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Length Biased Weighted Quasi Gamma Distribution with Characterizations and Applications

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***Abstract:** we introduce length biased technique for weighted quasi gamma to change the known distribution into new model called as the length biased weighted quasi gamma distribution. Lastly the newly developed model has been investigated with an application.*

***Keywords:** Reliability measures, quasi gamma distribution, Order statistics, Likelihood ratio test.*

I. INTRODUCTION

The weighted distribution gives an integrated approach to deal with the specification of models and data interpretation problems. The introduction of weighted distributions provides a platform for the researchers to introduce the new distributions by using the additional parameter in the known distribution. Weighted concept is a remarkable work was first emerged by Fisher (1934) then after that Rao (1965) formulate it in a general way for modeling statistical data because the existing standard distributions was found to be inappropriate. Length biased distribution is the reduced form of weighted distribution when the weight function considers only the length of the units and length biased concept was firstly introduced by Cox (1969) and Zelen (1974). Different authors have discussed regarding the length biased concept and give there different views. Roy et al. (2011) have obtained the length biased model of weighted weibull distribution. Mir et al.(2013) have achieved the length biased beta distribution of first kind. Reyad et al.(2017) have obtained length biased weighted erlang distribution. Mudasir and Ahmad (2018) obtained length biased version of Nakagami distribution. Recently, Rather and Subramanian (2019) have obtained the length biased erlang truncated exponential distribution with lifetime data.

Quasi gamma distribution was introduced by shanker et al. (2018) is a newly proposed two parametric probability distribution. The two parametric quasi gamma distribution is a particular case of one parameter quasi exponential distribution. The two parametric quasi gamma distribution gives better results over one parameter exponential and quasi exponential and two parameter Gompertz, Weibull and gamma distributions for modelling lifetime data from engineering.

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II. LENGTH BIASED WEIGHTED QUASI GAMMA DISTRIBUTION (LBWQGD)

The distribution of quasi gamma having probability density function

$$f(x;\theta,\alpha)=\frac{2\theta^{\alpha}}{\Gamma(\alpha)}e^{-\theta x^{2}}x^{2\alpha-1};\ x>0,\theta>0,\alpha>0. \quad (1)$$

and the distribution of quasi gamma having distribution of cumulative function

$$F(x;\theta,\alpha)=1-\frac{\Gamma(\alpha,\theta x^{2})}{\Gamma(\alpha)};x>0,\theta>0,\alpha>0 \quad (2)$$

Assume X is a randomly selected variable is non-negative with probability density function $f(x)$, then $w(x)$ be the non-negative weight function having probability density function

$$f_w(x)=\frac{w(x)f(x)}{E(w(x))},\quad x>0.$$

We have to obtain the length biased technique for the distribution of quasi gamma, so we will apply c =1 in weighted quasi gamma model to obtain the distribution of length biased weighted quasi gamma and its Pdf is given by

$$f_l(x;\theta,\alpha)=\frac{xf(x;\theta,\alpha)}{E(x)},x>0 \quad (3)$$

$$E(x)=\frac{\theta^{\alpha}\Gamma\frac{(2\alpha+2)}{2}}{\theta^{\frac{2\alpha+2}{2}}\Gamma(\alpha)} \quad (4)$$

By applying equation (1) and (4) in equation (3), we obtain the probability density function of the distribution of length biased weighted quasi gamma

$$f_l(x;\theta,\alpha)=\frac{2x^{2\alpha}\theta^{\frac{2\alpha+2}{2}}e^{-\theta x^{2}}}{\Gamma\frac{(2\alpha+2)}{2}} \quad (5)$$

Now, the cumulative function of the distribution of length biased weighted quasi gamma is

$$F_l(x;\theta,\alpha)=\int_0^x f_l(x;\theta,\alpha)dx$$

$$F_l(x;\theta,\alpha)=\int_0^x \frac{2x^{2\alpha}\theta^{\frac{2\alpha+2}{2}}e^{-\theta x^2}}{\Gamma\frac{(2\alpha+2)}{2}}dx$$

$$F_l(x;\theta,\alpha)=\frac{2}{\Gamma\frac{(2\alpha+2)}{2}}\int_0^x x^{2\alpha}\theta^{\frac{2\alpha+2}{2}}e^{-\theta x^2}dx$$

Put $\theta x^2=t \Rightarrow 2\theta x dx=dt \Rightarrow dx=\frac{dt}{2\theta x}=\frac{dt}{2\theta\left(\frac{t}{\theta}\right)^{\frac{1}{2}}}$

$Also\ \theta x^2=t \Rightarrow x^2=\frac{t}{\theta} \Rightarrow x=\left(\frac{t}{\theta}\right)^{\frac{1}{2}}$ After

$As\ x\to 0, t\to 0\ \ and\ \ x\to x, t\to \theta x^2$

simplification, we obtain cumulative distribution function of the distribution of length biased weighted quasi gamma

$$F_l(x;\theta,\alpha)=\frac{1}{\Gamma\frac{(2\alpha+2)}{2}}\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+2)}{2},\theta x^2\right)\right) \tag{6}$$

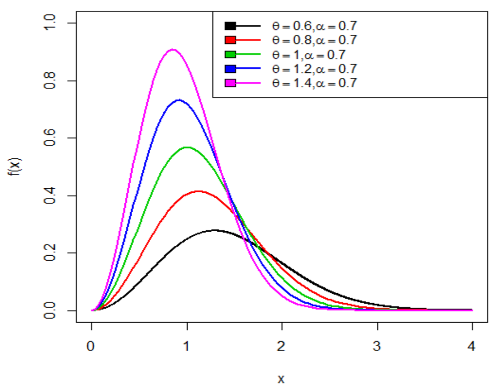

Fig.1:Pdf plot of LBWQGD

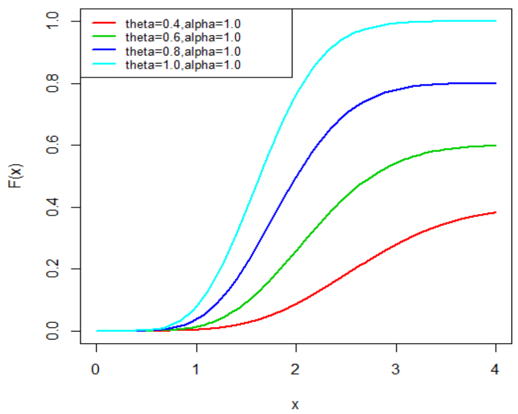

Fig.2 Cdf plot of LBWQGD

III. RELIABILITY MEASURES

A. Survival function

The survival function of the distribution of length biased weighted quasi gamma is given by

$$S(x)=1-F_l(x;\theta,\alpha)$$

$$S(x)=1-\frac{1}{\Gamma\frac{(2\alpha+2)}{2}}\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+2)}{2},\theta x^2\right)\right)$$

B. Hazard function

The hazard function is also known as failure rate and is given by

$$h(x)=\frac{f_l(x;\theta,\alpha)}{S(x)}$$

$$h(x)=\frac{2x^{2\alpha}\theta^{\frac{2\alpha+2}{2}}e^{-\theta x^2}}{\Gamma\frac{(2\alpha+2)}{2}-\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+2)}{2},\theta x^2\right)\right)}$$

C. Reverse hazard function

The reverse hazard function of the proposed distribution is given by

$$h^r(x)=\frac{f_l(x;\theta,\alpha)}{F_l(x;\theta,\alpha)}$$

$$h^r(x)=\frac{2x^{2\alpha}\theta^{\frac{2\alpha+2}{2}}e^{-\theta x^2}}{\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+2)}{2},\theta x^2\right)\right)}$$

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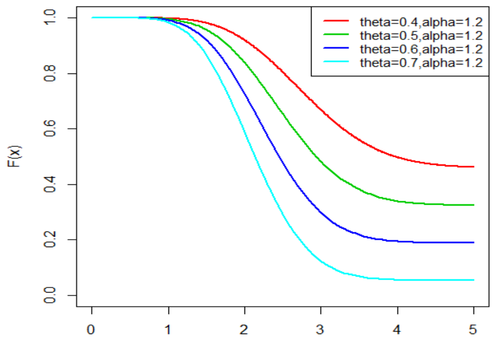

Fig.3 survival function of LBWQGD

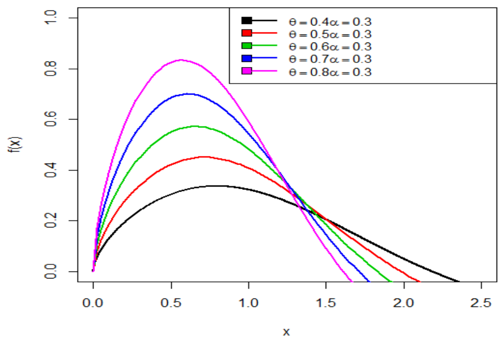

Fig.4:Showing Hazard function of LBWQGD

IV. STRUCTURAL PROPERTIES

A. Moments

Let X denotes the randomly selected variable of length biased weighted quasi gamma, then

$$E(X^r) = \int_0^\infty x^r f_l(x;\theta,\alpha)dx$$

$$E(X^r) = \int_0^\infty x^r \frac{2x^{2\alpha}\theta^{\frac{2\alpha+2}{2}} e^{-\theta x^2}}{\Gamma\frac{(2\alpha+2)}{2}}dx$$

$$= \frac{2\theta^{\frac{2\alpha+2}{2}}}{\Gamma\frac{(2\alpha+2)}{2}} \int_0^\infty x^{2\alpha+r} e^{-\theta x^2} dx \qquad (7)$$

Put $x^2 = t \Rightarrow x = t^{\frac{1}{2}}$

Also $2xdx = dt \Rightarrow dx = \frac{dt}{2x} = \frac{dt}{2t^{\frac{1}{2}}}$

After simplification, we obtain from equation (7)

$$= \frac{\theta^{\frac{2\alpha+2}{2}}}{\Gamma\frac{(2\alpha+2)}{2}} \left(\int_0^\infty t^{\frac{(2\alpha+r+2)-3}{2}} e^{-\theta t} dt \right)$$

$$E(X^r) = \mu_r' = \frac{\Gamma\frac{(2\alpha+r+2)}{2}}{\theta^{\frac{r}{2}}\Gamma\frac{(2\alpha+2)}{2}} \qquad (8)$$

Applying the values 1, 2, 3 and 4 in equation (8) for the purpose of getting the moments of the distribution of length biased weighted quasi gamma.

$$E(X) = \mu_1' = \frac{\Gamma\frac{(2\alpha+3)}{2}}{\theta^{\frac{1}{2}}\Gamma\frac{(2\alpha+2)}{2}}$$

$$E(X^2) = \mu_2' = \frac{\Gamma\frac{(2\alpha+4)}{2}}{\theta\Gamma\frac{(2\alpha+2)}{2}}$$

$$E(X^3) = \mu_3' = \frac{\Gamma\frac{(2\alpha+5)}{2}}{\theta^{\frac{3}{2}}\Gamma\frac{(2\alpha+2)}{2}}$$

$$E(X^4) = \mu_4' = \frac{\Gamma\frac{(2\alpha+6)}{2}}{\theta^2\Gamma\frac{(2\alpha+2)}{2}}$$

$$\text{Variance}(\mu_2) = \frac{\Gamma\frac{(2\alpha+4)}{2}}{\theta\Gamma\frac{(2\alpha+2)}{2}} - \left(\frac{\Gamma\frac{(2\alpha+3)}{2}}{\theta^{\frac{1}{2}}\Gamma\frac{(2\alpha+2)}{2}} \right)^2$$

$$S.D(\sigma) = \sqrt{\left(\frac{\Gamma\frac{(2\alpha+4)}{2}}{\theta\Gamma\frac{(2\alpha+2)}{2}} - \frac{\left(\Gamma\frac{(2\alpha+3)}{2}\right)^2}{\left(\theta^{\frac{1}{2}}\Gamma\frac{(2\alpha+2)}{2}\right)^2} \right)}$$

B. Harmonic mean

The harmonic mean is the reciprocal of the arithmetic mean of the reciprocals and the harmonic mean for the proposed distribution is

$$H.M = E\left(\frac{1}{x}\right) = \int_0^\infty \frac{1}{x} f_l(x;\theta,\alpha)dx$$

$$= \int_0^\infty \frac{2x^{2\alpha-1}\theta^{\frac{2\alpha+2}{2}} e^{-\theta x^2}}{\Gamma\frac{(2\alpha+2)}{2}} dx$$

$$= \frac{2}{\Gamma\frac{(2\alpha+2)}{2}}\left(\int_0^\infty x^{2\alpha-1}\theta^{\frac{2\alpha+2}{2}} e^{-\theta x^2} dx\right) \qquad (9)$$

Put $\theta x^2 = t \Rightarrow x^2 = \frac{t}{\theta} \Rightarrow x = \left(\frac{t}{\theta}\right)^{\frac{1}{2}}$

Also $2\theta x dx = dt \Rightarrow dx = \frac{dt}{2\theta x} \Rightarrow dx = \frac{dt}{2\theta\left(\frac{t}{\theta}\right)^{\frac{1}{2}}}$ After simplification, we obtain from equation (9)

$$= \frac{1}{\Gamma\frac{(2\alpha+2)}{2}}\left(\theta\int_0^\infty t^{\frac{(2\alpha+2)-4}{2}} e^{-t} dt\right)$$

$$H.M = \frac{1}{\Gamma\frac{(2\alpha+2)}{2}}\left(\theta\gamma\left(\frac{(2\alpha+2)}{2}, \theta x^2\right)\right)$$

C. Moment Generating Function

The moment generating function is the expectation of a function of the random variable and it is defined as

$$M_X(t) = E\left(e^{tx}\right) = \int_0^\infty e^{tx} f_l(x;\theta,\alpha)dx$$

$$= \int_0^\infty \left[1 + tx + \frac{(tx)^2}{2!} + ...\right] f_l(x;\theta,\alpha)dx$$

$$= \sum_{j=0}^\infty \frac{t^j}{j!}\mu_j'$$

$$= \sum_{j=0}^\infty \frac{t^j}{j!}\left[\frac{\Gamma\frac{(2\alpha+j+2)}{2}}{\theta^{\frac{j}{2}}\Gamma\frac{(2\alpha+2)}{2}}\right]$$

$$M_X(t) = \frac{1}{\Gamma\frac{(2\alpha+2)}{2}}\sum_{j=0}^\infty \frac{t^j}{j!\theta^{\frac{j}{2}}}\left(\Gamma\frac{(2\alpha+j+2)}{2}\right)$$

D. Characteristic Function

In probability theory and statistics characteristic function is the function of any real-valued random variable completely defines the probability distribution.

$$\varphi_X(t) = M_X(it)$$

$$M_X(it) = \frac{1}{\Gamma\frac{(2\alpha+2)}{2}}\sum_{j=0}^\infty \frac{(it)^j}{j!\theta^{\frac{j}{2}}}\left(\Gamma\frac{(2\alpha+j+2)}{2}\right)$$

V. ORDER STATISTICS

Order statistics are the sequence of samples placed in an increasing order, then the probability density function of r^{th} order statistics $X_{(r)}$ is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)\left(F_X(x)\right)^{r-1}\left(1 - F_X(x)\right)^{n-r}$$

Using equation (5) and (6), we obtain r^{th} model of length biased weighted quasi gamma

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!}\left(\frac{2x^{2\alpha}\theta^{\frac{2\alpha+2}{2}} e^{-\theta x^2}}{\Gamma\frac{(2\alpha+2)}{2}}\right)$$

$$\times\left(\frac{1}{\Gamma\frac{(2\alpha+2)}{2}}\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+2)}{2}, \theta x^2\right)\right)\right)^{r-1}$$

$$\times\left(1 - \frac{1}{\Gamma\frac{(2\alpha+2)}{2}}\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+2)}{2}, \theta x^2\right)\right)\right)^{n-r}$$

Therefore, the *Ist* order statistics $X_{(1)}$ of length biased weighted quasi gamma is

$$f_{x(1)}(x) = \frac{2nx^{2\alpha}\theta^{\frac{2\alpha+2}{2}} e^{-\theta x^2}}{\Gamma\frac{(2\alpha+2)}{2}}$$

$$\times\left(1 - \frac{1}{\Gamma\frac{(2\alpha+2)}{2}}\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+2)}{2}, \theta x^2\right)\right)\right)^{n-1}$$ and

the higher order statistics $X_{(n)}$ of length biased weighted quasi gamma is

$$f_{x(n)}(x) = \frac{2nx^{2\alpha}\theta^{\frac{2\alpha+2}{2}}e^{-\theta x^2}}{\Gamma\frac{(2\alpha+2)}{2}} \times \left(\frac{1}{\Gamma\frac{(2\alpha+2)}{2}}\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+2)}{2},\theta x^2\right)\right)\right)^{n-1}$$

VI. LIKELIHOOD RATIO TEST

The sample of size n randomly drawn from the distribution of quasi gamma or length biased weighted quasi gamma. We set up null and alternative hypothesis.

$H_o : f(x) = f(x;\theta,\alpha)$ vs $H_1 : f(x) = f_l(x;\theta,\alpha)$ For the purpose of testing whether random sample of size n comes from distribution of quasi gamma or length biased weighted quasi gamma, the following test procedure is used.

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n}\frac{f_l(x;\theta,\alpha)}{f(x;\theta,\alpha)}$$

$$\Delta = \prod_{i=1}^{n}\left[\frac{x_i\theta\Gamma\alpha}{\Gamma\frac{(2\alpha+2)}{2}}\right]$$

$$\Delta = \left(\frac{\theta\Gamma\alpha}{\Gamma\frac{(2\alpha+2)}{2}}\right)^n \prod_{i=1}^{n}x_i$$

We should not accept the null hypothesis if

$$\Delta = \left(\frac{\theta\Gamma\alpha}{\Gamma\frac{(2\alpha+2)}{2}}\right)^n \prod_{i=1}^{n}x_i > k$$

Equivalently, we should reject the null hypothesis

$$\Delta^* = \prod_{i=1}^{n}x_i > k\left(\frac{\Gamma\frac{(2\alpha+2)}{2}}{\theta\Gamma\alpha}\right)^n$$

$$\Delta^* = \prod_{i=1}^{n}x_i > k^* \text{ Where}$$

$$k^* = k\left(\frac{\Gamma\frac{(2\alpha+2)}{2}}{\theta\Gamma\alpha}\right)^n$$

Furthermore, null hypothesis should be rejected, when the probability value is given by

$p\left(\Delta^* > \lambda^*\right)$, Where $\lambda^* = \prod_{i=1}^{n}x_i$ is less than a particular level of the significance and $\prod_{i=1}^{n}x_i$ is the experimental value of the statistic Δ^*

VII. BONFERRONI AND LORENZ CURVES

The proposed measure for length biased weighted quasi gamma is

$$B(p) = \frac{1}{p\mu_1'}\int_0^q x f_l(x;\theta,\alpha)dx$$

and $L(p) = pB(p) = \frac{1}{\mu_1'}\int_0^q x f_l(x;\theta,\alpha)dx$

Where $\mu_1' = \frac{\Gamma\frac{(2\alpha+3)}{2}}{\theta^{\frac{1}{2}}\Gamma\frac{(2\alpha+2)}{2}}$ and $q = F^{-1}(p)$

$$B(p) = \frac{\theta^{\frac{1}{2}}\Gamma\frac{(2\alpha+2)}{2}}{p\left(\Gamma\frac{(2\alpha+3)}{2}\right)}\int_0^q \frac{x\,2x^{2\alpha}\theta^{\frac{2\alpha+2}{2}}e^{-\theta x^2}}{\Gamma\frac{(2\alpha+2)}{2}}dx$$

$$B(p) = \frac{\theta^{\frac{1}{2}}\Gamma\frac{(2\alpha+2)}{2}}{p\left(\Gamma\frac{(2\alpha+3)}{2}\right)}\int_0^q \frac{2x^{2\alpha+1}\theta^{\frac{2\alpha+2}{2}}e^{-\theta x^2}}{\Gamma\frac{(2\alpha+2)}{2}}dx$$

$$B(p) = \frac{2\theta^{\frac{1}{2}}}{p\left(\Gamma\frac{(2\alpha+3)}{2}\right)}\left(\int_0^q x^{2\alpha+1}\theta^{\frac{2\alpha+2}{2}}e^{-\theta x^2}dx\right) \quad (10)$$

Put $\theta x^2 = t \Rightarrow x^2 = \frac{t}{\theta} \Rightarrow x = \left(\frac{t}{\theta}\right)^{\frac{1}{2}}$

Also $2\theta x dx = dt \Rightarrow dx = \frac{dt}{2\theta x} = \frac{dt}{2\theta\left(\frac{t}{\theta}\right)^{\frac{1}{2}}}$

After simplification equation (10) become

$$B(p) = \frac{\theta^{\frac{1}{2}}}{p\left(\Gamma\frac{(2\alpha+3)}{2}\right)}\left(\int_0^q t^{\frac{(2\alpha+2)-2}{2}} e^{-t} dt\right)$$

$$B(p) = \frac{\theta^{\frac{1}{2}}}{p\left(\Gamma\frac{(2\alpha+3)}{2}\right)}\left(\gamma\left(\frac{(2\alpha+2)}{2}, \theta q\right)\right)$$

$$L(p) = \frac{\theta^{\frac{1}{2}}}{\left(\Gamma\frac{(2\alpha+3)}{2}\right)}\left(\gamma\left(\frac{(2\alpha+2)}{2}, \theta q\right)\right)$$

VIII. MAXIMUM LIKELIHOOD ESTIMATION AND FISHER'S INFORMATION MATRIX

In this portion, parameters of the newly proposed length biased weighted quasi gamma model are obtained, then the corresponding likelihood function is given by

$$L(x;\theta,\alpha) = \prod_{i=1}^{n} f_l(x;\theta,\alpha)$$

$$L(x;\theta,\alpha) = \prod_{i=1}^{n}\left[\frac{2x_i^{2\alpha}\theta^{\frac{2\alpha+2}{2}} e^{-\theta x_i^2}}{\Gamma\frac{(2\alpha+2)}{2}}\right]$$

$$L(x;\theta,\alpha) = \frac{2^n \theta^{n\left(\frac{2\alpha+2}{2}\right)}}{\left(\Gamma\frac{(2\alpha+2)}{2}\right)^n}\prod_{i=1}^{n}\left[x_i^{2\alpha} e^{-\theta x_i^2}\right]$$

The log likelihood function is given by

$$\log L = n\log 2 + n\left(\frac{2\alpha+2}{2}\right)\log\theta - n\log\left(\Gamma\frac{(2\alpha+2)}{2}\right) + 2\alpha\sum_{i=1}^{n}\log x_i - \theta\sum_{i=1}^{n} x_i^2 \qquad (11)$$

By differentiating the log likelihood equation (11) with respect to parameters. We obtain the normal equations

$$\frac{\partial \log L}{\partial\theta} = \frac{n(2\alpha+2)}{2\theta} - \sum_{i=1}^{n} x_i^2 = 0$$

$$\frac{\partial \log L}{\partial\alpha} = n\log\theta - n\psi\left(\frac{2\alpha+2}{2}\right) + 2\sum_{i=1}^{n}\log x_i = 0$$

ψ (.) is the function of digamma.

The likelihood equations are too complicated, so that it is very difficult to solve these systems of non-linear equations. Therefore we use R and wolfram mathematics for estimating the required parameters.

To obtain confidence interval, we use the asymptotic normality results. We have that, if $\hat{\eta} = (\hat{\theta}, \hat{\alpha})$ denotes the MLE of $\eta = (\theta, \alpha)$. We can state the result as:

$$\sqrt{n}(\hat{\eta} - \eta) \rightarrow N_2(0, I^{-1}(\eta))$$

Where $I(\eta)$ is the Fisher's Information matrix.

$$I(\eta) = -\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2 \log L}{\partial\theta^2}\right) & E\left(\frac{\partial^2 \log L}{\partial\theta\partial\alpha}\right) \\ E\left(\frac{\partial^2 \log L}{\partial\alpha\partial\theta}\right) & E\left(\frac{\partial^2 \log L}{\partial\alpha^2}\right) \end{pmatrix}$$

Where

$$E\left(\frac{\partial^2 \log L}{\partial\theta^2}\right) = -\frac{n(2\alpha+2)}{2\theta^2}$$

$$E\left(\frac{\partial^2 \log L}{\partial\alpha^2}\right) = -n\psi'\left(\frac{(2\alpha+2)}{2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial\theta\partial\alpha}\right) = E\left(\frac{\partial^2 \log L}{\partial\alpha\partial\theta}\right) = \frac{n}{\theta}$$

Where $\psi(.)'$ is the first order derivative of digamma function. Since η being unknown, we estimate $I^{-1}(\hat{\eta})$ by $I^{-1}(\eta)$ and this can be used to obtain asymptotic confidence intervals for θ and α

IX. DATA EVALUATION

In the portion of application, we apply data sets for purpose of showing that the distribution of length biased weighted quasi gamma shows better fit over quasi gamma, exponential and one parameter lindley distribution.. The following two data sets are given below as

Table-I: Data by Nichols and Padgettcit[14] (2006) regarding breaking stress of carbon fibre 50mm length (GPa)

0.39	0.85	1.08	1.25	1.47	1.57	1.61	1.61	1.69
2.03	2.03	2.05	2.12	2.35	2.41	2.43	2.48	2.50
2.59	2.67	2.73	2.74	2.79	2.81	2.82	2.85	2.87
2.97	3.09	3.11	3.11	3.15	3.15	3.19	3.22	3.22
3.33	3.39	3.39	3.56	3.60	3.65	3.68	3.70	3.75
4.90	1.80	2.53	2.88	3.27	4.20	1.84	2.55	2.93
3.28	4.38	1.87	2.55	2.95	3.31	4.42	1.89	2.56
2.96	3.31	4.70						

Table-II: Data from one of ministry of health hospitals in Saudi Arabia regarding blood cancer (leukaemia) patients

0.315	0.496	0.616	1.145	1.208	1.263	1.414	2.025	2.036
2.162	2.211	2.37	2.532	2.693	2.805	2.91	2.912	3.192
3.263	3.348	3.348	3.427	3.499	3.534	3.767	3.751	3.858
3.986	4.049	4.244	4.323	4.381	4.392	4.397	4.647	4.753
4.929	4.973	5.074	5.381					

In order to compare the length biased weighted quasi gamma model with quasi gamma, exponential and one parameter lindley distribution, we are using the criterion values such as AIC, BIC and AICC. The best distribution corresponds to minimum criterion values. The formulas for calculation of such criterion values are

$$AIC = 2k - 2\log L \quad AICC = AIC + \frac{2k(k+1)}{n-k-1}$$

$$\text{and} \quad BIC = k\log n - 2\log L$$

Actually k is the number of parameters, n is the sample size and $-2\log L$ is the maximized function of log likelihood .The parameters of the proposed distribution are obtained by the maximum likelihood technique. Finally, it is found from results given in table 3 below, that the distribution of length biased weighted quasi gamma have minimum AIC, AICC, BIC and -2logL values as compared to the quasi gamma, exponential and one parameter lindley distribution then, it is clear that the distribution of length biased weighted quasi gamma fits better than quasi gamma, exponential and one parameter lindley distribution.

Table-III: Comparison of Length biased weighted quasi gamma distribution Vs quasi gamma, Exponential and one parameter Lindley distribution

Data sets	Distribution	MLE	S.E	-2logL	AIC	BIC	AICC
1	Length biased weighted quasi gamma	$\hat{\alpha} = 1.88$ $\hat{\theta} = 0.23$	$\hat{\alpha} = 0.13$ $\hat{\theta} = 0.02$	155.6	159.6	164.0	159.8
	Quasi gamma	$\hat{\alpha} = 2.28$ $\hat{\theta} = 0.27$	$\hat{\alpha} = 0.37$ $\hat{\theta} = 0.04$	175.7	179.7	184.0	179.9
	Exponential	$\hat{\theta} = 0.36$	$\hat{\theta} = 0.04$	265.9	267.9	270.1	268.0
	Lindley	$\hat{\theta} = 0.59$	$\hat{\theta} = 0.05$	244.7	246.7	248.9	246.8
2	Length biased weighted quasi gamma	$\hat{\alpha} = 1.88$ $\hat{\theta} = 0.17$	$\hat{\alpha} = 0.17$ $\hat{\theta} = 0.01$	139.4	143.4	146.8	143.7
	Quasi gamma	$\hat{\alpha} = 1.20$ $\hat{\theta} = 0.10$	$\hat{\alpha} = 0.24$ $\hat{\theta} = 0.02$	140.7	144.7	148.1	145.1
	Exponential	$\hat{\theta} = 0.31$	$\hat{\theta} = 0.05$	171.5	173.5	175.2	173.6
	Lindley	$\hat{\theta} = 0.52$	$\hat{\theta} = 0.06$	160.5	162.5	164.1	162.6

X. CONCLUSION

The present manuscript approaches with the distribution of length biased weighted quasi gamma. The newly introduced distribution is generated by taking the two parameter quasi gamma distribution as the base distribution. The newly developed distribution is tested with application and then after that results are finally compared over quasi gamma, exponential and one parameter lindley distribution and the obtained results proved that the length biased weighted quasi gamma gives best results and fits good over quasi gamma, exponential and one parameter lindley distribution.

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