

# Geometric Mean Method to Solve Multi-Objective Transportation Problem Under Fuzzy Environment



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**Abstract:** *Transportation problem is a very common problem for a businessman. Every businessman wants to reduce cost, time and distance of transportation. There are several methods available to solve the transportation problem with single objective but transportation problems are not always with single objective. To solve transportation problem with more than one objective is a typical task. In this paper we explored a new method to solve multi criteria transportation problem named as Geometric mean method to Solve Multi-objective Transportation Problem Under Fuzzy Environment. We took a problem of transportation with three objectives cost, time and distance. We converted objectives into membership values by using a membership function and then geometric mean of membership values is taken. We also used a procedure to find a pareto optimal solution. Our method gives the better values of objectives than other methods. Two numerical examples are given to illustrate the method comparison with some existing methods is also made.*

**Keywords:** *Multicriteria distribution problem, Membership function, Geometric mean, Fuzzy environment.*

## I. INTRODUCTION

Operations research is a scientific approach of decision making which seeks to determine how best to design and operate a system under condition requiring allocation of scarce resources. Operations research is a field, primarily has a set or collection of algorithms which act as tools for problem solving in chosen application areas. O.R. has extensive applications in Engineering, business and public systems. O.R. is also used by manufacturing and service industries to solve their day to day problem. Transportation problem is one of them. The transportation problem is a distribution problem.

To distribute the several measures of lonely homogenous items that are initially available at different places to various destinations so that the complete transportation cost/ time/ distance is minimum. All the distribution problems are not having single objective. The distribution problems having multiple objective functions are taken here. The multi-objective transportation model is set to tackle the transportation problem all the while related with a few destinations. Typically, existing multi-target transportation models utilize a minimization of the all out cost objective as one of their destinations. Different goals may worry about conveyance time, amount of products conveyed, underused limit, unwavering quality of conveyance, vitality utilization, security of conveyance, and so forth. The basic T. P. was initially developed by Hitchcock (1941). Profitable table of solution was derived from the simplified algorithm and were progressive in (1947). Dantzig (1963) used the primal simplex T.P. of simplified method in the T.P. An IBFS for the TP can be obtained by using the N.W.C.R, row minimum, column minimum, Vogel's Method. ARSHAM et al (1989) found a new algorithm for determining the solution of TP. This method re-examined the use of LP to solve the problem of transportation. LI et al (2000) proposed a fuzzy compromised programming approach to MOTP. The decision makers preference is taken in to account by assigning the weights to objectives. Surapati et al (2008) presented a preference based fuzzy GPA for solving a MOTP with fuzzy coefficients. They explain the membership function for the fuzzy goal. This approach transforms membership functions into membership goals. To give the appropriate preference structure of goals, the Euclidean distance function is used. K.B. Jagtap et al (2017) presented a review on different techniques used in MOTP. All possible work on MOTP has been collected and an overview on different approaches like goal programming, fuzzy techniques and genetic algorithm has been published in this paper. P. Pandian (2012) gave a new approach to find a fair solution of MOTP. In this method it is proposed to make a sum of objectives. New row maxima method was given by Patel et al (2018) to solve MOTP. M. Afwat et al (2018) proposed product approach to solve MOTP using fuzzy membership function. Kaur et al (2018) proposed a simple approach to obtain the best compromise solution of linear MOTP. Khilendra et al (2019) gave Matrix maxima method

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with a pareto optimality criteria to solve MOTP.

## II. RESEARCH METHODOLOGY

### A. Mathematical Description

Mathematically the MOTP can be defined as:

$$\text{To min } G^K(x) = \sum_{i=1}^r \sum_{j=1}^s C_{ij} x_{ij}$$

$$K = 1, 2, 3, \dots, m$$

subject to the constraints

$$\sum_{j=1}^s x_{ij} = a_i, \quad i = 1, 2, 3, \dots, r$$

$$\sum_{i=1}^r x_{ij} = b_j, \quad j = 1, 2, 3, \dots, s$$

$$x_{ij} \geq 0, \quad i = 1, 2, 3, \dots, r; \quad j = 1, 2, 3, \dots, s$$

$$\text{and } \sum_{i=1}^r a_i = \sum_{j=1}^s b_j$$

where

$a_i (i = 1, 2, 3, \dots, r)$  denotes the no. of units present at various origins.

$b_j (j = 1, 2, 3, \dots, q)$  denotes the no. of demands at various destinations.

$x_{ij}$  is the no. of units distributed from the  $i^{\text{th}}$  origin to  $j^{\text{th}}$  destination.

The objectives (cost, time and distance) are converted into membership values by using a fuzzy membership function to minimize a set of objectives. The membership function is defined as

$$M_r(X_{ij}^r) = \begin{cases} 1, & x_{ij}^r \leq L_r \\ \frac{U_r - x_{ij}^r}{U_r - L_r}, & L_r \leq x_{ij}^r \leq U_r \\ 0, & x_{ij}^r \geq U_r \end{cases}$$

where

$L_r$  is the lowest crisp value of  $x_{ij}^r$  &  $U_r$  is the highest crisp value of  $x_{ij}^r$ .

### B. Proposed Algorithm

Step 1: Calculate the membership value for each cell & each objective using the membership function.

Step 2: Find the geometric mean of membership values of corresponding cells of objectives using the formula  $M = (M_1 M_2 M_3)^{1/3}$  and make a new matrix.

Step 3: Make the difference D of highest and second highest element for each row and each column.

Step 4: Select the minimum difference and corresponding to this minimum difference, select the maximum element and at this maximum element allocate as much as possible (min of  $a_i$  &  $b_j$ ).

Step 5: Again repeat step 4 for remaining matrix.

Step 6: Repeat these steps until all the deficiencies are removed. Now we find an initial solution.

Step 7: Find the matrix for the average of objectives (cost, time and distance).

Step 8: We put the initial solution (Values of  $X_{ij}$ ) at this matrix of average of objectives.

Step 9: Now we check for optimality by using uv method.

Step 10: If solution is not optimal then we amend the solution until optimality condition is satisfied.

Step 11: Now a Pareto Optimal Solution is obtained.

## III. RESULT AND DISCUSSION

In section II Mathematical description is given and thereafter we defined our proposed algorithm. Our defined algorithm is applied on two multi-criteria transportation problem and the results are shown by two numerical examples given in section III. In numerical example 1, table 1, 2 & 3 give the data for cost, time and distance respectively. Table 4, 5, 6 show the membership values for the objectives. Table 7 gives the geometric mean of objectives for each cell. In table 8 we made the allocations according our proposed algorithm and an initial solution is found. From table 9 to 12, remaining steps of our algorithm are applied to check the optimality and to improve the solution and a pareto optimal solution is found. Table 13 gives the comparison among our method and some other methods available in literature. It is clear that values of cost and time obtained by our method are sufficiently less than two other methods and the value of distance covered is nearly equal to the product approach. In numerical example 2, table 14, 15 & 16 show the data for objective 1, 2 & 3 respectively. Our algorithm is applied from table 17 to 27 and we obtained a pareto optimal solution. Table 28 gives the comparison between our method and other existing methods. We observe that our method gives the minimum values for cost, time and distance.

### A. Numerical Example 1

We take a distribution problem in which a single homogeneous item is to be distributed from three stores (A, B, C) to four different warehouses (P, Q, R, S). Cost, time and distance for each unit transported is given in the table. Find the minimum time, cost and distance.

Table 1: Cost

To \ From	P	Q	R	S	Supply (a <sub>i</sub> )
A	21	16	15	13	11
B	17	18	24	23	13
C	32	27	18	41	19
Demand (b <sub>j</sub> )	6	10	12	15	43

**Table 2: Time**

To \ From	P	Q	R	S	Supply (a <sub>i</sub> )
A	1	2	1	4	11
B	3	3	2	1	13
C	4	2	5	9	19
Demand (b <sub>j</sub> )	6	10	12	15	43

**Table 3: Distance**

To \ From	P	Q	R	S	Supply (a <sub>i</sub> )
A	11	13	17	14	11
B	16	18	14	10	13
C	21	24	13	10	19
Demand (b <sub>j</sub> )	6	10	12	15	43

Now using the membership function we calculate the membership values as:

**Table 4: Membership values for cost**

To \ From	P	Q	R	S	Supply
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Now applying remaining steps of our proposed method

**Table 8: Making allocations using our algorithm**

To \ From	P	Q	R	S	Supply (a <sub>i</sub> )	D		
A	6(0.8721)	5(0.8499)	0.7743	0.7643	11/5/0	0.0222	0.0222←	0.0756←
B	0.7162	0.6415	0.8447	13(0.8631)	13/0	0.0184←	----	----
C	0.3505	5(0)	12(0.6859)	2(0)	19/7/2/0	0.3354	0.3354	0.6859
Demand (b <sub>j</sub> )	6/0	10/5/0	12/0	15/2/0	43			
D	0.1559	0.2084	0.0704	0.0988				
	0.5216	0.8499	0.0884	0.7643				
	----	0.8499↑	0.0884↑	0.7643↑				

The solution is:

$$X_{11} = 6, X_{12} = 5, X_{24} = 13, X_{32} = 5, X_{33} = 12, X_{34} = 2.$$

Minimum Cost = 938 units, Minimum Time = 117 units, Minimum distance = 457 units

Now we will find pareto optimal solution

**Table 9: Matrix for average of objectives**

To \ From	P	Q	R	S	Supply (a <sub>i</sub> )
A	11	10.33	11	10.33	11
B	12	13	10	11.33	13
C	19	17.67	12	20	19
Demand (b <sub>j</sub> )	6	10	12	15	43

**Table 10: To put our initial solution on table no. 9**

To \ From	P	Q	R	S	Supply (a <sub>i</sub> )
A	0.7143	0.8929	0.9286	1	11
B	0.8571	0.8214	0.9643	0.6429	13
C	0.3214	0.5	0.8214	0	19
Demand (b <sub>j</sub> )	6	10	12	15	43

**Table 5: Membership values for time**

To \ From	P	Q	R	S	Supply (a <sub>i</sub> )
A	1	0.875	1	0.625	11
B	0.75	0.75	0.875	1	13
C	0.625	0.875	0.5	0	19
Demand (b <sub>j</sub> )	6	10	12	15	43

**Table 6: Membership values for distance**

To \ From	P	Q	R	S	Supply (a <sub>i</sub> )
A	0.9286	0.7857	0.5	0.7143	11
B	0.5714	0.4286	0.7143	1	13
C	0.2143	0	0.7857	1	19
Demand (b <sub>j</sub> )	6	10	12	15	43

**Table 7: Geometric mean of membership values**

To \ From	P	Q	R	S	Supply (a <sub>i</sub> )
A	0.8721	0.8499	0.7743	0.7643	11
B	0.7162	0.6415	0.8447	0.8631	13
C	0.3505	0	0.6859	0	19
Demand (b <sub>j</sub> )	6	10	12	15	43

To \ From	P	Q	R	S	Supply (a <sub>i</sub> )
A	6(11)	5(10.33)			11
B				13(11.33)	13
C		5(17.67)	12(12)	2(20)	19
Demand (b <sub>j</sub> )	6	10	12	15	43

**Table 11: To test for optimality of this**

**solution by UV method**

To \ From	P	Q	R	S	$(u_i)$
A	11	10.33	4.66  6.34	12.66  -2.33	-7.34
B	9.67  2.33	9  4	3.33  6.67	11.33	-8.67
C	18.34  0.66	17.67	12	20	0
$(v_j)$	18.34	17.67	12	20	

Since  $D_{14} < 0$ , the given solution is not optimum. Now we will amend the solution.

**Table 12: Improved Solution**

To \ From	P	Q	R	S	Supply $(a_i)$
A	6(11)	3(10.33)		2(10.33)	11
B				13(11.33)	13
C		7(17.67)	12(12)		19
Demand $(b_j)$	6	10	12	15	43

Again we applied the test for optimality & the solution is found to be optimal.

The improved solution is given as:

$$X_{11} = 6, X_{12} = 3, X_{14} = 2, X_{24} = 13, X_{32} = 7, X_{33} = 12.$$

Following are the values of objectives:

Minimum Cost = 904 units, Minimum Time = 107 units, Minimum distance = 587 units

**Table 13: Comparison between different methods**

Method	Minimum cost	Minimum time	Minimum Distance
New row Maxima method [9]	938	117	457
Product Approach [10]	938	132	552
Our method (Geometric mean method)	904	107	587

## B. Numerical Example 2

Now we take one more example with following characteristics:

**Table 14: Cost**

To \ From	P	Q	R	S	Supply $(a_i)$
A	6	4	1	5	14
B	8	9	2	7	16

Now applying remaining steps of our proposed method

C	4	3	6	2	5
Demand $(b_j)$	6	10	15	4	35

**Table 15: Time**

To \ From	P	Q	R	S	Supply $(a_i)$
A	13	11	15	20	14
B	17	14	12	13	16
C	18	18	15	12	5
Demand $(b_j)$	6	10	15	4	35

**Table 16: Distance**

To \ From	P	Q	R	S	Supply $(a_i)$
A	6	3	5	4	14
B	5	9	2	7	16
C	5	7	8	6	5
Demand $(b_j)$	6	10	15	4	35

**Table 17: Membership values for cost**

To \ From	P	Q	R	S	Supply $(a_i)$
A	0.375	0.625	1	0.5	14
B	0.125	0	0.875	0.25	16
C	0.625	0.75	0.375	0.875	5
Demand $(b_j)$	6	10	15	4	35

**Table 18: Membership values for time**

To \ From	P	Q	R	S	Supply $(a_i)$
A	0.7778	1	0.5556	0	14
B	0.3333	0.6667	0.8889	0.7778	16
C	0.2222	0.2222	0.5556	0.8889	5
Demand $(b_j)$	6	10	15	4	35

**Table 19: Membership values for distance**

To \ From	P	Q	R	S	Supply $(a_i)$
A	0.4286	0.8571	0.5714	0.7143	14
B	0.5714	0	1	0.2857	16
C	0.5714	0.2857	0.1429	0.4286	5
Demand $(b_j)$	6	10	15	4	35

**Table 20: Geometric mean of membership values**

To \ From	P	Q	R	S	Supply $(a_i)$
A	0.5	0.8122	0.6822	0	14
B	0.2877	0	0.9196	0.3816	16
C	0.4297	0.3624	0.3099	0.6934	5
Demand $(b_j)$	6	10	15	4	35

**Table 21: Making allocations using our algorithm**

To From	P	Q	R	S	Supply ( $a_i$ )	D			
A	6(0.5)	8(0.8122)	0.6822	0	14/8/0	0.13	0.13←	----	----
B	0.2877	1(0)	15(0.9196)	0.3816	16/15/0	0.538	0.538	0.538	0.9196
C	0.4297	1(0.3624)	0.3099	4(0.6934)	5/1/0	0.2637	0.331	0.331	0.0525←
Demand ( $b_j$ )	6/0	10/2/1/0	15/0	4/0	35				
D	0.0703↑	0.4498	0.2374	0.3118					
	----	0.4498	0.2374	0.3118					
	----	0.3624	0.6097	0.3118↑					
	----	0.3624↑	0.6097↑	----					

The solution is

$$X_{11} = 6, X_{12} = 8, X_{22} = 1, X_{23} = 15, X_{32} = 1, X_{34} = 4.$$

Minimum Cost = 118 units, Minimum Time = 426 units, Minimum distance = 130 units

Now we will find pareto optimal solution

**Table 22: Matrix for average of objectives**

To \ From	P	Q	R	S	Supply ( $a_i$ )
A	8.33	6	7	9.67	14
B	10	10.67	5.33	9	16
C	9	9.33	9.67	6.67	5
Demand ( $b_j$ )	6	10	15	4	35

**Table 23: To put our initial solution on table no. 22**

To \ From	P	Q	R	S	Supply ( $a_i$ )
A	6(8.33)	8(6)			14
B		1(10.67)	15(5.33)		16
C		1(9.33)		4(6.67)	5
Demand ( $b_j$ )	6	10	15	4	35

**Table 24: To test for optimality of this solution by uv method**

To \ From	P	Q	R	S	( $u_i$ )
A	8.33	6	<u>0.66</u>    <u>6.34</u>	<u>3.34</u>    <u>6.33</u>	-3.33
B	<u>13</u>    <u>-3</u>	10.67	5.33	<u>8.01</u>    <u>0.99</u>	1.34
C	<u>11.66</u>    <u>-2.66</u>	9.33	<u>3.99</u>    <u>5.68</u>	6.67	0
( $v_j$ )	11.66	9.33	3.99	6.67	

Since  $D_{21} < 0$  and  $D_{31} < 0$  the given solution is not optimum.

Now we will amend the solution.

**Table 25: Improved Solution**

To \ From	P	Q	R	S	Supply ( $a_i$ )
A	5(8.33)	9(6)			14
B	1(10)		15(5.33)		16
C		1(9.33)		4(6.67)	5
Demand ( $b_j$ )	6	10	15	4	35

**Table 26: Again test for optimality by uv method**



From \ To	P	Q	R	S	( $u_i$ )
A	8.33	6	<u>3.66</u>   <u>3.34</u>	<u>3.34</u>   <u>6.33</u>	-3.33
B	10	<u>10.99</u>   <u>-0.32</u>	5.33	<u>5.01</u>   <u>3.99</u>	-1.66
C	<u>11.66</u>   <u>-2.66</u>	9.33	<u>6.99</u>   <u>2.68</u>	6.67	0
( $v_j$ )	11.66	9.33	6.99	6.67	

Since  $D_{22} < 0$  and  $D_{31} < 0$  the given solution is not optimum. Again we will improve the solution.

Now the pareto optimal solution is given as:

$$X_{11} = 4, X_{12} = 10, X_{21} = 1, X_{23} = 15, X_{31} = 1, X_{34} = 4.$$

Now the values of objectives are as follows:

Minimum Cost = 114 units, Minimum Time = 425 units, Minimum distance = 118 units

**Table 28: Comparison between different methods**

Method	Minimum cost	Minimum time	Minimum Distance
New row	122	461	130
Maxima method [10]			
Product Approach [11]	114	425	128
Our method (Geometric mean method)	114	425	118

#### IV. CONCLUSION

In this we introduced a new method to solve MOTP named as Geometric mean method to solve MOTP under fuzzy environment. We have taken a problem with three objectives. We also gave a procedure to improve the solution by which a pareto optimal solution can be determined. The method very simple and easy to use. Our proposed method gives better values than other methods available in literature for some of the objectives.

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