Numerical Modeling of the Process of Thermoplastic Deformation of Transversally Isotropic Parallelepipeds

M. R. Babajanov, A. A. Kalandarov, U. E. Adambaev

Abstract*: *The paper proposes a modified version of the iterative method for numerically solving a three-dimensional uncoupled boundary-value problem that describes the process of thermoplastic deformations of a transversely isotropic parallelepiped. A discrete analogue of the boundary value problem is compiled on the basis of the finite-difference method. A recurrent finite-difference relation is written which allows one to find the desired components of the displacement vector in combination with the iterative method. It is assumed that, at a first approximation, the values of the sought displacements in the internal nodes are trivial. The essence of the method is demonstrated by solving the thermoplastic boundary-value problem for a transversely isotropic parallelepiped. The proposed method can be applied to solve related problems of dynamic thermoplasticity.

Keywords*: *Coupled problems, displacement, iterative method strain, stress, thermoplasticity.

I. INTRODUCTION

Mathematical models describing the process of linear deformation of solids with temperature taken into account, were first considered by Duhamel – Neumann [1]-[3], in which it was assumed that the total deformation consists of elastic deformation and thermal expansion. Questions of the theory of thermoplasticity of deformable solids are investigated in the following papers [4]-[6].

The issues of plastic deformation of solids with regard to the temperature, were first considered in more detail in the works of P.M.Nakhdi[7] and others. Further, these studies were continued in the works of Yu.N. Shevchenko [8]-[10], D. Kolarov [11], J.Casey[12], Aboudi [13],[14], W.F.Chen[15] and others [16],[17].

In many applied engineering and technical problems, the process of plastic deformation of structures and their elements, taking into account temperature influences, can be described by model equations of two types, namely "connected" and "unconnected" thermomechanical problems.

In an unconnected problem, the heat equation responsible for thermal factors is solved separately, and its results are used as a well-known parameter in solving the basic model equations of "thermoplasticity" considered in combination with the equation of motion. Note that in "coupled problems", in contrast to "uncoupled problems", a simultaneous solutions of the equations of "thermoplasticity" and the equation of "heat flux" is assumed. Since deformation cause the appearance of temperature, this approach allows one to more adequately describe the process of thermoplastic deformation of structures and their elements under the influence of mechanical and thermal influences. To formulate the above thermoplastic model equations, we need to know the nonlinear relationship between the stress and strain tensors and taking into account temperature [18]-[23].

II. PROBLEM STATEMENT

Let us consider an unrelated static boundary-value problem describing the process of thermoplastic deformation of a transversely isotropic parallelepiped, based on the deformation theory, which consists of the equilibrium equation

$$\sum_{j=1}^{3}\frac{\partial\sigma_{ij}}{\partial x_j}+X_i=0,\quad i=1,2,3 \qquad (1)$$

and, which determines the relation of the deformation theory of transversely isotropic bodies [24], [27], [28]

$$\sigma_{ij}=C_{ijke}\varepsilon_{kl}-\beta_{ij}(T-T_0)\delta_{ij}-2(\lambda_2-\lambda_2^{'})(1-\frac{p^*}{p})p_{ij}-$$
$$-2(\lambda_5-\lambda_5^{'})(1-\frac{q^*}{q})q_{ij}\quad for\quad p\ge p^*,\ q\ge q^* \qquad (2)$$

Cauchy relations

$$\varepsilon_{kl}=\frac{1}{2}\left(u_{k,l}+u_{l,k}\right) \qquad (3)$$

and boundary conditions

$$u_i\big|_{\Sigma_1}=u_i^0,\quad \sum_{j=1}^{3}\sigma_{ij}n_j\Big|_{\Sigma_2}=S_i^0 \qquad (4)$$

Where C_{ijke} - tensor of elastic constants of a transversally isotropic material, p^*, q^* - yield strength in the longitudinal and transverse directions of a transversally isotropic body, u_i - displacement vector components; X_i, S_i - bulk and surface forces, respectively, Σ_1,Σ_2 - surface parts, $\Sigma-$ volume V, n_j - external normal to the surface Σ_2- volume V, T –

Revised Manuscript Received on April 30, 2020.
* Correspondence Author
M.R.Badadjanov*, Tashkent University of Information Technologies, Tashkent, Uzbekistan. Email: mum1975@mail.ru
A.A.Kalandarov, Gulistan State University, Gulistan, Uzbekistan. Email: aziz_510@bk.ru
U.E.Adambaev, Faculty of Math. National University of Uzbekistan named after Mirzo Ulugbek, Tashkent, Uzbekistan. Email: a_uchqun@mail.ru

absolute temperature, δ_{ij} - Kronecker symbol, α_{kl} - thermal expansion tensor.

$$\lambda_1 = C_{2211},\ \lambda_2 = C_{1212},\ \lambda_3 = C_{1133},\ \lambda_4 = C_{3333},\ \lambda_5 = C_{1313},$$
$$\lambda_6 = \lambda_7 = \lambda_1 + 2\lambda_2 = C_{2211} + 2C_{1212},$$
$$\lambda_8 = \lambda_3 = C_{1133},\ \lambda_9 = \lambda_5 = C_{1313}.$$
$$\beta_{ij} = C_{ijkl}\alpha_{kl}\ ,\quad \alpha_{ij} = \alpha_1(\delta_{ij} - \delta_{i3}\delta_{j3}) + \alpha_3\delta_{i3}\delta_{j3}$$

Substituting (3) into (2) and obtained in (1), the boundary-value problem (1-4) can be written down with respect to the displacements u_i ($u = u_1$, $v = u_2$, $w = u_3$) in the following form (for simplicity, we neglect volume forces)

$$\begin{cases} C_{1111}\dfrac{\partial^2 u}{\partial x^2} + C_{1212}\dfrac{\partial^2 u}{\partial y^2} + C_{1313}\dfrac{\partial^2 u}{\partial z^2} + (C_{1122} + C_{1212})\times \\ \quad\times\dfrac{\partial^2 v}{\partial x\partial y} + (C_{1133} + C_{1313})\dfrac{\partial^2 w}{\partial x\partial z} - \beta_{11}\dfrac{\partial T}{\partial x} - F_1 = 0 \\ C_{1212}\dfrac{\partial^2 v}{\partial x^2} + C_{2222}\dfrac{\partial^2 v}{\partial y^2} + C_{2323}\dfrac{\partial^2 v}{\partial z^2} + (C_{2211} + C_{1212})\times \\ \quad\times\dfrac{\partial^2 u}{\partial x\partial y} + (C_{2233} + C_{2323})\dfrac{\partial^2 w}{\partial y\partial z} - \beta_{22}\dfrac{\partial T}{\partial y} - F_2 = 0 \\ C_{1313}\dfrac{\partial^2 w}{\partial x^2} + C_{2323}\dfrac{\partial^2 w}{\partial y^2} + C_{3333}\dfrac{\partial^2 w}{\partial z^2} + (C_{3311} + C_{1313})\times \\ \quad\times\dfrac{\partial^2 u}{\partial x\partial z} + (C_{3322} + C_{2323})\dfrac{\partial^2 v}{\partial y\partial z} - \beta_{33}\dfrac{\partial T}{\partial z} - F_3 = 0 \end{cases} \qquad (5)$$

$$u_i\big|_{\Sigma_1} = u_i^o,$$
$$\sum_{j=1}^{3}\left\{\begin{matrix} \lambda\delta_{ij}(\sum_{k=1}^{3}\dfrac{\partial u_k}{\partial x_k}) + \mu(\dfrac{\partial u_i}{\partial x_j} + \dfrac{\partial u_j}{\partial x_i}) - \\ -(3\lambda + 2\mu)\alpha(T - T_0)\delta_{ij} - \sigma_{ij}^p \end{matrix}\right\} n_j\Bigg|_{\Sigma_2} = S_i^0 \qquad (6)$$

where

$$\sigma_{ij}^p = \begin{cases} 0\ , & for\quad p < 0 \quad and \quad q < 0, \\ 2(\lambda_2 - \lambda_2^{'})(1 - \dfrac{p^*}{p})p_{ij}\ , & for\quad p \ge p^*, \\ 2(\lambda_5 - \lambda_5^{'})(1 - \dfrac{q^*}{q})q_{ij}\ , & for\quad q \ge q^* \end{cases} \qquad (7)$$

$$F_i = \sum_{j=1}^{3}\frac{\partial \sigma_{ij}^p}{\partial x_j} 2(\lambda_2 - \lambda_2')(1 - \frac{p^*}{p})\frac{\partial p_{ij}}{\partial x_j} + 2(\lambda_5 - \lambda_5')(1 - \frac{q^*}{q})\frac{\partial q_{ij}}{\partial x_j}$$
$$for\ \ p \ge p^*,\ q \ge q^*$$

Value σ_{ij}^p represents the nonlinear part of the defining relation (2), and F_i nonlinear part of equation (1).

III. NUMERICAL SOLUTION METHOD

Replacing the derivatives in equations (5-6) with the corresponding finite-difference relations, we find the finite-difference analogue [25],[26] of the equilibrium equation (5)

$$\begin{aligned} & C_{1111}\frac{u_{i+1,j,k} - 2u_{i,j,k} + u_{i-1,j,k}}{h_1^2} + \\ & + C_{1212}\frac{u_{i,j+1,k} - 2u_{i,j,k} + u_{i,j-1,k}}{h_2^2} + \\ & + C_{1313}\frac{u_{i,j,k+1} - 2u_{i,j,k} + u_{i,j,k-1}}{h_3^2} + \\ & (C_{1122} + C_{1212})\frac{v_{i+1,j+1,k} - v_{i-1,j+1,k} - v_{i+1,j-1,k} + v_{i-1,j-1,k}}{4h_1h_2} + \\ & (C_{1133} + C_{1313})\frac{w_{i+1,j,k+1} - w_{i-1,j,k+1} - w_{i+1,j,k-1} + w_{i-1,j,k-1}}{4h_1h_3} - \\ & - \frac{\beta_{11}(t_{i+1,j,k} - t_{i+1,j,k})}{2h_1} = F_1 \end{aligned} \qquad (8)$$

$$\begin{aligned} & C_{1212}\frac{v_{i+1,j,k} - 2v_{i,j,k} + v_{i-1,j,k}}{h_1^2} + \\ & + C_{2222}\frac{v_{i,j+1,k} - 2v_{i,j,k} + v_{i,j-1,k}}{h_2^2} + \\ & + C_{2323}\frac{v_{i,j,k+1} - 2v_{i,j,k} + v_{i,j,k-1}}{h_3^2} + \\ & (C_{2211} + C_{1212})\frac{u_{i+1,j+1,k} - u_{i-1,j+1,k} - u_{i+1,j-1,k} + u_{i-1,j-1,k}}{4h_1h_2} + \\ & (C_{2233} + C_{2323})\frac{w_{i,j+1,k+1} - w_{i,j-1,k+1} - w_{i,j+1,k-1} + w_{i,j-1,k-1}}{4h_2h_3} - \\ & - \frac{\beta_{22}(t_{i,j+1,k} - t_{i,j-1,k})}{2h_2} = F_2 \end{aligned} \qquad (9)$$

$$\begin{aligned} & C_{1313}\frac{w_{i+1,j,k} - 2w_{i,j,k} + w_{i-1,j,k}}{h_1^2} + \\ & + C_{2323}\frac{w_{i,j+1,k} - 2w_{i,j,k} + w_{i,j-1,k}}{h_2^2} + \\ & + C_{3333}\frac{w_{i,j,k+1} - 2w_{i,j,k} + w_{i,j,k-1}}{h_3^2} + \\ & +(C_{3311} + C_{1313})\frac{u_{i+1,j,k+1} - u_{i-1,j,k+1} - u_{i+1,j,k-1} + u_{i-1,j,k-1}}{4h_1h_3} + \\ & (C_{3322} + C_{2323})\frac{v_{i,j+1,k+1} - v_{i,j-1,k+1} - v_{i,j+1,k-1} + v_{i,j-1,k-1}}{4h_1h_2} - \\ & - \frac{\beta_{33}(t_{i,j,k+1} - t_{i,j,k-1})}{2h_3} = F_3 \end{aligned} \qquad (10)$$

and boundary conditions, on two opposite faces of the parallelepiped perpendicular to the axis OX

$$\sigma_{11}\big|_{x=0,l_1} = \begin{bmatrix} C_{1111}\dfrac{\partial u}{\partial x} + C_{1122}\dfrac{\partial v}{\partial y} + C_{1133}\dfrac{\partial w}{\partial z} - \\ -2(\lambda_2 - \lambda_2^{'})(1 - \frac{p^*}{p_u})p_{11} - \beta_{11}\dfrac{\partial T}{\partial x} \end{bmatrix}\Bigg|_{x=0,l_1} = S_1^0$$

$$\sigma_{12}\big|_{x=0,l_1} = \left[\begin{matrix} C_{1212}(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x})- \\ -2(\lambda_2-\lambda_2^{'})(1-\frac{p^*}{p_u})p_{12}\end{matrix}\right]\Bigg|_{x=0,l_1} = S_2^0 \quad (11)$$

$$\sigma_{13}\big|_{x=0,l_1} = \left[\begin{matrix} C_{1313}(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x})- \\ -2(\lambda_5-\lambda_5^{'})(1-\frac{q^*}{q_u})q_{13}\end{matrix}\right]\Bigg|_{x=0,l_1} = S_3^0$$

The remaining faces of the parallelepiped are free of loads. The boundary conditions can be set with ease with respect to the displacement vector.

Further, resolving the finite-difference equations (8-10) relatively $u_{i,j,k}$, $v_{i,j,k}$, $w_{i,j,k}$ accordingly, it is possible to organize the following iterative process:

$$u_{i,j,k}^{(n+1)} = \Bigg[\frac{C_{1111}}{h_1^2}(u_{i+1,j,k}^{(n)}+u_{i-1,j,k}^{(n)})+ +\frac{C_{1212}}{h_2^2}(u_{i,j+1,k}^{(n)}+u_{i,j-1,k}^{(n)})+\frac{C_{1313}}{h_3^2}(u_{i,j,k-1}^{(n)}+u_{i,j,k+1}^{(n)})+ +\frac{C_{1122}+C_{1212}}{4h_1h_2}(v_{i+1,j+1,k}^{(n)}-v_{i+1,j-1,k}^{(n)}-v_{i-1,j+1,k}^{(n)}+v_{i-1,j-1,k}^{(n)})+ +\frac{C_{1133}+C_{1313}}{4h_1h_3}(w_{i+1,j,k+1}^{(n)}-w_{i+1,j,k-1}^{(n)}-w_{i-1,j,k+1}^{(n)}+ +w_{i-1,j,k-1}^{(n)})-b_{11}\frac{(t_{i+1,j,k}-t_{i-1,j,k})}{2h_1}-F_{1\ i,j,k}\Bigg]/ \Bigg/\left(\frac{2C_{1111}}{h_1^2}+\frac{2C_{1212}}{h_2^2}+\frac{2C_{1313}}{h_3^2}\right) \quad (12)$$

$$v_{i,j,k}^{(n+1)} = \Bigg[\frac{C_{1212}}{h_1^2}(v_{i+1,j,k}^{(n)}+v_{i-1,j,k}^{(n)})+ +\frac{C_{2222}}{h_2^2}(v_{i,j+1,k}^{(n)}+v_{i,j-1,k}^{(n)})+\frac{C_{2323}}{h_3^2}(v_{i,j,k+1}^{(n)}+v_{i,j,k-1}^{(n)})+ +\frac{C_{2211}+C_{1212}}{4h_1h_2}(u_{i+1,j+1,k}^{(n)}-u_{i+1,j-1,k}^{(n)}-u_{i-1,j+1,k}^{(n)}+u_{i-1,j-1,k}^{(n)})+ +\frac{C_{2233}+C_{2323}}{4h_2h_3}(w_{i,j+1,k+1}^{(n)}-w_{i,j-1,k+1}^{(n)}-w_{i,j+1,k-1}^{(n)}+ +w_{i,j-1,k-1}^{(n)})-b_{22}\frac{(t_{i,j+1,k}-t_{i,j-1,k})}{2h_2}-F_{2\ i,j,k}\Bigg]/ \Bigg/\left(\frac{2C_{1212}}{h_1^2}+\frac{2C_{2222}}{h_2^2}+\frac{2C_{1313}}{h_3^2}\right) \quad (13)$$

$$w_{i,j,k}^{(n+1)} = \Bigg[\frac{C_{1313}}{h_1^2}(w_{i+1,j,k}^{(n)}+w_{i-1,j,k}^{(n)})+ +\frac{C_{2323}}{h_2^2}(w_{i,j+1,k}^{(n)}+w_{i,j-1,k}^{(n)})+\frac{C_{3333}}{h_3^2}(w_{i,j,k+1}^{(n)}+w_{i,j,k-1}^{(n)})+ +\frac{C_{3311}+C_{1313}}{4h_1h_3}(u_{i+1,j,k+1}^{(n)}-u_{i+1,j,k-1}^{(n)}-u_{i-1,j,k+1}^{(n)}+ +u_{i-1,j,k-1}^{(n)})+\frac{C_{3322}+C_{2323}}{4h_2h_3}(v_{i,j+1,k+1}^{(n)}-v_{i,j-1,k+1}^{(n)}- -v_{i,j+1,k-1}^{(n)}+v_{i,j-1,k-1}^{(n)})-b_{33}\frac{(t_{i,j,k+1}-t_{i,j,k-1})}{2h_3}-F_{3\ i,j,k}\Bigg]/ \left(\frac{2C_{1313}}{h_1^2}+\frac{2C_{2323}}{h_2^2}+\frac{2C_{3333}}{h_3^2}\right). \quad (14)$$

The values of displacements in the zeroth approximation of the iterative process are considered as a trivial. Recall that, as a first approximation, the terms $F_{1\ i,j,k}$, $F_{2\ i,j,k}$, $F_{3\ i,j,k}$ are considered zero and the linear problems of the theory of elasticity are solved. Based on the results obtained, the condition for the transition of the elastic limit, i.e. $p \geq p^*$, $q \geq q^*$ and when they are executed, they are calculated $F_{1\ i,j,k}$, $F_{2\ i,j,k}$, $F_{3\ i,j,k}$ and, again the linear problem is solved with the new right-hand side, and continues until the required accuracy is achieved. This is the essence of the method of elastic solutions.

IV. TEST TASK

When solving the model problem numerically, we considered a transversely isotropic thermoplastic parallelepiped pinched on all sides, inside which a temperature field is given by the following formula:

$$T(x_1,x_2,x_3) = T_0 \sin\frac{\pi x_1}{l_1}\sin\frac{\pi x_2}{l_2}\sin\frac{\pi x_3}{l_3}$$

Elastic modules, hardening modules and elastic limits had the following dimensionless values:

$C_{1111}=5.68$, $C_{1212}=2.735$, $C_{1313}=2.39$, $C_{1122}=0.21$, $C_{1133}=0.19$, $C_{2222}=5.68$, $C_{2233}=0.19$, $C_{2323}=2.39$, $C_{3333}=5.35$, $\lambda_2^{'}=2.5$, $\lambda_5^{'}=2.24$, $p^*=0.14$, $q^*=0.11$, $T_0=20$.

Below we consider the propagation of plastic zones in various sections of the transversally isotropic parallelepiped under consideration.

Table 1. The values of the intensity of the strain tensor p for z=0,7

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
y=0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
y=0.1	0.00000	0.10830	0.12893	0.14592	0.15313	0.15491	0.15355	0.14648	0.12922	0.10838	0.00000
y=0.2	0.00000	0.12884	0.10128	0.09329	0.09148	0.09124	0.09198	0.09387	0.10153	0.12911	0.00000
y=0.3	0.00000	0.14590	0.09377	0.06079	0.04546	0.04197	0.04561	0.06090	0.09367	0.14604	0.00000
y=0.4	0.00000	0.15299	0.09207	0.04567	0.01788	0.01067	0.01789	0.04561	0.09188	0.15297	0.00000
y=0.5	0.00000	0.15453	0.09153	0.04213	0.01073	0.00002	0.01073	0.04213	0.09153	0.15453	0.00000

y=0.6	0.00000	0.15297	0.09188	0.04561	0.01789	0.01067	0.01788	0.04567	0.09207	0.15299	0.00000
y=0.7	0.00000	0.14604	0.09367	0.06090	0.04561	0.04197	0.04546	0.06079	0.09377	0.14590	0.00000
y=0.8	0.00000	0.12911	0.10153	0.09387	0.09198	0.09124	0.09148	0.09329	0.10128	0.12884	0.00000
y=0.9	0.00000	0.10838	0.12922	0.14648	0.15355	0.15491	0.15313	0.14592	0.12893	0.10830	0.00000
y=1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

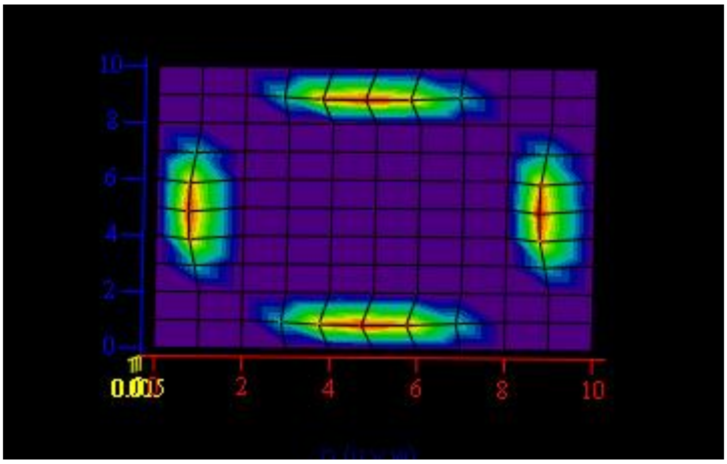

Fig. 1. The zone of plasticity according to the intensity of the tensor deformations p *in the XOY plane for z*=0,7 ($p \geq p^*$)

Table 2. The values of the intensity of the strain tensor q for z=0,7

	y=0	y=0.1	y==0.2	y=0.3	y=0.4	y=0.5	y=0.6	y=0.7	y==0.8	y=0.9	y=1
z=0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
z=0.1	0.00000	0.05489	0.07683	0.09445	0.10553	0.10926	0.10544	0.09433	0.07678	0.05491	0.00000
z=0.2	0.00000	0.07660	0.08492	0.09123	0.09449	0.09544	0.09449	0.09125	0.08499	0.07688	0.00000
z=0.3	0.00000	0.09410	0.09114	0.08400	0.07547	0.07147	0.07548	0.08404	0.09123	0.09434	0.00000
z=0.4	0.00000	0.10517	0.09436	0.07543	0.05198	0.03842	0.05199	0.07545	0.09441	0.10530	0.00000
z=0.5	0.00000	0.10899	0.09532	0.07142	0.03840	0.00000	0.03840	0.07142	0.09532	0.10899	0.00000
z=0.6	0.00000	0.10530	0.09441	0.07545	0.05199	0.03842	0.05198	0.07543	0.09436	0.10517	0.00000
z=0.7	0.00000	0.09434	0.09123	0.08404	0.07548	0.07147	0.07547	0.08400	0.09114	0.09410	0.00000
z=0.8	0.00000	0.07688	0.08499	0.09125	0.09449	0.09544	0.09449	0.09123	0.08492	0.07660	0.00000
z=0.9	0.00000	0.05491	0.07678	0.09433	0.10544	0.10926	0.10553	0.09445	0.07683	0.05489	0.00000
z=1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

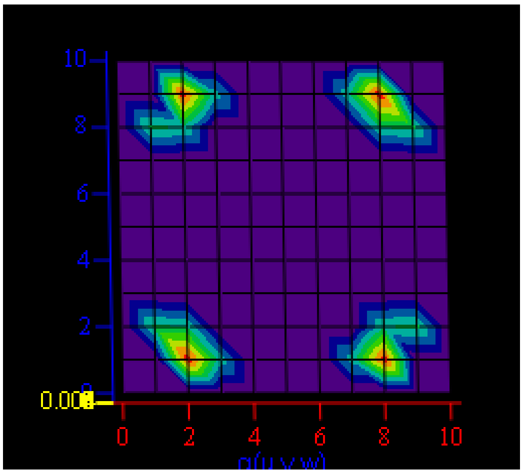

Fig. 2. The zone of plasticity according to the intensity of the tensor deformations *q* in the XOY plane for *z=0,7* ($q \geq q^*$)

The followings are numerical results and 3D function graphs. $u(x,y,z)$, $v(x,y,z)$, $w(x,y,z)$, $t(x,y,z)$, $p(x,y,z)$, $q(x,y,z)$, in the area in question.

Table 3.The values of movement $u(x, y, z)$ *for z*=0,5

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
y=0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
y=0.1	0.00000	-0.01078	-0.01436	-0.01261	-0.00731	0.00000	0.00699	0.01243	0.01435	0.01080	0.00000
y=0.2	0.00000	-0.01768	-0.02385	-0.02110	-0.01216	0.00000	0.01209	0.02107	0.02389	0.01783	0.00000
y=0.3	0.00000	-0.02245	-0.03058	-0.02720	-0.01570	0.00000	0.01569	0.02721	0.03062	0.02257	0.00000
y=0.4	0.00000	-0.02539	-0.03469	-0.03093	-0.01787	0.00000	0.01787	0.03093	0.03472	0.02545	0.00000
y=0.5	0.00000	-0.02641	-0.03609	-0.03219	-0.01861	0.00000	0.01861	0.03219	0.03609	0.02641	0.00000
y=0.6	0.00000	-0.02545	-0.03472	-0.03093	-0.01787	0.00000	0.01787	0.03093	0.03469	0.02539	0.00000
y=0.7	0.00000	-0.02257	-0.03062	-0.02721	-0.01569	0.00000	0.01570	0.02720	0.03058	0.02245	0.00000
y=0.8	0.00000	-0.01783	-0.02389	-0.02107	-0.01209	0.00000	0.01216	0.02110	0.02385	0.01768	0.00000
y=0.9	0.00000	-0.01080	-0.01435	-0.01243	-0.00699	0.00000	0.00731	0.01261	0.01436	0.01078	0.00000
y=1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

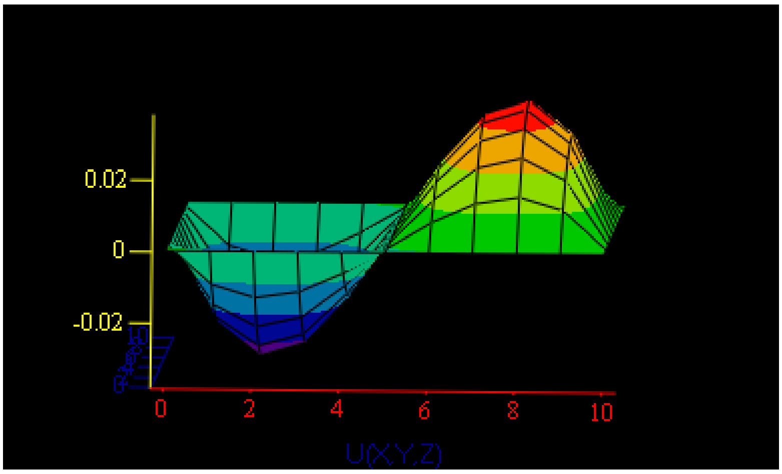

Fig. 3: Distribution graph of the function $u(x, y, z)$ in the plane XOY for *z*=0,5

Table 4. Values of displacement $v(x, y, z)$ for *z*=0,8

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
y=0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
y=0.1	0.00000	-0.00762	-0.01250	-0.01595	-0.01806	-0.01878	-0.01806	-0.01595	-0.01250	-0.00762	0.00000
y=0.2	0.00000	-0.00992	-0.01674	-0.02155	-0.02448	-0.02547	-0.02448	-0.02155	-0.01674	-0.00992	0.00000
y=0.3	0.00000	-0.00859	-0.01470	-0.01901	-0.02163	-0.02252	-0.02163	-0.01901	-0.01470	-0.00859	0.00000
y=0.4	0.00000	-0.00488	-0.00842	-0.01091	-0.01243	-0.01295	-0.01243	-0.01091	-0.00842	-0.00488	0.00000
y=0.5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
y=0.6	0.00000	0.00488	0.00842	0.01091	0.01243	0.01295	0.01243	0.01091	0.00842	0.00488	0.00000
y=0.7	0.00000	0.00859	0.01470	0.01901	0.02163	0.02252	0.02163	0.01901	0.01470	0.00859	0.00000
y=0.8	0.00000	0.00992	0.01674	0.02155	0.02448	0.02547	0.02448	0.02155	0.01674	0.00992	0.00000
y=0.9	0.00000	0.00762	0.01250	0.01595	0.01806	0.01878	0.01806	0.01595	0.01250	0.00762	0.00000
y=1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

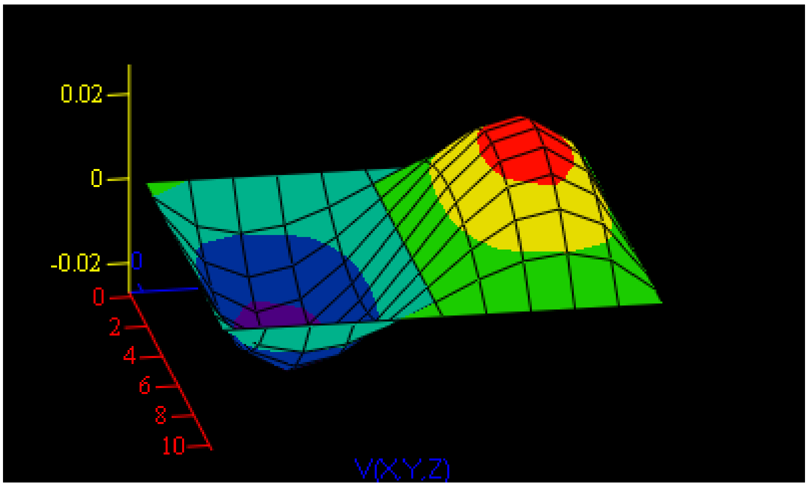

Fig. 4.Graph of the distribution of the function $v(x, y, z)$ in the plane XOY for *z*=0,8

Table 5. The displacement values $w(x,y,z)$ for z=0,3

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
y=0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
y=0.1	0.00000	-0.00714	-0.01273	-0.01690	-0.01950	-0.02038	-0.01949	-0.01688	-0.01272	-0.00713	0.00000
y=0.2	0.00000	-0.01272	-0.02284	-0.03043	-0.03517	-0.03679	-0.03517	-0.03042	-0.02283	-0.01272	0.00000
y=0.3	0.00000	-0.01689	-0.03043	-0.04055	-0.04689	-0.04905	-0.04689	-0.04055	-0.03043	-0.01689	0.00000
y=0.4	0.00000	-0.01950	-0.03518	-0.04689	-0.05424	-0.05674	-0.05423	-0.04689	-0.03517	-0.01950	0.00000
y=0.5	0.00000	-0.02039	-0.03679	-0.04905	-0.05674	-0.05937	-0.05674	-0.04905	-0.03679	-0.02039	0.00000
y=0.6	0.00000	-0.01950	-0.03517	-0.04689	-0.05423	-0.05674	-0.05424	-0.04689	-0.03518	-0.01950	0.00000
y=0.7	0.00000	-0.01689	-0.03043	-0.04055	-0.04689	-0.04905	-0.04689	-0.04055	-0.03043	-0.01689	0.00000
y=0.8	0.00000	-0.01272	-0.02283	-0.03042	-0.03517	-0.03679	-0.03517	-0.03043	-0.02284	-0.01272	0.00000
y=0.9	0.00000	-0.00713	-0.01272	-0.01688	-0.01949	-0.02038	-0.01950	-0.01690	-0.01273	-0.00714	0.00000
y=1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

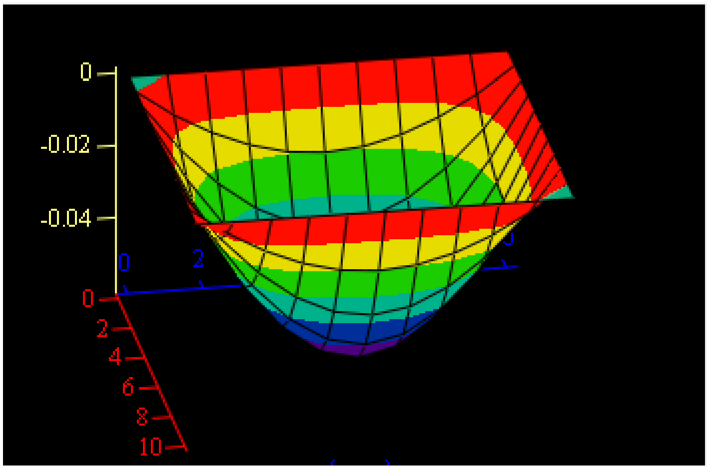

Fig. 5. Distribution graph of the function $w(x,y,z)$ in the plane XOY for z=0,3

Table 6. Temperature $t(x,y,z)$ for z=0,4

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
y=0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
y=0.1	0.00000	1.81636	3.45492	4.75528	5.59017	5.87785	5.59017	4.75528	3.45492	1.81636	0.00000
y=0.2	0.00000	3.45492	6.57164	9.04508	10.63314	11.18034	10.63314	9.04508	6.57164	3.45492	0.00000
y=0.3	0.00000	4.75528	9.04508	12.44949	14.63525	15.38842	14.63525	12.44949	9.04508	4.75528	0.00000
y=0.4	0.00000	5.59017	10.63314	14.63525	17.20477	18.09017	17.20477	14.63525	10.63314	5.59017	0.00000
y=0.5	0.00000	5.87785	11.18034	15.38842	18.09017	19.02113	18.09017	15.38842	11.18034	5.87785	0.00000
y=0.6	0.00000	5.59017	10.63314	14.63525	17.20477	18.09017	17.20477	14.63525	10.63314	5.59017	0.00000
y=0.7	0.00000	4.75528	9.04508	12.44949	14.63525	15.38842	14.63525	12.44949	9.04508	4.75528	0.00000
y=0.8	0.00000	3.45492	6.57164	9.04508	10.63314	11.18034	10.63314	9.04508	6.57164	3.45492	0.00000
y=0.9	0.00000	1.81636	3.45492	4.75528	5.59017	5.87785	5.59017	4.75528	3.45492	1.81636	0.00000
y=1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

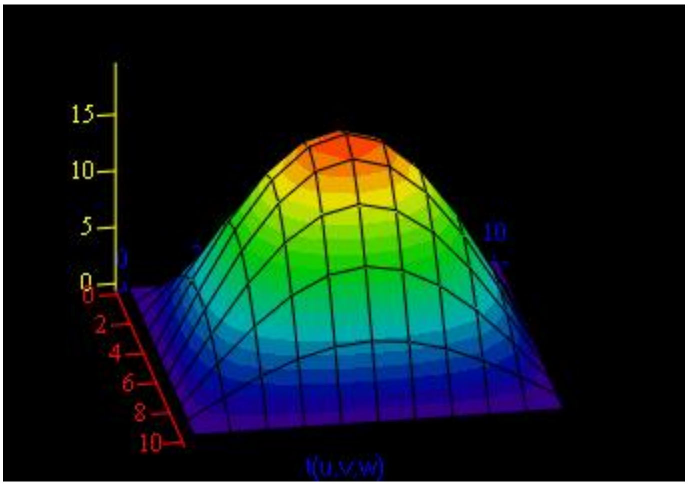

Fig. 6. The distribution graph of the function $t(x,y,z)$ in the plane XOY for z=0,4

Since the boundary conditions are given symmetrically, the numerical results given in tables (1-6) and figures (1-6) obtained by the above iterative method are distributed symmetrically in the parallelepiped under consideration, which ensures the validity of the numerical results and the proposed iterative approach for the numerical solution of thermoplastic problems for transversely isotropic bodies.

V. CONCLUSION

A three-dimensional thermoplastic boundary value problem for a transversely isotropic parallelepiped is formulated. A discrete analogue of the problem is compiled by the finite-difference method. Finite-difference equations are resolved with respect to the main nodal displacements and a recurrence relation is obtained that is solved by the iterative method in combination with the elastic solution method. The thermoplastic boundary-value problem of a clamped transversally isotropic parallelepiped is numerically solved.

3D graphs of the distribution of the displacement functions $u(x,y,z)$, $v(x,y,z)$, $w(x,y,z)$ and temperature $T(x,y,z)$ are constructed. The propagation of plasticity zones in various sections of the transversally isotropic parallelepiped under the influence of a temperature field is studied.

REFERENCES

1. V.Novatskiy. Dynamic problems of thermoelasticity. Moscow, Mir, 1970.p.256. In Russian.
2. A.Kovalenko. Thermoelasticity. – Kiev, 1975. –p. 216. In Russian.
3. A.Kovalenko. Development of researches in the field of thermoelasticity, thermoplasticity and thermoviskoelasticity. // Applied Mathematics. –1969. – P.5, № 12. – p. 1-16.
4. Biot, M. Thermoelasticity and irreversible thermo-dynamics. J. Appl. Phys., 27, 240–253. 1956
5. Youssef H. M., Al-Felali A.S. Generalized Thermoelasticity Problem of Material Subjected to Thermal Loading Due to Laser Pulse. Applied Mathematics, 2012, 3, 142-146
6. A.Lykov. Theory of thermal conductivity. – MOSCOW: Vis.shkola, 1967. – p.599.
7. P. Naghdi. Relations of strain and space in plasticity, thermoplasticity. Collection «Mechanics». -1962. 1, 71,p.87-133. In Russian.
8. Yu.Shevchenko, M.Babashko, V.Piskun, V.Savchenko. Area problems of thermoplasticity. -Kiev: Nauk. dumka, 1980. –p.262.
9. Yu.Shevchenko, R.Terekhov. Experimental check of a hypothesis of existence of a thermomechanical surface in the thermoplasticity theory. -1973. V.13. -p.43-47.
10. Senchenkov I.K., Andrushko N.F. Thermal Effects in a Physically Nonlinear Cylinder under Impulsive Loading // Int. Appl. Mech. – 2005. – 41, N 8. – P. 890 – 894.
11. Kolarov D., Baltov A., Boncheva N. Mechanics of plastic environments. Mir. 1979. P.302.
12. Casey J. On elastic-thermo-plastic materials at finite deformations. Int. J. of Plasticity .V. 14, Issues 1–3, 1998, p. 173–191
13. Aboudi J. The effective thermomechanical behavior of inelastic fiber-reinforced materials // Int. J. Eng. sci. -1985.- Vol. 23, -No 7. - P. 773-787.
14. Aboudi J., Benveniste Y. Finite amplitude one-dimensional wave propagation in a thermo-elastic half-space // Int. J. Solids structures. -1974. Vol. 10. - P. 293 – 308.
15. W. Chen, D. Han, Plasticity for Structural Engineers, Springer Verlag, New York, 1988, 415 p.
16. Maughin G.A. The Thermomechanics of Plasticity and Fracture. Cambridge Univ. Press, Cambridge (1992).
17. Miehe C. A theory of large-strain isotropic thermoplasticity based on metric transformation tensor. Archive Appl. Mech.66 (1995) 45–64.
18. Bezukhov N., O.Luzhin. Applied methods of elasticity and plasticity theories for solving engineering problems. -Moscow, 1974. –p. 200. In Russian.
19. V. Rekach. Management to the decision of problems of the applied theory of elasticity. Moscow, 1973, p.384. In Russian.
20. Khaldjigitov A.A., Nik M.A. Asri Long., Qalandarov A., Eshquvatov Z. Mathematical and numerical modelling of the thermoplastic coupled problem. International conference on mathematical sciences and statistics 2013. Singapore, Springer, pp. 69-75, (2014).
21. Khaldjigitov A.A., Khudazarov R.S., Sagdullaeva D.A. Theories of plasticity and thermoplasticity of anisotropic bodies.-Tashkent.:"Fan va texnologiya ", 2015, p.320.
22. AM. Polatov, A.M. Ikramov, A.A. Khaldjigitov. Computer Modeling of Elastoplastic Stress state of Fibrous Composites with hole//Coupled Systems Mechanics, An International Journal, Techno-Press Ltd., Yuseong-gu Daejeon, Korea, 2019.Vol.8, №4, pp. 299-313.
23. V.Pavlichko. The decision of a three-dimensional thermoplastic problem at simple burden. Jour. Acta durability problems, 1986. № 1. -p. 77-81.
24. B. Pobedria, Mechanics of Composite Materials, Moscow State University Press, RU, 1984. in Russian.
25. Yusupov Y.S., Babadjanov M.R. Numerical solution of a Three-Dimensional Coupled thermoplastic problem based on deformation theory. The international journal of science & technology. Vol 7 Issue 9, DOI.:10.24940/theijst/2019/v7/i9/ST1909-012 September,2019
26. Kalandarov A.A., Babadjanov M.R. Numerical simulation of the coupled dynamic thermoelastic problem for orthotropic bodies. International journal of computer science and Mobile computing, Vol 8 Issue 9, September 2019,pg. 182-189
27. Khaldjigitov A. A., Yusupov Y. S., Khudazarov R. S. & D. A. Sadullaeva. On the thermoplasticity constitutive relations for isotropic and transversely isotropic materials. International Journal of Mechanical and Production Engineering Research and Development (IJMPERD) ISSN(P):2249-6890; ISSN(E):2249-8001 Vol.9, Issue 4, Aug 2019, 467-478
28. Khaldjigitov A. A. , Nik Long N.M., Adambaev U.E. On the constitutive relations for isotropic and transversely isotropic materials. Int. J. Appl. Math. Modeling. 2013, 37 p. 7726-7740

AUTHORS PROFILE

Babajanov Mumin – graduated Tashkent State university (presently National university of Uzbekistan). Senior Lecturer, Department of Computer Science Basics at the Tashkent University of Information Technologies named after Muhammad al-Khwarizmi. 2017-2019 Doctoral student (Ph.D.) University Information Technology named after Muhammad al-Khwarizmi. Has 20 years of experience teaching the basics of computer science and the basics of programming. Author of more than 25 scientific researches.

Research Area: Numerical modeling of the process of nonlinear deformation of anisotropic bodies.

Selected Publications: 1. Numerical simulation of the coupled dynamic thermoelastic problem for orthotropic bodies. International journal of computer science and Mobile computing, Vol 8 Issue 9, September 2019, pp. 182-189.

2. Numerical solution of a Three-Dimensional Coupled thermoplastic problem based on deformation theory. The international journal of science & technoledge. Vol 7 Issue 9, DOI.:10.24940/theijst/2019/v7/ i9/ST1909-012 September, 2019

Kalandarov Aziz – graduated Gulistan state University for Bachelor degree (1997 to 2001). Studied Master's degree in National University of Uzbekistan (2001 to 2003). PhD in math. and physics (2019 year). Author of more than 30 scientific researches. Chief of department of "Information technology" in Gulistan State university. Has 16 years of teaching experience in applied mathematics, numerical methods and computer modeling.

Research Area: Numerical modeling of the process of nonlinear deformation of anisotropic bodies.

Selected Publications: 1. Mathematical and numerical modelling of the thermoplastic coupled problem. International conference on mathematical sciences and statistics 2013. Singapore, Springer, pp. 69-75, (2014).

2. Numerical solution of 1D and 2D thermoelastic coupled problems. International journal of modern physics. Vol. 9, pp. 503-510, (2012).

3. Computer simulation of the coupled dynamic thermoelasticity problem for a two-dimensional isotropic bodies. The international journal of science and technoledge, 2019. Vol. 7, Issue 5, pp. 24-32. DOI:10.24940/theijst/2019/v7/i5/ST1905-020.

4. Numerical Simulation of the Coupled Dynamic Thermoelastic Problem for Orthotropic Bodies. International Journal of Computer Science and Mobile Computing, Vol.8 Issue.9, September- 2019, pp. 182-189

Adambaev Uchkunbek – graduated Tashkent State university (presently National university of Uzbekistan). PhD in math. and physics (2008). Author of more than 30 scientific researches. 2018-2020 Doctoral student (DSc). Senior lecturer in the National university of Uzbekistan. Has 18 years of teaching experience in Applied Mathematics, IT and software programming.

Research Area: Numerical modeling of the process of nonlinear deformation of anisotropic bodies.

Selected Publications: 1. On the constitutive relations for isotropic and transversely isotropic materials. Applied Mathematical Modelling, Original Research Article, Volume 37, Issues 14–15, 1 August 2013, Pages 7726-7740 (USA).

2. Finite difference method for solving the elastoplastic problems of anisotropic bodies. Applied mathematics and control sciences». 2019. № 4, pp. 9-25. (Russia).