

# Convergence and BER Approximation of HIC Detector for DS-CDMA System in Rayleigh Fading Multipath Environment

Monica Khanore, Srija Unnikrishnan



**Abstract:** In this paper we present mathematic analysis and implementation of hybrid interference canceller (HIC) for direct sequence-code division multiple access (DS-CDMA) system in multipath environment over Rayleigh channel. The HIC detector is examined for convergence. The bit error rate (BER) for the system is obtained using Simplified Improved Gaussian Approximation (SIGA). Using this expression, BER is plotted for the Rayleigh channel having exponentially decaying power delay profile (PDP) as well as flat PDP. The theoretical results are compared with the simulated system results.

**Keywords :** DS-CDMA, PDP, MAI, Multipath fading, SIGA, Convergence.

## I. INTRODUCTION

The use of spread spectrum technique gives DS-CDMA system, the ability to support simultaneous access to multiple users, with immunity to multipath fading. The DS-CDMA system is, to some extent, capable of combating multipath fading and the multiple access interference (MAI).

Another factor that contributes to the degradation of performance is the Doppler shift in the carrier frequency. Since Doppler shift is directly proportional to the carrier frequency as well as the relative velocity of the transmitter and the receiver, it will be larger for fast-moving users and higher carrier frequencies. Doppler shift affects the synchronization of users, channel estimation and data recovery. Moreover, it introduces carrier frequency offset, leading to an increase in the MAI.

Among the various algorithms employed for interference cancellation (IC) in DS-CDMA, the two widely used subtractive type of interference cancellation algorithms are Successive Interference Cancellation (SIC) and Parallel Interference Cancellation (PIC) algorithms, both having their own merits and demerits. SIC is simple and has very good error performance at the cost of being slow while PIC is fast but complex.

An improved algorithm called Hybrid Interference canceller (HIC) combines the best characteristics of PIC and SIC. In addition to being fast, HIC retains the simplicity of SIC, delivering performance at par with the SIC. In HIC, the users are divided into groups of equal size. PIC is performed within the groups, and among the groups SIC is performed, thus, making the HIC fast while maintaining simplicity.

To analyze the system performance, many techniques have been employed, such as Standard Gaussian Approximation (SGA), Improved Gaussian Approximation (IGA), and Simplified Improved Gaussian Approximation (SIGA). The simplest of them is the SGA. It is based on the Central Limit Theorem (CLT) to approximate the aggregate of the MAI signals in addition to the channel noise. It is widely used for its simplicity. However, it has the limitation that it is not very accurate, especially for a small number of users.

The accuracy of approximation is further improved by IGA method [1]. In order to obtain IGA, the values of probability of data bit errors are averaged by finding the distribution of the variance of the MAI for all combinations of spreading codes and interfering signal delays and phases.

IGA is complex [1], owing to the numerical integrations and multiple numerical convolutions required. The method is simplified by Holtzman [2], such that the complex computations required in SGA and IGA can be avoided [2] while maintaining the BER at an acceptable level. It is further simplified by Morrow keeping the same BER accuracy [5].

The paper is organized as follows. Section II gives a brief literature survey. In section III, we introduce the receiver model for the multipath propagation of the DS-CDMA signal over Rayleigh fading channel, considering mobility of the users. Mathematical analysis of the HIC detector is presented in section IV. In section V, the detector is analyzed for convergence. The SIGA approximation for the error performance of the system is summarized in section VI. The impact of flat PDP as well as exponentially decaying PDP of the channel, and the number of significant paths on the error performance is presented through simulations in section VII. Finally, section VIII gives conclusions.

## II. LITERATURE SURVEY

[3] presented the mathematical analysis of linear HIC detector using matrix algebra. This mathematical analysis forms the basis for the mathematical analysis of the HIC scheme presented in this paper. The detector is further analyzed for convergence.

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As discussed in the previous section, several methods for approximation of the BER calculation of DS-CDMA, such as SGA, IGA, and SIGA, have been reported in the literature. The decision statistic of a correlation receiver in the DS-CDMA system employing randomly chosen spreading sequences has been expressed in a simplified form in [4].

The authors have also illustrated an approach for obtaining upper and lower bounds on the average probability of error without using Gaussian approximation. The approach used reduces the number of computations as well as the computational complexity.

An accurate approximation for BER calculation of a DS-CDMA system using binary phase-shift keying (BPSK) modulation is presented, assuming all interfering users employing random signature sequences in [5]. The performance of the desired signals with deterministic sequences is also evaluated for various values of B, which is the number of times the sequence changes state over the duration of a bit. The MAI variance is expressed as a random variable which is a function of the chip delay, carrier phase offset and the signal spreading sequence. BER is evaluated for different signal structures.

In [6], a closed-form expression for the overall BER is obtained, which can be used to compute the BER to any desired accuracy with minimal computational complexity. Moreover, the authors have provided a novel solution which can also be used to assess the accuracies of SGA, IGA, and SIGA methods for the DS-CDMA system in Rayleigh fading environment. The solution is found to be rapid for small and moderate values of processing gain. The performance of the synchronous and asynchronous systems is compared in the same environment in terms of BER.

The references cited above consider only single path propagation. A simple method of error analysis for the DS-CDMA system over independent Rayleigh faded multipath channels is presented in [7]. The system uses SRake receiver and performance analysis is based on Holtzman's SIGA. This paper investigates the impact of power delay profile (PDP) of the channel and the number of selective combined paths of the SRake on the multiple access performance of the system and on the validity of the SGA. Though SGA is accurate, its approximation reliability reduces with increases in the number of multipaths. It is shown that SIGA gives a better approximation, without increasing the computational complexity.

In this paper, we present mathematical analysis for the proposed HIC scheme for multipath propagation in as low fading, frequency-nonselective Rayleigh fading channel. Furthermore, closed form expressions for outputs from the interference cancellation unit (ICU) of each user at various stages of interference cancellation are worked out. The scheme is also examined for convergence. The expression for the probability of error for the system is derived using the SIGA approach.

### III. RECEIVED SIGNAL MODEL

This section presents the mathematical model for the signal at the receiver input of a multiuser BPSK modulated CDMA communication system, through a flat fading frequency non-selective Rayleigh channel, on the reverse

link. It is assumed that all user signals experience mutually independent fading and is free of interference from other base stations. The system uses coherent detection and is symbol-synchronous.

The received signal,  $g(t)$ , encompassing multipath signals of all users is given by;

$$g(t) = \sum_{i=1}^N \sum_{p=1}^P \{d_i(t - \tau_{ip}) w_i(t - \tau_{ip}) \cos(\omega_0 + \omega_{di}) + \eta_{ip}\} \quad (1)$$

where, N: the total number of active users,

P: the number of multipaths,

$d_i$ : the data sequence of the  $i^{\text{th}}$  user,

$w_i$ : the  $i^{\text{th}}$  user spreading code of length G,

$\tau_{ip}$ : the transmission delay for the  $i^{\text{th}}$  user in the  $p^{\text{th}}$  path,

$\eta_{ip}$ : the noise added by the Rayleigh channel in the  $p^{\text{th}}$  path of the  $i^{\text{th}}$  user, which is a zero-mean Gaussian random variable with variance  $\sigma_n^2 = \eta_0 G/4$ ,  $\eta_0$  being the noise power, and

$\omega_{di}$ : the Doppler frequency of the  $i^{\text{th}}$  user, assumed between 60 to 100Hz for practical systems.

The  $i^{\text{th}}$  user data signal  $d_i(t)$  is a rectangular waveform which takes values  $\{+1, -1\}$  at a rate of  $R_b=1/T_b$  and is expressed as

$$d_i(t) = \sum_{j=-\infty}^{\infty} b_j^i P_{T_b}(t - jT_b), \quad (2)$$

where,  $P_{T_b}(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq T_b \\ 0 & \text{elsewhere,} \end{cases}$

and  $T_b$  is the data bit duration, and  $b_j^i$  denotes the  $j^{\text{th}}$  data bit of user  $i$ . We assume the data sources to be uniform so that the +1 and -1 symbols are equiprobable. The spreading signal  $w_i(t)$  can be expressed as

$$w_i(t) = \sum_{j=-\infty}^{\infty} w_j^i P_{T_c}(t - jT_c), \quad (3)$$

where,  $P_{T_c}(t) = \begin{cases} \pm 1 & \text{for } 0 \leq t \leq T_c \\ 0 & \text{elsewhere} \end{cases}$

with equal probability,  $T_c$  is the chip duration, and  $T_b = GT_c$ .  $\tau_{ip}$  includes the relative delay between the users as well as the channel delay.  $\{\tau_{ip}\}_{i=1}^N$  are assumed to be uniform over  $[0, T)$  and are independent random variables. Since the physical sources of generation of the random variables  $\{\tau_{ip}\}$  and  $\{b_j^i\}$  are different, they are mutually independent.

At the receiver,  $g(t)$  is coherently demodulated and passed through low pass filter. The signal at the output of the filter gives the combined base-band signal of all users with interference from all active users.

### IV. MATHEMATICAL EXPRESSIONS FOR HIC

In this section, the expressions for the output signals and the decision variables at the output of the interference cancellation unit (ICU) are derived at every stage of interference cancellation. Figure 1 shows the receiver structure [8]. The low pass filtered received signal is applied to the banks of matched filters. Each bank of filters consists of  $P$  matched filters. The outputs of the matched filters are used for ranking the users.

Figure 2 illustrates the multistage HIC detector for  $N$  users,  $S$  stages, and  $N$  groups. The internal structure of the ICU is demonstrated in Figure 3.

Before starting the interference cancellation process, all users are arranged in the decreasing order of their strength. For convenience, we refer the strongest user as user1, the second strongest user as user2, and so on. The input signal to the ICU of the user1 at the first stage is

$$\mathbf{s}_{1,1} = \mathbf{r}. \quad (4)$$

As depicted in figure 2, the decision variables,

$$\mathbf{b}_{1,1} = \mathbf{W}_1^T \mathbf{r} \quad (5)$$

are obtained in parallel by performing matched– filtering on the input signal,  $\mathbf{r}$ , which is a vector of size  $G \times 1$ .  $\mathbf{W}_i$  is the code vector matrix of the  $i^{\text{th}}$  user, of size  $P \times G$  and vector  $\mathbf{b}_{s,i}$  of size  $P \times 1$  contains the estimates of the multipaths of the  $i^{\text{th}}$  user, and  $\mathbf{s}_{s,i}$  of size  $G \times 1$  is the input signal to the  $i^{\text{th}}$  user at the  $s^{\text{th}}$  stage. The MAI incurred by this user is

$$\Delta \mathbf{I}_{1,1} = \mathbf{W}_1 \mathbf{b}_{1,1}. \quad (6)$$

The MAI is subtracted (cancelled) from  $\mathbf{s}_{1,1}$  to obtain the input to the ICU of user2. Hence,

$$\begin{aligned} \mathbf{s}_{1,2} &= \mathbf{s}_{1,1} - \mathbf{W}_1 \mathbf{b}_{1,1} \\ &= (\mathbf{I} - \mathbf{W}_1 \mathbf{W}_1^T) \mathbf{r}. \end{aligned} \quad (7)$$

The decision variables for user2 can be expressed as

$$\begin{aligned} \mathbf{b}_{1,2} &= \mathbf{W}_2^T \mathbf{s}_{1,2} \\ &= \mathbf{W}_2^T (\mathbf{I} - \mathbf{W}_1 \mathbf{W}_1^T) \mathbf{r}. \end{aligned} \quad (8)$$

The MAI experienced due to this user is

$$\Delta \mathbf{I}_{1,2} = \mathbf{W}_2 \mathbf{b}_{1,2} = \mathbf{W}_2 \mathbf{W}_2^T (\mathbf{I} - \mathbf{W}_1 \mathbf{W}_1^T) \mathbf{r}. \quad (9)$$

Cancelling this MAI from  $\mathbf{s}_{1,2}$ , the input residual to the ICU of the next user is

$$\begin{aligned} \mathbf{s}_{1,3} &= \mathbf{s}_{1,2} - \Delta \mathbf{I}_{1,2} \\ &= (\mathbf{I} - \mathbf{W}_2 \mathbf{W}_2^T) (\mathbf{I} - \mathbf{W}_1 \mathbf{W}_1^T) \mathbf{r} \\ &= \prod_{j=2}^1 (\mathbf{I} - \mathbf{W}_j \mathbf{W}_j^T) \mathbf{r} \\ &= \boldsymbol{\varphi}_2 \mathbf{r}, \end{aligned} \quad (10)$$

the decision variables are

$$\mathbf{b}_{1,3} = \mathbf{W}_3^T \mathbf{s}_{1,3} = \mathbf{W}_3^T \boldsymbol{\varphi}_2 \mathbf{r}, \quad (11)$$

and the MAI is given by

$$\Delta \mathbf{I}_{1,3} = \mathbf{W}_3 \mathbf{b}_{1,3} = \mathbf{W}_3 \mathbf{W}_3^T \boldsymbol{\varphi}_2 \mathbf{r}. \quad (12)$$

Thus, the above process of parallel estimation and serial interference cancellation continues till the last (weakest) user is cancelled as in (13a)-(13c). For the last user,

$$\mathbf{s}_{1,N} = \prod_{j=N-1}^1 (\mathbf{I} - \mathbf{W}_j \mathbf{W}_j^T) \mathbf{r}$$

$$= \boldsymbol{\varphi}_{N-1} \mathbf{r}, \quad (13a)$$

$$\begin{aligned} \mathbf{b}_{1,N} &= \mathbf{W}_N^T \mathbf{s}_{1,N} \\ &= \mathbf{W}_N^T \prod_{j=N-1}^1 (\mathbf{I} - \mathbf{W}_j \mathbf{W}_j^T) \mathbf{r} \\ &= \mathbf{W}_N^T \boldsymbol{\varphi}_{N-1} \mathbf{r}, \end{aligned} \quad (13b)$$

$$\Delta \mathbf{I}_{1,N} = \mathbf{W}_N \mathbf{b}_{1,N} = \mathbf{W}_N \mathbf{W}_N^T \boldsymbol{\varphi}_{N-1} \mathbf{r}, \quad (13c)$$

In general, at stage1,

$$\mathbf{s}_{1,n} = \boldsymbol{\varphi}_{n-1} \mathbf{r}, \quad (14a)$$

$$\mathbf{b}_{1,n} = \mathbf{W}_n^T \boldsymbol{\varphi}_{n-1} \mathbf{r}, \quad (14b)$$

$$\Delta \mathbf{I}_{1,n} = \mathbf{W}_n \mathbf{W}_n^T \boldsymbol{\varphi}_{n-1} \mathbf{r}, \quad (14c)$$

Where  $\boldsymbol{\varphi}_n = \prod_{j=n}^1 (\mathbf{I} - \mathbf{W}_j \mathbf{W}_j^T)$  and  $n=1,2,\dots,N$ ,

$$\boldsymbol{\varphi}_0 = \mathbf{I},$$

and  $\mathbf{I}$  is an identity matrix, size  $G \times G$ .

As shown in figure 2, after the cancellation of the  $N^{\text{th}}$  user at the first stage, the residual signal is directed as an input signal to the user1 at the second stage along with the MAI due to the same user at the previous stage and this residual signal is determined as

$$\begin{aligned} \mathbf{s}_{2,1} &= \mathbf{s}_{1,N+1} = \mathbf{s}_{1,N} - \Delta \mathbf{I}_{1,N} + \Delta \mathbf{I}_{1,1} \\ &= (\boldsymbol{\varphi}_N + \mathbf{W}_1 \mathbf{W}_1^T) \mathbf{r}, \end{aligned} \quad (15)$$

the decision variables for the user can be evaluated as,

$$\begin{aligned} \mathbf{b}_{2,1} &= \mathbf{W}_1^T \mathbf{s}_{2,1} + \mathbf{b}_{1,1} \\ &= \mathbf{W}_1^T (\boldsymbol{\varphi}_N + \mathbf{I} + \mathbf{W}_1 \mathbf{W}_1^T) \mathbf{r}, \end{aligned} \quad (16)$$

and the MAI due to this user as,

$$\begin{aligned} \Delta \mathbf{I}_{2,1} &= \mathbf{W}_1 (\mathbf{b}_{2,1} - \mathbf{b}_{1,1}) \\ &= \mathbf{W}_1 \mathbf{W}_1^T (\boldsymbol{\varphi}_N + \mathbf{W}_1 \mathbf{W}_1^T) \mathbf{r}. \end{aligned} \quad (17)$$

The baseband estimate of the  $i^{\text{th}}$  group at the  $(s-1)^{\text{th}}$  stage is given by,

$$\Delta \mathbf{I}_{s-1,i} = \mathbf{W}_i \mathbf{b}_{s-1,i}, \quad (18)$$

is reinserted at the  $s^{\text{th}}$  stage in order to improve the estimate of the  $i^{\text{th}}$  user. Also, the decision variables at  $s^{\text{th}}$  stage are obtained by adding the decision variables of the same user from the previous stage.

The process of parallel estimation and serial cancellation for the user2 at stage2 is expressed as,

$$\begin{aligned} \mathbf{s}_{2,2} &= \mathbf{s}_{2,1} - \Delta \mathbf{I}_{2,1} + \Delta \mathbf{I}_{1,2} \\ &= (\boldsymbol{\varphi}_N + \mathbf{W}_1 \mathbf{W}_1^T + \mathbf{W}_2 \mathbf{W}_2^T) \boldsymbol{\varphi}_1 \mathbf{r}, \end{aligned} \quad (19a)$$

$$\begin{aligned} \mathbf{b}_{2,2} &= \mathbf{W}_2^T \mathbf{s}_{2,2} + \mathbf{b}_{1,2} \\ &= \mathbf{W}_2^T (\boldsymbol{\varphi}_N + \mathbf{W}_1 \mathbf{W}_1^T + \mathbf{W}_2 \mathbf{W}_2^T + \mathbf{I}) \boldsymbol{\varphi}_1 \mathbf{r}, \end{aligned} \quad (19b)$$

$$\begin{aligned} \Delta \mathbf{I}_{2,2} &= \mathbf{W}_2 (\mathbf{b}_{2,2} - \mathbf{b}_{1,2}) \\ &= \mathbf{W}_2 \mathbf{W}_2^T (\boldsymbol{\varphi}_N + \mathbf{W}_1 \mathbf{W}_1^T + \mathbf{W}_2 \mathbf{W}_2^T) \boldsymbol{\varphi}_1 \mathbf{r}. \end{aligned} \quad (19c)$$

$$\begin{aligned} \mathbf{b}_{2,3} &= \mathbf{W}_3^T \mathbf{s}_{2,3} + \mathbf{b}_{1,3} \\ &= \mathbf{W}_3^T (\mathbf{I} + \boldsymbol{\varphi}_N + \mathbf{W}_1 \mathbf{W}_1^T + \mathbf{W}_2 \mathbf{W}_2^T + \\ &\quad \mathbf{W}_3 \mathbf{W}_3^T) \boldsymbol{\varphi}_2 \mathbf{r}, \end{aligned} \quad (20b)$$

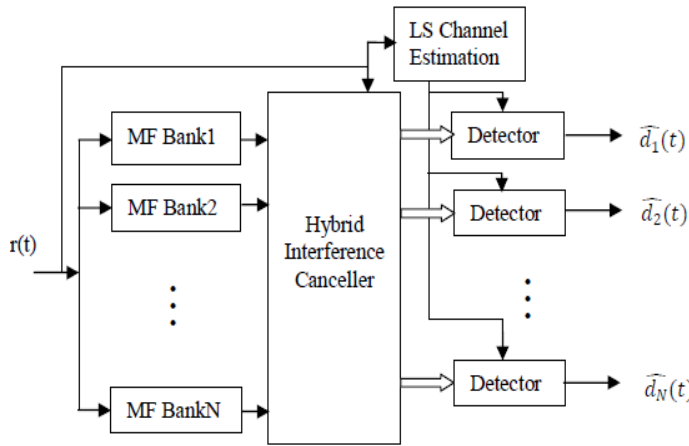


Fig. 1: Receiver block diagram.

$$\begin{aligned} \text{and } \Delta I_{2,3} &= W_1(b_{2,3} - b_{1,3}) \\ &= W_3 W_3^T (\varphi_N + W_1 W_1^T + W_2 W_2^T + W_3 W_3^T) \varphi_2 r. \end{aligned} \quad (20c)$$

Thus, continuing with the cancellation process, these signals for the last user can be written as,

$$s_{2,N} = (\varphi_N + W_1 W_1^T + W_2 W_2^T + W_3 W_3^T + \dots + W_N W_N^T) \varphi_{N-1} r \quad (21a)$$

$$\begin{aligned} b_{2,N} &= W_N^T s_{2,N} + b_{1,N} \\ &= W_N^T (I + \varphi_N + W_1 W_1^T + W_2 W_2^T + \dots + W_N W_N^T) \varphi_{N-1} r, \end{aligned} \quad (21b)$$

$$\begin{aligned} \text{And } \Delta I_{2,N} &= W_N(b_{2,N} - b_{1,N}) \\ &= W_N W_N^T (\varphi_N + W_1 W_1^T + W_2 W_2^T + W_3 W_3^T + \dots + W_N W_N^T) \varphi_{N-1} r \end{aligned} \quad (21c)$$

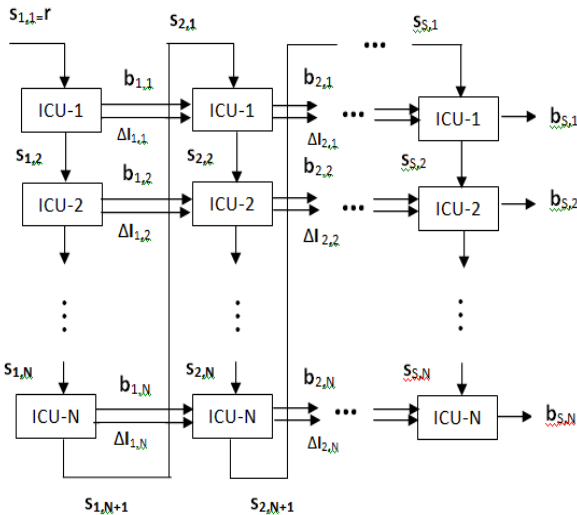


Fig. 2: Multi-stage HIC detector for N-users, N-groups, and S-stages.

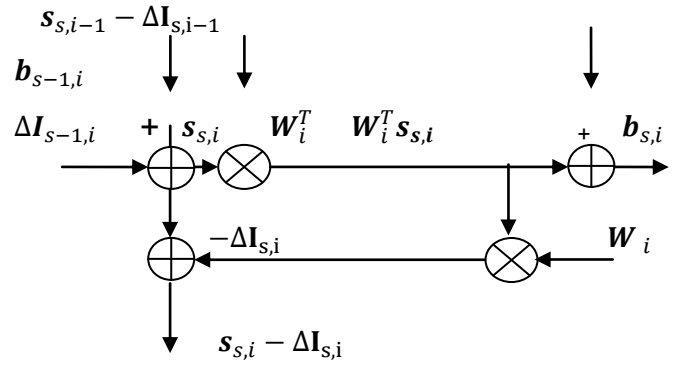
Therefore, the general expressions for the input signal to the ICU, the decision variables and the MAI at the second stage are

$$s_{2,n} = (\varphi_N + \psi_n) \varphi_{n-1} r, \quad (22a)$$

$$b_{2,n} = W_n^T (I + \varphi_N + \psi_n) \varphi_{n-1} r, \quad (22b)$$

$$\Delta I_{2,n} = W_n W_n^T (\varphi_N + \psi_n) \varphi_{n-1} r, \quad (22c)$$

respectively, where,



$s_{s,i}$ : Input residual to the  $i^{\text{th}}$  user group at the  $s^{\text{th}}$  stage

Fig. 3: Interference cancellation unit (ICU) for the  $i^{\text{th}}$  group at the  $s^{\text{th}}$  stage.

$$\begin{aligned} \psi_n &= W_1 W_1^T + W_2 W_2^T + W_3 W_3^T + \dots + W_n W_n^T \\ &= \sum_{j=1}^n (W_j W_j^T), \end{aligned}$$

and  $\psi_0 = 0$ .

Repeating the same process for obtaining the input residual signal, the decision variables and finding the MAI, the respective signals at the third stage are expressed as,

$$\begin{aligned} s_{3,1} &= s_{2,N} - \Delta I_{2,N} + \Delta I_{2,1} \\ &= (\varphi_N + 2W_1 W_1^T + W_2 W_2^T + W_3 W_3^T + \dots + W_N W_N^T) \varphi_N r + (W_1 W_1^T)^2 r; \end{aligned} \quad (23a)$$

$$\begin{aligned} b_{3,1} &= W_1^T s_{3,1} + b_{2,1} \\ &= W_1^T (I + \varphi_N + 2W_1 W_1^T + W_2 W_2^T + W_3 W_3^T + \dots + W_N W_N^T) \varphi_N r + W_1^T (W_1 W_1^T)^2 r + W_1^T \varphi_1 r \end{aligned} \quad (23b)$$

$$\begin{aligned} \text{and } \Delta I_{3,1} &= W_1 W_1^T (\varphi_N + 2W_1 W_1^T + W_2 W_2^T + W_3 W_3^T + \dots + W_N W_N^T) \varphi_N r + W_1 W_1^T (W_1 W_1^T)^2 r \end{aligned} \quad (23c)$$

Following the same approach for the user2,

$$\begin{aligned} s_{3,2} &= s_{3,1} - \Delta I_{3,1} + \Delta I_{2,2} \\ &= (\varphi_N + 2W_1 W_1^T + 2W_2 W_2^T + W_3 W_3^T + \dots + W_N W_N^T) \varphi_1 \varphi_N r + (W_1 W_1^T)^2 \varphi_1 r + (W_2 W_2^T)^2 \varphi_1 r + W_2 W_2^T W_1 W_1^T \varphi_1 r \end{aligned} \quad (24a)$$

$$\begin{aligned} b_{3,2} &= W_2^T s_{3,2} + b_{2,2} \\ &= W_2^T (I + \varphi_N + 2W_1 W_1^T + 2W_2 W_2^T + W_3 W_3^T + \dots + W_N W_N^T) \varphi_1 \varphi_N r + W_2^T (W_1 W_1^T)^2 \varphi_1 r + W_2^T (W_2 W_2^T)^2 \varphi_1 r + W_2^T W_2 W_2^T W_1 W_1^T \varphi_1 r + W_2^T (I + W_1 W_1^T + W_2 W_2^T) \varphi_1 r \end{aligned} \quad (24b)$$

$$\begin{aligned} \Delta I_{3,2} &= W_2 W_2^T (\varphi_N + 2W_1 W_1^T + 2W_2 W_2^T + W_3 W_3^T + \dots + W_N W_N^T) \varphi_1 \varphi_N r + W_2 W_2^T (W_1 W_1^T)^2 \varphi_1 r + W_2 W_2^T (W_2 W_2^T)^2 \varphi_1 r + (W_2 W_2^T)^2 W_1 W_1^T \varphi_1 r \end{aligned} \quad (24c)$$

Similarly, for the next user,

$$\begin{aligned} s_{3,3} &= s_{3,2} - \Delta I_{3,2} + \Delta I_{2,3} \\ &= (\varphi_N + 2W_1 W_1^T + 2W_2 W_2^T + 2W_3 W_3^T + \dots \end{aligned}$$



$$+W_N W_N^T \phi_1 \phi_N r + (W_1 W_1^T)^2 \phi_2 r + (W_2 W_2^T)^2 \phi_2 r + (W_3 W_3^T)^2 \phi_2 r + W_3 W_3^T (W_1 W_1^T + W_2 W_2^T) \phi_2 r, \quad (25a)$$

$$b_{3,3} = W_3^T s_{3,3} + b_{2,3} \\ = W_3^T (I + \phi_N + 2W_1 W_1^T + 2W_2 W_2^T + 2W_3 W_3^T + \dots + W_N W_N^T) \phi_1 \phi_N r + W_3^T (W_1 W_1^T)^2 \phi_2 r + W_3^T (W_2 W_2^T)^2 \phi_2 r + W_3^T (W_3 W_3^T)^2 \phi_2 r + W_3^T W_2 W_2^T W_1 W_1^T \phi_2 r + W_3^T W_3 W_3^T (W_1 W_1^T + W_2 W_2^T) \phi_2 r + W_3^T (I + W_1 W_1^T + W_2 W_2^T + W_3 W_3^T) \phi_2 r, \quad (25b)$$

$$\Delta I_{3,2} = W_2 W_2^T (\phi_N + 2W_1 W_1^T + 2W_2 W_2^T + W_3 W_3^T \dots + W_N W_N^T) \phi_1 \phi_N r + W_2 W_2^T (W_1 W_1^T)^2 \phi_1 r + W_2 W_2^T (W_2 W_2^T)^2 \phi_1 r + (W_2 W_2^T)^2 W_1 W_1^T \phi_1 r. \quad (25c)$$

Thus, for the weakest user at the third stage,

$$s_{3,N} = (\phi_N + 2\psi_N) \phi_{N-1} \phi_N r + \Lambda_N \phi_{N-1} r + Z_N \phi_{N-1} r, \quad (26a)$$

$$b_{3,N} = W_N^T (I + \phi_N + 2\psi_N) \phi_{N-1} \phi_N r + W_N^T \Lambda_N \phi_{N-1} r + W_N^T Z_N \phi_{N-1} r + W_N^T (I + \psi_N) \phi_{N-1} r, \quad (26b)$$

$$\Delta I_{3,N} = W_N W_N^T (\phi_N + 2\psi_N) \phi_{N-1} \phi_N r + W_N W_N^T \Lambda_N \phi_{N-1} r + W_N W_N^T Z_N \phi_{N-1} r, \quad (26c)$$

where,  $\Lambda_n = \sum_{j=1}^n (W_j W_j^T)^2$ ,  
and  $Z_n = \sum_{j=2}^n (W_j W_j^T \psi_{j-1})$ .

Equation (26a)-(26c) can be expressed in general as

$$s_{3,n} = (\phi_N + \psi_N + \psi_n) \phi_{n-1} \phi_N r + \Lambda_n \phi_{n-1} r + Z_n \phi_{n-1} r, \quad (27a)$$

$$b_{3,n} = W_n^T (I + \phi_N + \psi_N + \psi_n) \phi_{n-1} \phi_N r + W_n^T \Lambda_n \phi_{n-1} r + W_n^T Z_n \phi_{n-1} r + W_n^T (I + \psi_n) \phi_{n-1} r, \quad (27b)$$

and

$$\Delta I_{3,n} = W_n W_n^T (\phi_N + \psi_N + \psi_n) \phi_{n-1} \phi_N r + W_n W_n^T \Lambda_n \phi_{n-1} r + W_n W_n^T Z_n \phi_{n-1} r. \quad (27c)$$

The process of reinsertion of baseband estimates from the previous stage, parallel estimation and interference cancellation further continues for the desired number of stages.

## V. CONVERGENCE ANALYSIS

The proposed HIC is analyzed for convergence in this section. As discussed in section III, the decision variables at  $(s+1)^{\text{st}}$  stage are expressed as

$$b_{s+1,i} = W_i^T s_{s+1,i} + b_{s,i} \quad (28a)$$

$$s_{s+1,i} = r - \sum_{k=1}^N W_k b_{s,k} + \sum_{j=1}^i W_j b_{s,j} - \sum_{m=1}^{i-1} W_m b_{s+1,m} \\ = r - \sum_{k=i+1}^N W_k b_{s,k} - \sum_{m=1}^{i-1} W_m b_{s+1,m}. \quad (28b)$$

As a result,

$$b_{s+1,i} = W_i^T [r - \sum_{k=i+1}^N W_k b_{s,k} - \sum_{m=1}^{i-1} W_m b_{s+1,m}] + b_{s,i}. \quad (28c)$$

For convergence at  $s^{\text{th}}$  stage,

$$b_{s+1,i} = b_{s,i}, \quad (29a)$$

$$\therefore W_i^T [r - \sum_{k=i+1}^N W_k b_{s,k} - \sum_{m=1}^{i-1} W_m b_{s,m}] = 0, \quad (29b)$$

$$W_i^T r = W_i^T W b_s + W_i^T W_i b_{s,i}, \quad \text{for all } 1 \leq i \leq N \quad (29c)$$

Hence,

$$W^T r = W^T W b_s + W^T W_i b_s \\ = 2W^T W_i b_s. \quad (30)$$

Therefore, the decision statistics at the  $s^{\text{th}}$  stage are expressed as,

$$b_s = 2(W^T W)^{-1} W^T r. \quad (31)$$

Thus,  $b_s$  converges to a decorrelator [3].

## VI. PROBABILITY OF ERROR

Assuming  $j=1$  as the desired user, the variance of MAI can be represented as

$$\chi = \sum_{j=2}^N X_j \quad (32)$$

where,

$$X_j = \sum_{l=1}^P a_l^2 [2(D_{j,l}^2 - D_{j,l})(2C + 1) + G] \cos^2 \theta_{j,l}$$

are independent identically distributed (iid) random variables,  $D_{j,l}$  are chip delays uniformly distributed over  $[0,1]$ ,  $\theta_j$  are carrier phases of the remaining users with respect to user1 and  $C$  is the number of times the chip boundaries change state in one bit duration for user1.  $C$  is binomially distributed with a minimum value of 0, an average value of  $(N-1)/2$ , and a maximum value of  $N-1$ .  $P$  is the number of paths for each user with independent fading over different paths.

Since the SGA of MAI is not so accurate and the IGA method requires the knowledge of the distribution of  $X_j$ , we use SIGA method, which is more accurate and needs only the mean and variance of  $\chi$ , hence is a better choice for the BER analysis.

The mean of  $\chi$  using SIGA can be obtained as

$$\mu = \frac{(N-1)G}{3} \sum_{l=1}^P E[a_l^2], \quad (33)$$

$$\text{where, } E[a_l^2] = \begin{cases} \frac{1}{P} & \text{for flat PDP,} \\ \frac{1 - \exp(-\frac{\kappa}{P})}{1 - \exp(-\kappa)} & \text{for decaying PDP} \end{cases}$$

and  $\kappa > 0 \quad l=1,2,\dots,P$ .

For paths having independent fading, the variance of  $\chi$  can be obtained as

$$\sigma^2 = \text{var}[\chi] = (N-1)[E[X_j^2] - E[X_j]^2] \\ \approx \frac{23(N-1)N^2}{360} (\sum_{l=1}^P E[a_l^2])^2. \quad (34)$$

From the overall BER in a Rayleigh fading channel [6,7], the BER performance can be expressed as

$$P_{e,SIGA} = \frac{2}{3}P(\mu) + \frac{1}{6}P(\mu + \sqrt{3}\sigma) + \frac{1}{6}P(\mu - \sqrt{3}\sigma)$$

$$\approx \frac{2}{3} \left[ 1 - \frac{G}{\sqrt{\mu + \frac{G\eta_0}{4} + G^2}} \right] + \frac{1}{6} \left[ 1 - \frac{G}{\sqrt{\mu + \sqrt{3}\sigma + \frac{G\eta_0}{4} + G^2}} \right] + \frac{1}{6} \left[ 1 - \frac{G}{\sqrt{\mu - \sqrt{3}\sigma + \frac{G\eta_0}{4} + G^2}} \right] \quad (35)$$

## VII. RESULT AND DISCUSSION

This section presents the theoretical and simulation results of the BER performance of the proposed HIC scheme. Simulation is performed, considering three multipaths per user. It is assumed that all paths are significant, and the processing gain is assumed as  $G=64$ . First, the users are ranked based on their total received strengths, using the outputs of the matched filters. Then the users are arranged in the decreasing order of their strength and cancelled using HIC. Channel estimation is implemented using the LS algorithm.

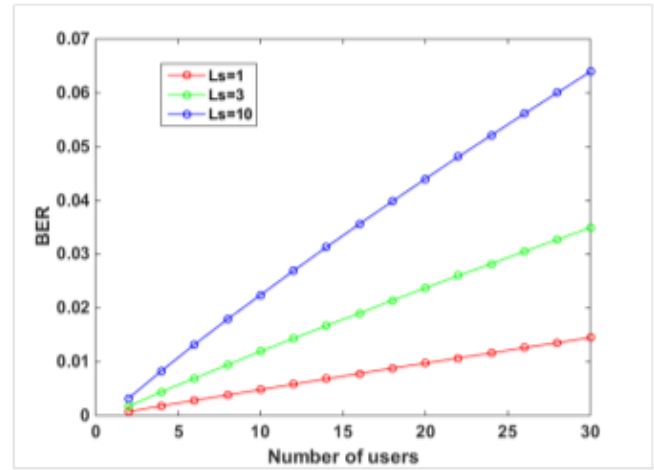
Figures 4 and 5 show the theoretical BER graphs using equation (35). Figure 4 illustrates the BER obtained for exponentially decaying PDP, assuming  $\kappa=10$ ,  $P=50$ , and the number of multipaths,  $L_s=1, 3$ , and  $10$ . Figure 5 shows the BER plot for flat PDP. It can be observed in both figures that the performance degrades with increasing  $L_s$ . As the number of multipaths increase, interference proportionally increases. BER for flat PDP shows better performance over exponentially decaying PDP. Also, for a channel having exponentially decaying PDP, the variation in the BER performance with variations in  $L_s$  for a fixed value of  $N$ , are larger than in the channel having flat PDP.

Figure 6 gives the theoretical and simulated system BER performance graphs for the designed DS-CDMA system using HIC. In both cases, three significant multipaths are considered for all users. Exponentially decaying PDP is considered since it is the practical choice for high data rate communication. It can be observed that the performances are comparable.

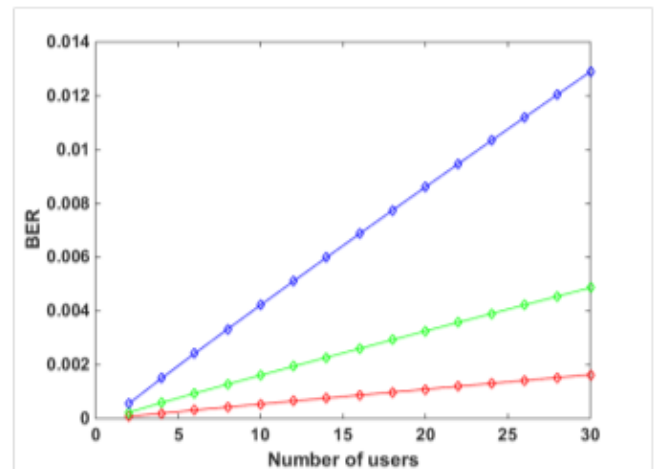
## VIII. CONCLUSION

This paper presents the mathematical expressions for the output signal, the decision variables, and the MAI for each user at various stages of interference cancellation of the proposed HIC detector. The closed-form expressions for these outputs are also derived. The detector is analyzed for convergence. It is proved that the interference canceller converges to a decorrelator.

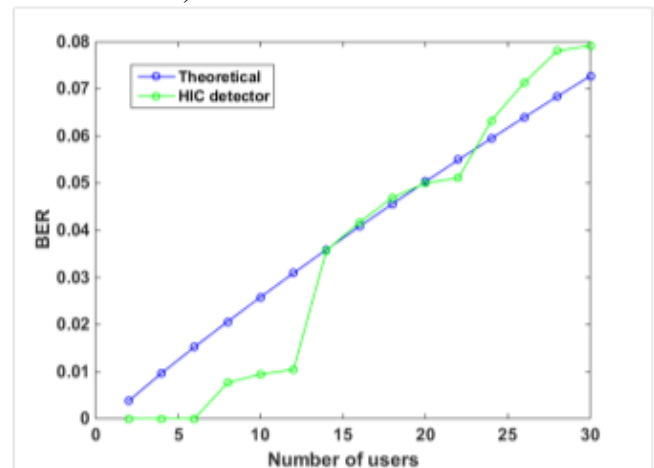
The expression for the probability of error is also obtained for the DS-CDMA system operating over slow-fading, non-frequency selective Rayleigh channel. The derived expression is used to compare the BER performance of the detector, theoretically and using system simulation, in a Rayleigh fading channel. The results are comparable with a deviation of 21%.



**Fig4: BER Vs Number of users for channels having exponentially decaying PDP, with  $C=10$ ,  $P=50$  at  $SNR=20dB$  for SIGA.**



**Fig.5: BER Vs Number of users for channels having flat PDP,  $P=50$  at  $SNR=20dB$  for SIGA.**



**Fig. 6: BER performance vs. number of users at  $SNR=20dB$ .**

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