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Applying Haar Wavelets in Tasks of Digital Processing of Two-Dimensional Signals

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Abstract: In this article, a system application of local basis functions discussed, defined on compact media.Applying formulas for estimating the accuracy of calculating the spectral energy by the method of two-dimensional Haar wavelets and main concepts of Haar fast transformation, spectral energy of Haar coefficients will be discussed in this article. Their effectiveness will be shown in order to solve problems of sampling of compact signals. Signals cannot only be functions of time, defined at finite intervals, but also functions of arguments of a different physical nature, for example, distances along surfaces.

Keywords: compact spectrum, finite function, signal energy, approximation, Haar wavelets, wavelet fast transform.

I. INTRODUCTION

Under digital processing of multidimensional signals (DPMS) understand the processing of information presented in the form of multidimensional arrays of numbers, for example, the results of measurements of data that continuously change in time and space and come from several sources [1,2,5,9,12,13].

Significant differences in the processing of multidimensional signals compared to one-dimensional ones can be reduced to the following main factors:

1) As the dimension of the data increases, the amount of numerical information increases dramatically.

2) Complicated mathematical processing methods, as result of which, as a rule, errors increase, the reliability of the results weakens.

The traditional and simplest way to organize multidimensional samples is rectangular discretization, when the information carriers on the plane are squares, rectangles, and in spaces of higher dimension - parallelepipeds, hypercube, etc.

As noted by most experts in the processing of one-dimensional signals – functions of time, the principle of finiteness of the spectrum is very effective [1, 3, 6, 7, 9, and 10]. Its limitations, usually satisfied when processing one-dimensional processes, work very differently in multidimensional areas, where physical quantities must be measured in the multidimensional “space-time”. In addition, there are research areas where time is specifically excluded from consideration in order to obtain reliable solutions to spatial problems.

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II. SPECTRAL ENERGY OF THE VECTOR OF HAAR COEFFICIENTS

Significant progress in the use of wavelets in various applications is connected, firstly, with the availability of fast discrete spectral transform algorithms, whose class is much wider than the set of fast transforms in the basis of complex exponential functions [8, 9, and 10]. In order to solve the problem of organizing the minimum signal samples providing the necessary accuracy of reconstruction, it is necessary to study the intrinsic spectra of wavelet coefficients. Such systems of wavelet functions, such as derivatives of the Gaussian function, Morlet wavelets, Shannon wavelets, etc. Theoretically, it is defined on the entire axis, but it can be considered as local. However, the main role in discrete wavelet fast transform (WFT) algorithms is played by orthonormal wavelet bases defined on compact media.

Applying the energy criterion for the accuracy of signal reconstruction from wavelet coefficients, two main operators we need: multi-scale analysis [5] and calculation of energy octave spectrum [9]. The advantage of the octave spectrum is that it, like the Fourier spectrum, is invariant with respect to time shifts of stationary signals. Several wavelets considered on the entire axis $t \in (-\infty, \infty)$, for example, Shannon wavelets, also possess the property of a small-scale analysis [10].

We transform the continuous signal f (x) to a discrete form - imagine it as a row vector containing n real numbers f_i, i = 0,1, ..., n-1. The fast wavelet transform algorithms actually use integer iterations of a single scaling operator D_σ (σ>1), describing stretching [8, 11]. Commonly used scale is σ=2, at which the mother wavelet satisfies the identity:

$$D_2\psi(t) = \sum_{k=0}^{n-1} c_k \psi(t-k). \qquad (1)$$

For functions $f \in L^2(R)$, the partial sum with wavelet coefficients c_k is interpreted as the difference between two approximations f – c resolutions 2^{-j+1} and 2^{-j}, and a multi-scale analysis uses sets of approximation grids. An approximation with a resolution of 2^{-j} contains all the necessary information for calculation with a coarser resolution of 2^{-j-1}.

Figure 1, shows the graph of the Haar fast transformation (HFT) for n = 2^3 = 8 samples with the addition of operators for calculating the components of the octave spectrum.

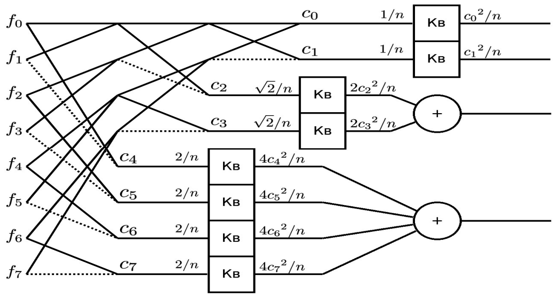

Fig. 1.Haar Fast transform graph: c_k – Haar coefficients, Кв – quadratics, E_s – squared values of the octave components of the spectral energy of a discrete signal

The coefficient values are obtained as a result of using a special wavelet transform program written in the C# programming language. In Fig. 2, the histogram is shown that half values of the coefficients of the Haar fast transform (HFT) of the initial vector $\{f_i\}$ containing n = 2^p = 128 samples. It is said, "the signal has a length of 128". The exponent p, which means the maximum number of iterations, is called the discrete transformation order. Expression for the total over all octaves of spectral energy of the vector of Haar coefficients $\{c_k\}$, represents the sum of quadratic form:

$$E_\varepsilon = ((c_0^2 + c_1^2) + 2^{-1}\left(c_2^2 + c_3^2\right) + 2^{-2}\sum_{k=4}^{2^{p-4}-1} c_k^2 + 2^{-3}\sum_{k=8}^{2^{p-3}-1} c_k^2 + \ldots + 2^{-p}\sum_{k=2^{p-1}}^{2^p-1} c_k^2)n. \quad (2)$$

Its numerical value is equal to Eε=136.887.

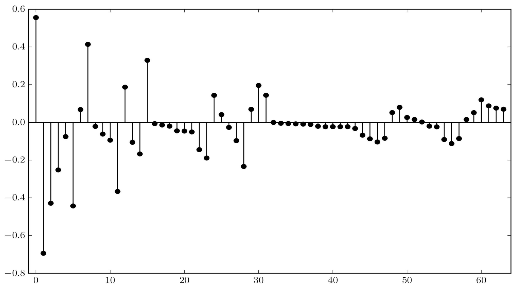

Fig. 2. Haar wavelet coefficients

We perform the Haar fast transformation again for the grid of samples, 2 times rarer, i.e. for even values of the previous signal vector (the signal length is 64). In this case, the energy value is E_ε = 136.375.

III. EXAMPLE FOR PROCESSING TWO-DIMENSIONAL SIGNALS

Wavelets are mathematical functions as a class were opened geophysicists Morlet and Grossman in the study of seismic waves. Seismic exploration is one of the areas of application of mathematical methods in solving geophysical problems of prospecting for minerals, which include gravity exploration, electrical exploration, magnetic exploration. Magnetic exploration is the most effective method from the point of view of high performance when measuring physical field parameters above the surface. The basis of mathematical transformations are methods for solving systems of partial differential equations written, for example, in the form:

$$\frac{\partial^2 F_i(x,y,z,t)}{\partial x \partial y} = f_i\left(x,y,z,t\right), \quad i = 1,2,\ldots,N, \quad (3)$$

Where x, y, z are the spatial coordinates of the four-dimensional continuum, and t is the time. Searching methods consist of detecting geophysical anomalies in signals. For this, it is necessary to exclude time from consideration as a factor that interferes with the constantly acting field components and leave only spatial variables in the equations. For example, from the data of magnetic measurements such effects on the readings of magnetometers as:

-a secular variation of the magnetic field (pole shift);

-magnetic field variations in time (fluctuations during the day);

-the effect of the altitude of the aircraft above the surface;

-deviation functional dependence caused by the influence of the magnetic field of the aircraft, etc.

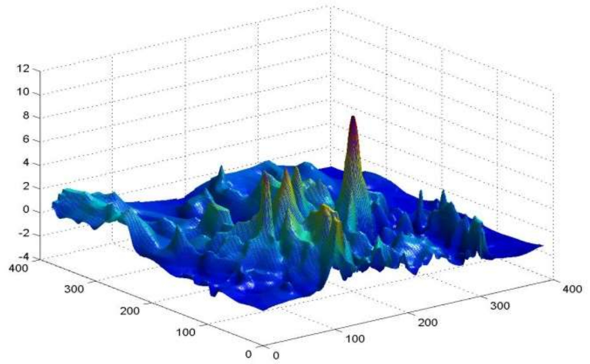

Fig. 3.Graph of the magnetic induction field, measured by aeromagnetic.

Figure 3 shows a picture of the complex field of the values of magnetic induction induced on one of the sections of the Earth's surface and measured by the method of aeromagnetic exploration. Unit of measurement – micro-Tesla. The two-dimensional array f (x, y) includes 300 x 300 samples with distances of 1 km between adjacent samples. On the graph (Fig. 3), we select a square section with a size of 16 x 16 samples containing one local maximum of the function, and apply formulas for estimating the accuracy of calculating the spectral energy by the method of two-dimensional Haar wavelets. Magnetometers can measure field intensity to the nearest 6 decimal places. In table 1, shows the results of magnetic data f (x, y). The graph of this field section is shown in Fig. 4.

Table 1.

$$CW_H = \begin{bmatrix} 1.9168 & 0.1198 & 0.1199 & 0.1198 & 0.1204 & 0.1206 & 0.1205 & 0.1206 & \dots \\ 0.0063 & -0.0029 & -0.0018 & -0.0029 & 0.0021 & 0.0011 & 0.0017 & 0.0026 & \dots \\ -0.0028 & -0.0047 & -0.0042 & -0.0047 & -0.0030 & -0.0025 & -0.0020 & -0.0018 & \dots \\ -0.0018 & 0.0022 & 0.0025 & 0.0022 & 0.0031 & 0.0033 & 0.0035 & 0.0037 & \dots \\ -0.0002 & -0.0018 & -0.0019 & -0.0019 & -0.0018 & -0.0017 & -0.0015 & -0.0014 & \dots \\ -0.0002 & -0.0002 & -0.0002 & -0.0002 & -0.0001 & -0.0001 & -0.0001 & -0.0001 & \dots \\ -0.0002 & 0.0001 & 0.0001 & 0.0001 & 0.0002 & 0.0001 & 0.0001 & 0.0002 & \dots \\ 0.0002 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

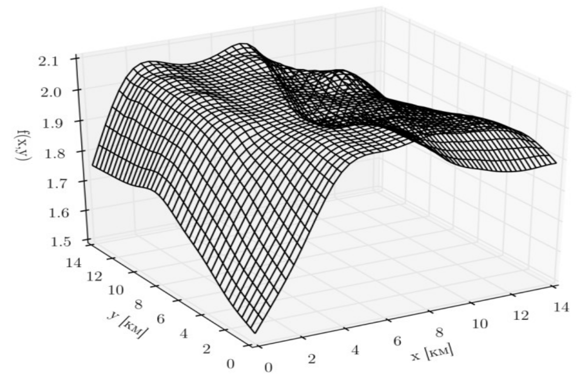

Fig. 4. Graph of the selected square magnetic field

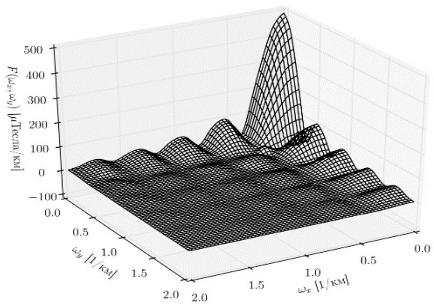

Fig. 5. Graph of the magnetic field's amplitude spectrum

The amplitude spectrum graph of the two-dimensional sequence of Haar wavelets is shown in Fig. 5. From the graph, it is obvious that these two components represent the main part of the high-frequency energy of the infinite spectrum of the sequence.

Let us estimate the accuracy of samples of the function f (x, y) given over an area of 16 x 16 km^2. The energy value of the sum of squares of 256 wavelet coefficients, taking into account the weights of octaves, turned out to be equal to $E_\varepsilon = 949.376$.

The error with respect to the total energy, which was calculated at the level $E_\varepsilon = 955.403$, was $\varepsilon = 0.6\%$. If we take samples of a signal with a sample that is 2 times rarer (only even samples on each of the horizontal axes), then the calculated value of the spectral energy is 943.976. This corresponds to an error $\varepsilon = 1.2\%$. Thus, the movement to full energy with increasing number of readings on a binary scale is obvious.

IV. CONCLUSION

The material shows that in the solution with compact support theory problems signal samples wavelet methods functions may provide certain advantages over theory, the principle of utilizing a finite spectrum. The principle of finiteness of signal carriers, which leads to the infiniteness of the spectrum and requires detailed estimates of the accuracy of the calculation of energy characteristics, is put at the forefront. A result of the article is obtained by using the Haar wavelets for two-dimensional signals. The results presented in this paper are mainly based on the quantitative processing of seismic signals and detection of earthquake center point by introducing a new algorithm for determining the values of the magnetic field's maximum and when used seismically, geophysical problems of prospecting for minerals can be solved.The advantage of the methods of finite basis functions, in contrast to the practice of interpolation approximations, considered in applications of the theory of functions with finite spectrum, is possible to restore signals according to various criteria - both interpolation and minimization criteria of quadratic functional and others.

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