & \text{if } n \geq 3 \end{cases}$

Proof. The Middle Path graph $M(P_n)$ has $(2n-1)$ nodes $v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_{n-1}$ and $2n-2$ edges. Here $u_1, u_2, \dots, u_{n-1}$ be the middle nodes.

Case (i) $n = 2$

The Middle Path graph $M(P_2)$ has three nodes v_1, u_1, v_2 and two edges v_1u_1, u_1v_2. Let us consider the total dominating set $\gamma_{td}(M(P_2)) = \{v_1, u_1\}$. Minimal cototal dominating set is obtained by $(V(M(P_2)) - \{v_1, u_1\}) \cap \{y\}$ where y= v_1 or v_2 is an isolated node. Hence $\gamma_{cctd}(M(P_2) = \{v_1, u_1\} \cup \{y\}$. Therefore $\gamma_{cc}(M(P_2)) = 3$.

Case (ii) $n \geq 3$

The Middle Path graph $M(P_n)$ has $(2n-1)$ nodes and $(2n-2)$ edges. Let us consider the total dominating set $\gamma_{td}(M(P_n)) = \{u_1, u_2, \dots, u_{n-1}\}$. Minimal cototal dominating set is obtained by $(V(M(P_n)) - \{u_1, u_2, \dots, u_{n-1}\}) \cap \{v_1, v_2, \dots, v_n\}$. Hence $\gamma_{cctd}(M(P_n) = \{u_1, u_2, \dots, u_{n-1}\} \cup \{v_1, v_2, \dots, v_n\}$. Therefore $\gamma_{cc}(M(P_n) = 2n-1$.

Theorem: 3.2 For a Cycle graph C_n, $\gamma_{cc}(M(C_n)) = 2n-3, n \geq 3$.

Proof. The Middle Cycle graph $M(C_n)$ has $2n$ nodes $v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n$ and $3n$ edges. Here $v_1, v_2, \dots, v_n$ be the outer nodes on cycle C_n and $u_1, u_2, \dots, u_n$ be the middle nodes on cycle C_n . Let us consider the total dominating set $\gamma_{td}(M(C_n)) = \{u_1, u_2, \dots, u_{n-1}\}$. Minimal cototal dominating set is obtained by $(V(M(C_n)) - \{u_1, u_2, \dots, u_{n-1}\}) \cap \{y\}$ where y are the isolated nodes. Hence $\gamma_{cctd}(M(C_n)) = \{u_1, u_2, \dots, u_{n-1}\} \cup \{y\}$.Therefore $\gamma_{cc}(M(P_n) = 2n-3$.

Theorem:3.3 For a Comb graph $P_n \odot K_1$, $\gamma_{cc}(M(P_n \odot K_1)) = 4n-1, n \geq 2$.

Proof. The Middle Comb graph $M(P_n \odot K_1)$ has $(4n-1)$ nodes $v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n, z_1, z_2, \dots, z_{n-1}, x_1, x_2, \dots, x_n$ and $(6n-4)$ edges. Here $v_1, v_2, \dots, v_n$ be the nodes on P_n and $u_1, u_2, \dots, u_n$ be the pendant nodes and $z_1, z_2, \dots, z_{n-1}$ be the middle nodes on P_n and x_i's are the middle nodes on v_iu_i , $1 \leq i \leq n$. Let us consider the total dominating set $\gamma_{td}(M(P_n \odot K_1)) = \{x_i, z_j\}$ where $1 \leq i \leq n, 1 \leq j \leq n-1$.

Minimal cototal dominating set is obtained by $(V(M(P_n \odot K_1)) - \{x_i, z_j\}) \cap \{y\}$ where y are isolated nodes with cardinality $2n$ so that $\gamma_{cctd}(M(P_n \odot K_1)) = \{x_i, z_j\} \cup \{y\}$. Therefore $\gamma_{cc}(M(P_n \odot K_1)) = 4n - 1$.

Theorem:3.4 For a n-sunlet graph $\gamma_{cc}(M(n - \text{sunlet})) = 4n - 3, n \geq 3$.

Proof. The middle $M(n - \text{sunlet})$ graph has $4n$ nodes $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$, $z_1, z_2, \dots, z_n, x_1, x_2, \dots, x_n$ and $7n$ edges. Here $u_1, u_2, \dots, u_n$ be the pendant nodes and $v_1, v_2, \dots, v_n$ be the nodes on cycle C_n and x_i and z_i be the middle nodes of $u_i v_i$ and cycle C_n respectively. Let us consider the total dominating set $\gamma_{td}(M(n - \text{sunlet})) = \{z_1, z_2, \dots, z_{n-1}, x_1, x_2, \dots, x_n\}$. Minimal cototal dominating set is obtained by $(V(M(n - \text{sunlet})) - \{z_1, z_2, \dots, z_{n-1}, x_1, x_2, \dots, x_n\}) \cap \{y\}$ where y are isolated nodes. Therefore $\gamma_{cctd}(M(n - \text{sunlet})) = \{z_1, z_2, \dots, z_{n-1}, x_1, x_2, \dots, x_n\} \cup \{y\}$ so that $\gamma_{cc}(M(n - \text{sunlet})) = 4n - 3$.

Theorem: 3.5 For a n -pan graph, $\gamma_{cc}(M(n - pan)) = 2n - 1, n \geq 3$.

Proof: The Middle $n - pan$ graph has $(2n + 2)$ nodes $v_1, v_2, \dots, v_n, v,$ $u, u_1, u_2, \dots, u_n$ and $(3n + 4)$ edges. Here $v_1, v_2, \dots, v_n$ be the nodes on cycle C_n and $u_1, u_2, \dots, u_n$ be the middle nodes on cycle C_n and v be the pendant node and u be the middle node on the pendant node. Let us consider the total dominating set γ_{td} = $\{u, u_1, u_2, \dots, u_{n-1}\}$. Minimal cototal dominating set is obtained by $(V(M(n - pan)) - \{u, u_1, u_2, \dots, u_{n-1}\}) \cap \{y\}$ where y are isolated nodes. Therefore $\gamma_{cctd}(M(n - pan))$ = $\{u, u_1, u_2, \dots, u_{n-1}\} \cup \{y\}$. Hence $\gamma_{cc}(M(n - pan)) = 2n - 1$.

IV. BOUNDS FOR $\gamma_{cc}(M(G))$

Theorem: 4.1 Let M(G) be a connected graph, then $\gamma_{cc}(M(G)) > \left\lceil \frac{n}{\Delta M(G)} \right\rceil$.

Proof: Let $S \subseteq V(M(G))$ be a γ_{cctd} - set in G. Every node in S dominates at most $\Delta(M(G)) - 1$ nodes of $V(M(G)) - S$ and dominate at least one of the nodes in S. Hence, $|S|(\Delta(M(G)) - 1) + |S| > n$. Since, S is an arbitrary γ_{cctd} – set, then $\gamma_{cc}(M(G)) > \left\lceil \frac{n}{\Delta(M(G))} \right\rceil$.

Theorem : 4.2 If $M(G)$ is a connected graph with the girth of length $g(M(G)) \geq 3$ and $\delta(M(G)) \geq 2$, then $\gamma_{cc}(M(G)) > n - \left\lceil \frac{g(M(G))}{2} \right\rceil + 1$.

Proof: Let $M(G)$ be a connected graph with $g(M(G)) \geq 3$ and let C be a cycle of length $g(M(G))$. Remove C from $M(G)$ to form a graph $M(G')$. Suppose an arbitrary node $v \in V(M(G'))$. Since $\delta(M(G)) \geq 2$, v has at least two neighbors say w and z Let $w, z \in C$. If $d(w, z) \geq 3$, then replacing the path from w to z on C with the path w, v, z, which reduces the girth of $M(G)$, a contradiction. If $d(w, z) \leq 2$, then w, z, v are on either C_3 or C_4 in $M(G)$, contradicting the hypothesis that $g(M(G)) \geq 3$. Hence, no node in $M(G')$ has two or more neighbours on C. Since $\delta(M(G)) \geq 2$, the graph $M(G')$ has minimum degree at least $\delta(M(G)) - 1 \geq 1$. Then $M(G)$ has no isolated nodes. Now let S' be a γ_{cc} – set for C. Then $S = S' \cup V(M(G'))$is a γ_{cctd} – set for $M(G)$. Hence, $\gamma_{cc}(M(G)) > n - \left\lceil \frac{g(M(G))}{2} \right\rceil + 1$.

Theorem : 4.3 Let $M(G)$ be a graph without isolated nodes. Then $\gamma_{cc}(M(G)) > \left\lceil \frac{n}{2} \right\rceil$.

Proof. Let $D \subseteq V(M(G))$ be a γ_{cctd}- set. Since $M(G)$ has no isolated nodes, every $v \in D$ has at least one neighbor in $V - D$. This means that $V - D$ is also a complete cototal dominating set. If $|D| < \left\lceil \frac{n}{2} \right\rceil$, then $V - D$ is the smallest γ_{cctd} – set, contradicting the choice of D as a γ_{cctd}- set. Thus $\gamma_{cc}(M(G)) = |D| > \left\lceil \frac{n}{2} \right\rceil$.

Theorem : 4.4 Let $M(G)$ be a graph with $\text{diam}(M(G)) \geq 1$, then $\gamma_{cc}(M(G)) > \delta(M(G)) + 1$.

Proof: Let $z \in V(M(G))$ and $\deg(z) = \delta(M(G))$. Since $\text{diam}(M(G)) \geq 1$, then $N(z)$ is a total dominating set for $M(G)$. Now $S = N(z) \cup \{z\}$ is a γ_{cctd} – set for $M(G)$ and $|S| = \delta(M(G)) + 1$. Hence, $\gamma_{cc}(M(G)) > \delta(M(G)) + 1$.

Theorem : 4.5 For any graph $M(G)$, $\gamma_{cc}(M(G)) > n - \Delta(M(G))$.

Proof. Let $M(G)$ be a γ_{cc} -set of $M(G)$. Let l be a node of maximum degree $\Delta(M(G))$. Then l dominates $N[r]$ and the nodes in $V - N[r]$ dominate themselves. Hence $V - N[r]$ is a γ_{cctd} - set of cardinality $n - \Delta(M(G))$, so $\gamma_{cc}(M(G)) > n - \Delta(M(G))$

V. CONCLUSION

In this paper, Complete Cototal Domination Number of Middle Path graph, Middle Cycle graph, Middle Comb graph, Middle n-sunlet graph, Middle $n-pan$ graph are found and new bounds for $\gamma_{cc}(M(G))$ are obtained.

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