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Analysis of the Arrival Geo/G/1queue with Geometric Distributed Multiple

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Abstract: The paper represents a batch appearance Geo/G/1 queuing structure with Geometric distribution numerous working arrival model is analysis. The organization distribution is inferred by utilizing the Markov chain process, different execution measures including expected organization size are determined. The distribution of the system states observed that such specific points is in reality indistinguishable with the distribution of the system states noticed at an capricious point on the continuous time domain. This is because an capricious point on the continuous time space falls somewhere in the halfway of a slot with probability 1. The organization state noticed at such a point is equivalent to that noticed immediately after the preceding slot boundary.

Keyword: Time Averages, Transmission Line, Signals, Network, Transportation, System States.

I. INTRODUCTION

The properties of the length of a full of activity phase and the numeral of messages served in a busy phase. The length of a delay cycle is also analyzed. The distribution's waiting time of a message in a last come first served system, and that of a message that arrives during a delay cycle in systems with various service disciplines. A separate treatment is given to special $Geo^x/G/1$ system in which the service time is always one slot, which is called a packet model. In a late arrival model, a busy period is started at a slot boundary following the arrival of messages when the system is empty, and is terminated when a repair is finished and the organization is vacant. An idle era in the time period between two successive busy period lengths of a busy period does not depend on the service discipline

II. DISCRETE-TIME SYSTEM

Consider various discrete-time systems, which the time pivot is fragmented into a sequence of equivalent intervals of unit term, called slots. It is constantly expected that administrations and vacations could be ongoing just slot limits and their spans are integral multiple of slot length. Hunter mentions that the machine cycle time's processor, the bit or byte length's signals on a channel or transmission line, and the pulse span of any fixed size information unit are instance of the normal elementary unit of time in the field of computers and communications,

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slot CSMA (carrier-sense multiple-access) and TDMA (time division multiple-access) have been widely studied as intermediate access control (MAC) protocols for packet radio network and local region computer networks. More recently in broadband included services digital networks (ISDN), asynchronous-transfer-mode (ATM) switches handle fixed-size data units of 53 octets provided by time discredited multiplexers summarize early applications of discrete-time queuing systems to dam models, fixed cycle traffic light problems and railroad carpool systems in the area of civil and transportation engineering. Let us call the entity of service a message, whose service time is constantly an integral multiple of slot length. Numbers of messages that arrive during successive slots constitute a sequence's independent and identically distributed random variables. If the number of message that arrive during a slot is 1 with probability α (0< α <l) and 0 with probability 1-α, plainly that the inter arrival time I is geometrically distributed with a mean of 1/ α, slots,

$\mathrm{Prob}[= l\ slots] = \alpha(1-\alpha)^{(l-1)}$……(1)

Where $l = 1,2, \dots \dots$

Such an arrival process is called Bernoulli process. A single-server discrete-time organization with a Bernoulli arrival process and usually distributed repair time is denoted by Geo/G/1. If more than one message can reach your destination during a slot, here a bunch arrival Geo/G/1 organization, which is defined by $Geo^x/G/1$. A $Geo^x/G/1$ system is the basis of all the extended models. In discrete-time queuing systems, one of the earliest investigations being provide surveys of applications to computer communication networks. Analyze many discrete-time techniques involve normalized ones with non independent arrivals, server adjourns & multiple servers.

III. GE0/G/1 and BUNCH ARRIVAL GEO/G/1

In Geox/G/1 techniques, define by β the numeral of message that arrive during a one slot.

The probability distribution of β is

$\alpha(k) = \mathrm{Prob}[\beta = k]$ (2)

Where $k = 0, 1, 2, \dots \dots \dots$

$\beta(z) = \sum_{k=0}^{\infty} \alpha(k)\, z^k$ (3)

β (z) is PGF (the probability generating function) of β and described by α and α (i) is mean & ith factorial moment, with respect of β that is

$\alpha = E[\beta] = \beta^{(1)}(1)$ (4)

$\alpha^2 = E[\beta(\beta-1)] = \beta^2(1)$ (5)

$\alpha^i = E[\beta(\beta-1) \dots \dots \dots (\beta-(i-1)] = \beta^i(1), i = 3,4 \dots \dots$ (6)

The Geo/G/1 system is a particular type of the Geox/G/1 system in which

β (z) = 1 - α□+ αz (6)

Let X be the service time (measured in slots) of each message. all service is begin and finished at exact slot limits. The probability distribution of X is

$B(u) = \sum_{l=1}^{\infty} b(l)u^l$, $|u| \leq 1$ (7)

Where $b(l) = prob(X = l)$, $l = 1, 2 \ldots \ldots \ldots$

The offered load or the traffic power ρ is suppose to be not as much as unity for the stability condition

$\rho = \alpha b < 1$ (8)

If this condition is satisfied, the offered load is equivalent to the carried load or the server employ in Geox/G/1 techniques.

IV. LATE ARRIVAL MODEL

In late arrival model, the message arrives delayed during a slot, only before the finish of the slot. Hence, arriving messages see a departing message going to leave, the withdrawing message abandons the message that have quite recently arrived. In the event that a message arrives in the nth slot and the organization is vacant toward the finish of the nth slot, its administration is begun toward the start of the (n+1)st slot.

Let L_n be the numeral of message present in the framework(queue size) following the administration fulfillment of the nth message, where n= 1, 2…. If A_n signifies the numeral of message that arrive during the administration time of the nth message.

$$L_{n+1} = \begin{cases} A_n + 1 & L_n = 0 \\ L_n + b_{n+1} - 1 & L_n \geq i \end{cases}$$

(8)

Since the sequence (An n = 1, 2….) comprises of independent and identically conveyed random variables, the sequence (Ln; n = 1,2….) constitutes a homogenous Markov chain.

consider a steady-state distribution exists for Markov chain {Ln; n = 1,2,….} and that it is denoted by

$$\pi_k = \lim_{n\to\infty} Prob[L_n = k], \ \ k = 0, 1, 2 \ldots \ldots \quad (9)$$

$$\pi(z) = \sum_{k=0}^{\infty} \pi_k z^k$$

After the service completion is identical with the PGF P (z) of the queue size L following each slot limit in the Geo/G/1 system

$$P(z) \triangleq \sum_{k=0}^{\infty} Prob\ [L = k]z^k$$

(10)

$$P(z) = \frac{(1-\rho)(1-z)B(1-\alpha+\alpha z)}{B(1-\alpha+\alpha z)-z} \quad (11)$$

Therefore, the first and second moments of the queue size distribution immediately after an arbitrary slot boundary as

$$E[L] = \frac{\alpha^2 b^{(2)} - \alpha\rho}{2(1-\rho)} + \rho \quad (12)$$

The Little's theorem of Discrete time version is

$$E[L] = \lambda E[T] \quad (13)$$

Where T is the time (measured in slots) that a message stays in the system for arrival to service completion, called the response time of a message. Therefore,

$$E[T] = \frac{\alpha b^{(2)} - \rho}{2(1-\rho)} + b \quad (14)$$

This is independent of the service discipline. Furthermore, a simple of the Poisson arrivals see time mean property in the continuous-time framework, there is a Bernoulli arrivals see time mean or Geometric arrivals see time mean property in the discrete-time framework. Hence, the PGF ∏(z) of the queue size seen by an arriving message is also given by

$$\prod(z) = \frac{(1-\rho)(1-z)B(1-\alpha+\alpha z)}{B(1-\alpha+\alpha z)-z} \quad (15)$$

If we take the first come first serve order, the numeral of messages left in the organization following the service consummation of messages is identical with the quantity of messages that arrive in the time interval during which that messages was in the system. Therefore, if T(u) defines the PGF for time T that a message stay in the system, then

$$\Pi(z) = T(1 - \alpha + \alpha z) \quad (16)$$

And

$$T(u) = \frac{(1-\rho)(1-u)B(u)}{1-\alpha-u+\alpha B(u)} \quad (17)$$

A loose argument to justify this inequality is that an arriving message does not wait if it finds only one message in the system which is about to leave

V. EARLY ARRIVAL MODEL

In an early arrival model the messages arrive early during a slot. If the system is empty when a message arrives, its service is started immediately; include the slot in which the message arrives when we calculate it's waiting time. A sequence {Ln; n=1, 2…} of the queue size is now measured following the service consummation of the nth message & before the following possible arrival point. The Markov chain {Ln; n=1,2….} then satisfies the relation

$$L_{n+1} = \begin{cases} A'_{n+1} & L_n = 0 \\ L_n + A_{n+1} - 1 & L_n \geq 1 \end{cases}$$

(18)

Where A_{n+1} defines the numeral of messages that arrive during the administration time of (n+1)st message minus one slot. The coefficient of b(l) in the probability that K messages arrive during the last (l-1) slot's service time of the (n+1)th message that consists of l slots. The PGF for A_{n+1} is given by

Since we count the slot in which a message arrives when we calculate the waiting time in the early arrival model,

$$\prod(z) = \frac{T(1-\alpha+\alpha z)}{1-\alpha+\alpha z} \quad (19)$$

Where both sides represent the numeral of messages present in the system following a service completion.

$$T(u) = \frac{(1-\rho)(1-u)B(u)}{1-\alpha-u+\alpha B(u)} \quad (20)$$

Observe the queue size of an early arrival model following possible arrival point following each slot limit, it is also the queue size observed at an arbitrary point during the same slot on the continuous time domain.

VI. OUTPUT PROCESS

A sequence of those slot boundaries at which service is completed is called an output process or a departure process. Consider the interdeparture time of a Geo/G/I system, which is defined as the time interval between two successive service completions. The mean interdeparture time is given by 1/α because no message are created or lost in the system. The correlation of two contiguous interdeparture times, denoted by τ and τ'. Assuming that three successive service completions occur at the end of the n0th, (n0 + 0)th and (n0 + τ + 0')th slots, obtain the joint probability.

$$\Delta(u,u') \triangleq \sum_{k=1}^{\infty} \sum_{\lambda=1}^{\infty} \Pr ob\ [\tau = k, \tau' = l]\ u^k u'^{\lambda} \quad (21)$$

There are five cases regarding the numeral of messages in the organization at times once before and after ends of the n0th and (n0 + τ + 0')th slots and the number of messages that arrive during τ slots,

Case	n_0- 0	n_0+ 0	No. of arrivals during τ slots	n_0 + τ - 0	n_0 + τ + 0
i	1	0	1	1	0
ii	1	0	>2	>2	>1
iii	2	1	0	1	0
iv	2	1	>1	>2	>1
v	>3	>2	any	>2	>1

Denote by $L^{(n)}$, the queue size following the nth slot .

Case (i) : There are no message in the system following the n_0th slot; this case occurs with probability π_0. The probability that τ= k and that there are no messages in the system again following the (n_0 + k) th slot is given by

Case (ii): There is no message in the system following the n_0th slot. The probability that τ = k and that there is at least one message in the system following the n_0th slot is given by

$$Prob[\tau = k, L^{(k+n_0)} \geq 1] = \sum_{k=0}^{n} \alpha(1-\alpha)^{j-1} b(k - j)[1-1-\alpha]k-j \quad (22)$$

Since the next service is started at the (n_0 + k + 1)th slot, then

$$\Pr ob[\tau' = \lambda] = b(\lambda) \quad (23)$$

Case (iii): There is exactly one message in the system following the north slot with probability π_1. There probability that τ = k and that there are no messages in the system following the (n_0 + k) th slot is then

$$\text{Prob}\ [\tau = k, L^{(n_0+k)} = 0] = b(k)\ (1-\alpha)^k \quad (24)$$

Since an idle period is started at the (n_o + k +1)st slot,

Case (iv): There is exactly one message in the system following the north slot. The probability that τ = k and that there is at least one message in the system immediately after the (no + k)th slot is

$$\text{Prob}[\tau = k, L^{(k+n_0)} \geq 1] = b(k)[1-(1-\alpha)]^k \quad (25)$$

Since the next service is started at the (n0 + k + 1)th slot,

VII. LATE ARRIVAL MODEL

For a late arrival model, first consider the Markov chain { L_n; n = 1, 2 …} where L_n is the queen size following the sow ice consumption of the n^{th} message. If an defines the numeral of messages that arrive during the administration time of the n^{th} message & β' defines the numeral of messages that arrive in a slot given that in any one message arrives in that slot,

$$L_{n+1} = \begin{cases} A'_{n+1} & L_n = 0 \\ L_n + A_{n+1} - 1 & L_n \geq 1 \end{cases} \quad (26)$$

VIII. WAITING TIME IN THE FCFS SYSTEM

The distribution of the waiting time in the $Geo^x/G/1$ system with FCFS discipline could be handily gotten by accepting that a group of messages that arrive in the similar slot establish a Super message in a Geo/G/1framework. i.e., the PGF βg(z)& the average αg for the numeral of super messages that arrive in a slot in this Geo/G/1framework. The waiting time W of an discretionary message comprises of two independent parts. first is the waiting time Wg of a super message to which the arbitrary message belong the order, defined by J, in the add of the service time for the messages with in the similar super message that are served before the self-assertive message.

These parts are independent. If J(u) denotes PGF. So as to obtain J(u), the quantity of messages inside the super message that are served before the capricious message is similar to the forward repeat time in a discrete time restoration process when the interregnal time is given by the numeral of messages remembered for the super message. Consider a super message that includes L_n messages, and let An be the quantity of messages that arrive during the administration time of the nth message within the super message, where n = 1,2…. β.

The quantity L_n of messages in the system following the completion of service to the nth message with in the super message, excluding those that were there when the super message arrived, the numeral of messages in the system following the completion of service to an capricious message includes of the numeral of messages that arrive during the waiting time of the super message to which the arbitrary message belongs and the numeral of messages in the organization, following the completion of service to the arbitrary message.

IX. QUEUE SIZE AT AN ARBITRARY TIME

The PGF P(z) for the queue size observed immediately after every slot boundary. When observe the system follows a slot limit both possible arrivals and then a feasible service execution have already eventuated.

The distribution's system states distinguished at such specific points is in fact identical with the distribution's system states distinguished at an feasible point on the continuous time domain. This is because a feasible point on the continuous time domain falls somewhere in the mid of a slot with probability 1.

The framework state distinguished at such a point is the equivalent to that distinguished immediately after the preceding slot boundary, because noting can occur between the two points in the discrete time system.

X. RELATION TO THE CONTINUOUS TIME SYSTEM

Now consider the continuous time system by using the limit Δ→ ∞ firstly change the arrival and process. Since the numeral of slots per time unit is taken by 1/Δ, the PGF Δ_c (z) for the numeral of messages that arrive during a time unit is

$$\beta_c(z) = [\beta(z)]^{1/\Delta} \quad (27)$$

XI. EARLY ARRIVAL MODEL

For an early arrival model, let us first consider the Markov chain $\{L_n;\ n = 1,2....\}$ of the queue size L_n following the nth service completion and before a potential arrival in the start of the accompanying slot.

$$L_{n+1} = \begin{cases} \beta' - 1 + A'_{n+1}\ L_n = 0 \\ L_n + A_{n+1} - 1 \quad L_n \geq 1 \end{cases} \qquad (28)$$

where A' denotes the number of messages that arrive in a slot taken that in any one message arrives in that slot, and A_{n+1} indicates the numeral of messages that arrive during the service time of the (n+1)st message given that Ln= 0. A_{n+1} is the numeral of messages that arrive during the j-1 last slots if the service time of the (n+1)st message is j slots. Hence the PGF A'(z) for A_{n+1} is

$$A' = \sum_{j=1}^{\infty} b(j)[\beta(z)]^{j-1} = \frac{B[\beta(z)]}{\beta(z)} \qquad (29)$$

From the normalization condition ($\Pi_{(0)} = 1$)

$$\Pi_{(0)} = \frac{[1-\alpha(0)][1-\rho]}{\alpha(0)\alpha}$$

Hence, $$\Pi(z) = \frac{(1-\rho)[1-\beta]B[\beta(z)]}{\alpha\beta(z)\{B[\beta(z)]-z\}} \qquad (30)$$

The state of the system in terms of the queue size & the remaining service time observed at an arbitrary point on the continuous time domain is identical with that observed following a feasible arrival point following each slot limit in the early arrive model. Consider $L^{(n)}$ be the queue size following a possible arrival point during the nth slot and $X^{(n)}_+$ be the abiding service time of the message being served at the same instant where n = 1,2.... The limiting distribution of $\{L^{(n)}, X^{(n)}_+ ;\ n = 1,2....\}$ is used by P_0 and Π (z,u) in which is also the joint distribution's queue size and the remaining service time at an capricious point on the continuous time domain. The PGF P(z) of the queue size observed at an capricious point on the continuous time domain is given by .

Hence we reach the following conclusion about the comparison of the late and early arrival models. In the uniform state, the joint distribution's queue size and the remaining or progressed service time at an capricious point on the continuous time domain are the alike in both models. The distribution's waiting time of an arbitrary message is also the same in both models. The queue size following the service completion in the early arrival model is smaller than that in the late arrival model by the number of messages included in a possible arrival. This is because the observation point in the early arrival model is located before the possible arrival point.

XII. CONCLUSION

Since 2002, some authors have worked on working vacation queues utilizing geometric distribution method and Embedded Markov chain method. In any case, the methodology utilized by them isn't simple and adequate to get the outcomes in closed structure. Concerning the Non Markovian queues, the most broadly utilized apparatus is strengthening system. In the paper, we have investigated the bunch arrival Geo/G/1 queue with numerous working vacations. The probability generating function of the framework size probabilities are determined and exhibited in closed structure. Further different execution measures including anticipated framework length, probability that server is occupied & on vacation is acquired and numerical investigation is completed. The outcomes acquired in the paper can be extended to unreliable server in future research.

REFERENCES

1. Adar, Z., N. Ahituv, and O. Berman, "The value of an Information system for service Network," Computers and Operations Research 12(1), pp 72-85, 1985.
2. Berman and M.R. Rahman, "Optimal location-relocation decision on stochastic network," International Journal of Transportation of science, 19(3), pp203-221, 1985.
3. Berman and M.R. Rahman, " Optimal path of single service unit on a network to a Nonemergency destination," International Journal of Transportation of science, 17(2), pp 218-232, 1983.
4. Green, L., "A multiple dispatch queuing model of policy patrol Operation" Management science, 30 (6), pp 653-664, June2001.
5. Green, L., and Kolesar, "A comparison of the multiple dispatch and MIMIC priority queuing models of police patrols" Management science, 30(6), pp 665-670,June 1984.
6. Hall and R.W. "Direct versus terminal freight routing on a network with concave costs" Operation Research, 21, pp 287-298, 2009.
7. Jarvis, J.P., "Optimal assignment in a Markovian queuing system" Computers and Operations Research 8, pp 17-23, 1991.
8. Jordan and W.C. "Scale economics on multi-commodity distribution network" Computers and Operations Research, pp 27-35, 1986.
9. Larson and R.C. "A hypercube queuing model for facility location and redistricting in Urban emergency services" Computers and Operations Research 1, pp 11-23, 1999.
10. Halpern and J, "The accuracy of estimates for the performance criteria on certain emergency service queuing system" transportation Science 11, pp 223-242, 1999.
11. Hill and D.M. "Growth allocation model for the Boston" Journal of the American Institute of Planners 31, pp 11-120 1998.
12. Goldman and P.R. "A Domination theorem for optimal location" Journal of Operation Research 2, pp 213-220, 1999.
13. Green and L. "A Queuing system in which customers require a random number of services" Journal of Operation Research 1, pp 325-335, 1998
14. Geoffrion and A.M. "Proper efficiency and the theory of vector maximization" Journal of Mathematics analysis and applications 22, pp 618-623, 1999.
15. Chruch and R.L. "Locating on obnoxious facility on a network" Journal of Transportation of Sciences 12, pp 107-118, 2010.
16. Ompal Singh "Characterization of Model on Location Facilities" Pertinika Journal of science and technology 27(3), pp 1451-1458, 2019.
17. Ompal Singh "Characterization of Model on Zoning Process" Pertinika Journal of science and technology 27(4), pp 2373-2379, 2019.
18. Julia Rose Mary.K. and Afthab Begum.M.I., "MX/M/1 queue with working vacations", Acta Ciencia India, Vol.No.3, 429-439, 2010.
19. Servi.L.D and Finn .S.G., "M/M/1 queue with working vacations (M/M/1/wv)" Performance evaluations, Vol 50: p.p 41 -52, 2003.
20. Takagi.H., "M/G/1 queue with multiple working vacations" Performance Evaluations, 63 (7),p.p 654 -681, 2006.
21. 34. Mukesh Sharma, K.K.Bansal: "Numerical Solution for Reliability and Availability Analysis of Three Unit Standby System " International Journal of Engineering and Technology (IJET) Vol.9 Issue 3 (2017) pp 1804-1808
22. Ompal Singh, Bansal K.K.,Kumar S."Algorithm of Model on Zoning Process"IJEAT , Vol.-8, Issue-2S2, pp-26-28 2019