

Automation and Process Control of High Performance Concrete Production by using Visual Interactive Analysis Method



Minh-Hong Nguyen, Hoang-Minh Dang

Abstract: *High performance concrete (HPC) has been used widely for many applications such as high-rise building, highway bridges, tunnels, ect. Although up to the present time there are many recommendations and guidelines issued from research institution on the method how to achieve HPC mix proportion, around the world engineers often spent a lot of time to do preliminary experimental tests in order to select the optimal mix especially when dealing with tailored-made HPC. This paper presents an innovative visual interactive analysis method (VIAM) with the application of parameter space investigation, which helps engineer efficiently in automation and process control of proportioning HPC mix. Component content of tailored-made HPC is considered as a solution of a mathematical model with six objective functions controlling by ten constraints and seven parameters. Indeed, VIAM has played an important role in successfully determining 18 Pareto optimal solutions of this complex model for three particular scenarios set by engineers.*

Keywords: *Automation and process control, High performance concrete, Mix proportion, Multi-objective optimization, Visual interactive analysis method, VIAM.*

I. INTRODUCTION

Over the years, concrete has been used for building construction widely, whether in moderate aggressive environments, or in strongly aggressive environments. It is attributed to the fact that it possesses excellent water resistance, can be moulded into any shapes, and for being cheaper and more easily available in the field [1,2]. To illustrate such statement, it is estimated that more than 20 billion tonnes of Portland cement concrete is produced annually making it the world's most widely used manufactured material [3]. On the other hand, due to the recent trends in construction industries (i.e., increased number of heavily reinforced concrete structures), construction of large and taller structures, the industries and companies in general strive to cast massive volume of concrete.

When this large volume of concrete is used for construction, the safety and durability of cast concrete become fundamental issues [4,5]. To ensure these issues, much effort has been focused on the developments of high performance concrete (HPC) [6].

Tailored-made HPC is designed to give optimized performance characteristics for a given set of materials, usage, and exposure conditions, consistent with strength, workability, service life, and durability. Around the world, engineers are finding that using HPC allows them to build more serviceable structures at comparable cost. HPC is being used for structures in severe environments: marine structures, highway bridges and viaducts, tunnels, etc. [7,8]. The major difference between conventional concrete and HPC is essentially the use of chemical and mineral admixtures. The use of chemical admixtures reduces the water content, thereby at the same time reduces the porosity within the hydrated cement paste. While, mineral admixtures, also called as cement replacement materials, act as pozzolanic materials as well as fine fillers; thereby, the microstructure of hardened cement matrix becomes denser and stronger. Thus, the combined use of superplasticizer and cement replacement materials can lead to economical HPC with enhanced strength, workability, and durability [9].

To obtain the special combinations of performance, uniformity requirements and reasonable cost of 1m³, at present engineers have mainly to use guidelines and recommendations issued from official research institution and carry out many preliminary tests.

The most important step is a process control in which engineers have to select the optimal mix that suits properly the technical requirements. To do that several authors have attempted to use advanced optimization methods such as genetic algorithm [10], artificial neural networks [11,12], and/or multi-objective optimization [13], in which only two criteria including chlorine ion diffusion coefficient and concrete cost were considered.

However, in order to achieve tailored-made HPC, several criteria such as the amounts of costly components like cement, fly ash, slag and, chemical admixtures need to be involved when proportioning HPC mix [14-16].

As a result of that, an innovative visual interactive analysis method (VIAM) with application of parameter space investigation is proposed to deal with automation and process control in this paper.

Revised Manuscript Received on April 30, 2020.

* Correspondence Author

Minh-Hong Nguyen*, PhD, Lecturer at Le Quy Don Technical University, 236 Hoang Quoc Viet Str., Hanoi, Vietnam.

Hoang-Minh Dang, PhD, Lecturer at Industrial University of Ho Chi Minh City, 12 Nguyen Van Bao, W. 4, D. Go Vap, Ho Chi Minh City, Vietnam.

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an [open access](https://creativecommons.org/licenses/by-nc-nd/4.0/) article under the CC-BY-NC-ND license ([http://creativecommons.org/licenses/by-nc-nd/4.0/](https://creativecommons.org/licenses/by-nc-nd/4.0/))

II. PROBLEM STATEMENT

In this work, HPC consists of six constituent materials such as Portland cement, fly ash, slag, sand, stone, chemical admixtures, and water. Hence, a mathematical model is developed for optimizing HPC mix. It includes 7 control parameters ($x_1 - x_7$). Symbols, meaning, units and admissible values of these parameters are given in Table I.

With every vector $\{x\} = (x_1, x_2, K, x_7)$ of these 7 parameters, it needs to define 10 functional constraints vector $\{f\} = (f_1, x_2, K, f_{10})$. Functions, expressions, type of

constraints, and their meaning are described in Table II. If only one of those 10 functional constraints $f_i(x)$, $i = 1 \dots 10$ does not comply with the requirement, parameter vector will be considered invalid, and it needs to find for another valid vector. Any vector, which "passes" through those 10 functional constraints, will be used for determining 6 function values $\{\Phi\} = (\Phi_1, \Phi_2, K, \Phi_6)$, which is a goal of HPC mix proportion. Criteria or objective functions, optimization tendency (MIN or MAX), expressions, and meaning of criteria are included in Table III.

Table- I: Control parameters and their ranges

Control parameter	Constituent material	Units	Initial lower admissible value	Initial upper admissible value
x_1	Portland cement	kg/m ³	300	500
x_2	Water	kg/m ³	130	210
x_3	Fly ash	kg/m ³	45	155
x_4	Fine slag	kg/m ³	60	200
x_5	Sand	kg/m ³	500	1000
x_6	Stone	kg/m ³	900	1400
x_7	Chemical admixtures	kg/m ³	2.5	12

Table- II: Functional constraints

Function	Expression	Type of constraint	Meaning
f_1	$-\frac{x_2}{x_1 + x_3 + x_4} + 0.2$	≤ 0	The range of water to binder ratio
f_2	$\frac{x_2}{x_1 + x_3 + x_4} - 0.4$	≤ 0	
f_3	$-\frac{x_5}{x_5 + x_6} + 0.35$	≤ 0	The range of sand ratio, which is the ratio of the amount of sand to the amount of overall aggregates
f_4	$\frac{x_5}{x_5 + x_6} - 0.4$	≤ 0	
f_5	$450 - (x_1 + x_3 + x_4)$	≤ 0	The range of the amount of cementitious material including cement, fly ash and slag.
f_6	$x_1 + x_3 + x_4 - 600$	≤ 0	
f_7	$-\frac{x_7}{x_1 + x_3 + x_4} + 0.01$	≤ 0	Superplasticizer is used to improve the workability and micro-structure of concrete. These are its ratio to cement
f_8	$\frac{x_7}{x_1 + x_3 + x_4} - 0.02$	≤ 0	
f_9	$\sum_{i=1}^7 \frac{x_i}{\rho_i} - 990$	$= 0$	The volume of concrete mixture is made up of the absolute volume of each constituent material and the volume of the air captured in the mixture
f_{10}	$-0.304\lambda_c f_{ce,k} \left(\frac{x_1 + x_3 + x_4}{x_2} + 0.62 \right) + f_{cu,k} - t\sigma$	≤ 0	The strength of concrete, which is affected by various factors, is the most important parameter in concrete design

Where, ρ_i ($i = 1..7$) represents the density of each ingredient (ton/m³): $\rho_1 = 3.11$; $\rho_2 = 1$; $\rho_3 = 2.11$; $\rho_4 = 2.45$; $\rho_5 = 2.61$; $\rho_6 = 2.76$; $\rho_7 = 1.08$. λ_c is the affluence coefficient of the strength class of concrete. It should be determined according to statistics and in general cases it can be 1.13; $f_{ce,k}$ represents the grading strength of cement and $f_{ce,k} = 60$; $f_{cu,k}$ is the

standard value of compressive strength of concrete and $f_{cu,k} = 68$; t is the degree of probability and $t = -1.64$; σ is the standard deviation of concrete strength. It is determined according to the national standard code for acceptance of constructional quality of concrete structure and $\sigma = 5$.

Table- III: Performance criteria or objective functions

Criteria	Expression	Meaning
$\Phi_1 \rightarrow$ MIN	$\left\{ \left[5.760 + 5.81 \cdot \left(\frac{x_2}{x_1 + x_3 + x_4} - 0.45 \right) \right] / 0.2 - 0.567 \cdot (x_1 + x_3 + x_4 - 425) / 175 + 1.323 \right.$ $+ 0.74 \cdot \left(\frac{x_3}{x_1 + x_3 + x_4} \cdot 100 - 22.5 \right) / 22.5 - 2.117 \cdot \left(\frac{x_4}{x_1 + x_3 + x_4} \cdot 100 - 35 \right) / 35$ $- (2.78 \cdot 0.472) - (0.254 \cdot 0.286) - (0.368 \cdot 1) + 1.171 \cdot \frac{\frac{x_2}{x_1 + x_3 + x_4} - 0.45}{0.2} \cdot \frac{\frac{x_3}{x_1 + x_3 + x_4} \cdot 100 - 22.5}{22.5}$ $\left. - 2.891 \cdot \frac{\frac{x_2}{x_1 + x_3 + x_4} - 0.45}{0.2} \cdot 0.472 - 1.053 \cdot \frac{\frac{x_3}{x_1 + x_3 + x_4} \cdot 100 - 22.5}{22.5} \cdot 0.472 \right\}$ $\frac{365 \cdot 24 \cdot 3600 \cdot 10^6}{}$	Chlorine ion diffusion coefficient on the 28th day for concrete without micro silica under a molding temperature of 21 Celsius degree, m^2/s
$\Phi_2 \rightarrow$ MIN	$\sum_{i=1}^7 (y_i \cdot x_i)$	Per cubic meter cost, (cu/ m^3)
$\Phi_3 \rightarrow$ MIN	x_1	Amount of Portland cement per cubic meter, kg/m^3
$\Phi_4 \rightarrow$ MIN	x_3	Amount of Fly ash per cubic meter, kg/m^3
$\Phi_5 \rightarrow$ MIN	x_4	Amount of Fine slag per cubic meter, kg/m^3
$\Phi_6 \rightarrow$ MIN	x_7	Amount of Chemical Admixtures per cubic meter, kg/m^3

Where y_i ($i = 1 \dots 7$) the unit price of each component (currency unit/kg), taking into account that currency is the one of the local country-Vietnam, where the research is taken place: $y_1 = 1500$; $y_2 = 12$; $y_3 = 550$; $y_4 = 5050$; $y_5 = 118$; $y_6 = 135$; $y_7 = 21000$.

The second approach is prone to determine optimization solution in “one-way” manner. It means that after setting a model based on the certain algorithms, the obtained solutions correspond to the fixed function value. Although, they could be Pareto optimal solutions, they might not comply with the requirements. Let’s consider a simple example such as for an obtained solution (with - a vector with N parameters of a Pareto optimal solution, - function value i , $i = 1..M$ derived from Pareto solution, M is a number of functions), a single-objective function might have a very good value and it excels the expectation ($j = 1..M$), but the another function might have yield a bad value and it does not comply with the engineer’s requirement ($k = 1..M$, $k \neq j$). Thus, though in this case the solution is Pareto solution, it might not be used for optimizing HPC mix.

Therefore, VIAM is proposed to deal with the existing drawbacks in solving the multi-objective optimization problem in general and in particular for proportioning HPC mix [18]. The main idea of this method includes: i) set up an interactive panel, containing the domain function values, which complies with all constraints; ii) based on the current circumstance and determined production demand, it needs to define the threshold values of these functions (the threshold is within the domain value in the interactive panel); iii) the final step is to define the parameter vectors in compliance with the threshold values.

In practice, there are several ways to set an interactive panel, and indeed VIAM with a single-objective optimization technique was used in the latest studies of the authors [16].

Although this technique results in valid solutions, it does not consider the correlation between parameter and function. Besides, an application of a single-objective optimization techniques within VIAM might cause difficulties of determining global solutions, because most of them need to have good initial points. Due to this, VIAM with an application of parameter space investigation (PSI), as shown in Fig 1, is used in this work for dealing with multi-objective optimization problem. In fact, PSI approach, developed by Statnikov [17, 19-23], is used for HPC as follows: first, it needs to build a model of HPC mix proportion {1}, including objects such as parameters, functions and constraints which ensures technical requirements of HPC. Next, it has to fill N-dimensional space of parameters with a huge amount of test point vector by using uniformly distributed sequences {2}, like LPTau. Then, it defines function values to check every constraint {3}. Like “a filter”, invalid values are abolished, and only valid vectors are left and they will be placed into the interactive panel {4}. For the vectors passing through the filter, it needs to define function values and place them with a diminishing order from the top to the bottom (On top there are best values, like the minimum value of function i , at the bottom there are least values, like the maximum value of function i). With this interactive panel, engineer are able to determine a threshold value for every function, as they are highlighted in the panel {4} – Fig 1. Complying with the threshold value, the solution will go inside a “desired area” in the M -dimensional space {5}.

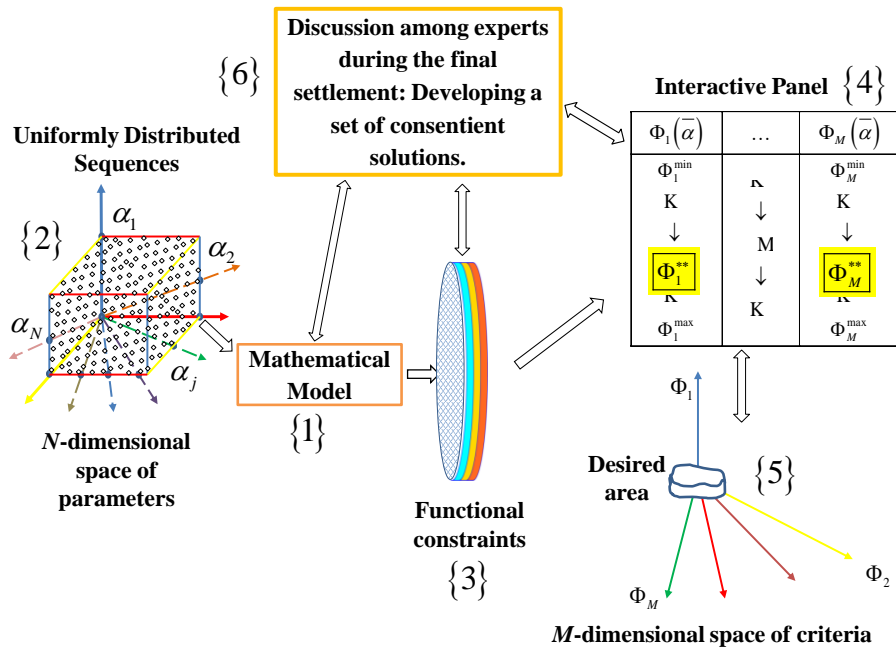


Fig. 1. Parameter space investigation approach within VIAM

However, it is noteworthy that in many cases there are few valid solutions or even not exists. The aim of filling parameter space is to investigate the global circumstance, but with a limited number of test points {1}, it is occasionally impossible to find valid solution. PSI approach focuses on uniformly distributed sequences LPtau that frequently fix the position of test points in the parameter space, thus in order to increase the possibility to be closer to the “desired area” there is no alternative rather than augmenting a number of test points. Besides, according to algorithm feature of sequence LPtau, the test points are quite far from the marginal values.

Fig 2a shows that there are few test points being at margins. In addition, test point position created by the sequence LPtau in Sobol’s algorithm is always fixed at each initialization [1-2]. Thus, the distribution image (Fig 2a) is unchangeable with a determined amount of test points. This is considered to be a major disadvantage for mathematical problems, in which the solution lies at the boundary. Since for the random uniformly distributed sequences like Windows, Halton the position of test points in parameter space is not fixed at each initialization, the multiple use of this sequence can deal with the mentioned issue in the sequence LPtau. While looking into Fig 2b at the boundary, it is seen that there are more solutions at the sequence Windows in comparison with the sequence LPtau. Moreover, the solution distribution rule of the sequence Windows changes constantly that allows for testing and storing calculation results many times with not so many test points, that it is possible to obtain valid solutions. In other words, for instance, in order to fill the parameter space according to the sequence LPtau, it needs to fill it with a huge amount of test points ($\approx 2^{17} \approx 130\,000$), while within the same parameter space in accordance with the sequences Windows, Halton it is possible to fill it up to 10 times. Every time with approximately 13 000 test points could result in the equivalent filling efficiency, besides calculation and data acquisition of test points according to the random sequences might be carried out conveniently and less time-consuming.

Therefore, in this work the authors propose space-filling techniques with several methods for dealing with scenarios of solution non-existence, as follows:

- Increase amount of point to fill the space;
- Alter the uniformly distributed sequences (LPtau, Windows, Halton);
- Alter the mathematical model itself such as boundary contour of parameter, constraints and criteria;
- Step by step with a gradual analysis and determine accurately valid solution domain by resolve the problem repeatedly while adjusting the mathematical model.

Next section presents a detailed process of determining solution for selecting optimal HPC mix.

III. RESULTS AND DISCUSSION

Based on the VIAM algorithm with filling parameter space, the calculation is presented as follows:

Step 1: Determine an initial valid domain complying with the functional constraints

It is necessary to fill the space of 7 parameters with $2^{16} = 65\,536$ test point vectors of the uniformly distributed sequence LPtau. Based on this, it is possible to determine the values of 10 functional constraints f_i ($i = 1..10$), nevertheless none of those vectors passes through the filter of these 10 functional constraints and goes into the interactive panel. Next, the other 90000 test point vectors of uniformly distributed sequence Windows are evaluated under the 10 functional constraints. But, there is still no valid solution. At this point, it needs to analyze the constraints, which are not suitable, that causes an exclusion of initial test point vectors for both of calculations ($65\,536 + 90\,000 = 155\,536$). The numbers of test point vectors infringe upon each constraint and the closest values to the constraint condition that can be achieved for each constraint are included in Table IV.

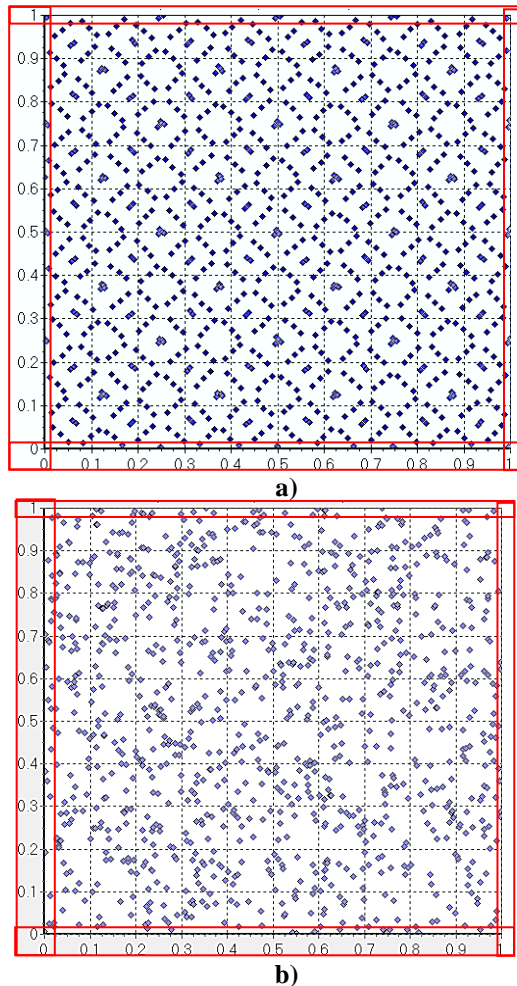


Fig. 2. a) Fixed uniformly distributed sequence LPtau of 2 parameters with 1024 test points; b) Windows random uniformly distributed sequence of 2 parameters with 1024 test points

Table IV shows that the first nine constraints are not satisfactory, i.e. there is no valid solution going into the interactive panel. As a matter of fact, a significant increase of test point can be done, however this does not provide a way to determine the required amount in the general case, it means that the search time is uncontrollable. Herein, it might need to change the mathematical model, or namely the threshold value of the constraint conditions. According to Table IV, a significant infringement occurs at the constraints 3 and 4, though the obtained value is still close to zero, thus it decides to modify these two constraints such as $f_3 \leq 10^{-4}$ and $f_4 \leq 10^{-4}$. While, a moderate infringement occurs at the constraints 1, 2, 5, 7, 8, and similarly the value is close to zero, hence preliminarily these constraints would be: $f_1 \leq 10^{-5}$, $f_2 \leq 10^{-5}$, $f_5 \leq 10^{-5}$, $f_7 \leq 10^{-5}$, $f_8 \leq 10^{-5}$. Besides, the constraint f_9 , which

implies a constant design concrete volume of 1m^3 or 1000 litre, rarely occurs since after 155 536 test points the closest value is 0.021 instead of zero. Thus, it has to modify this constraint such as $f_9 = \pm 1$, i.e. the total design concrete volume (1000 litre) can be varied in the range of 1 litre.

Table- IV: List of test point vectors violating against the constraint condition

Constraint	Unsatisfied test point vector [LPtau] +[Sequence Windows]	The closest values to the constraint condition [LPtau] /[Sequence Windows]	Percentage of solutions infringing upon the constraint
$f_1 \leq 0$	3711 + 5020 = 8731	2.66E-06/4.79E-07	5.61%
$f_2 \leq 0$	906 + 1313 = 2219	5.40E-05/3.12E-05	1.43%
$f_3 \leq 0$	14569 + 19931 = 34500	3.94E-07/1.37E-06	22.18%
$f_4 \leq 0$	28437 + 39303 = 67740	1.15E-06/6.56E-06	43.55%
$f_5 \leq 0$	57 + 82 = 139	3.14E-01/5.22E-01	0.09%
$f_6 \leq 0$	11237 + 15363 = 26600	9.16E-04/1.64E-02	17.1%
$f_7 \leq 0$	2085 + 2882 = 4967	1.11E-06/1.49E-07	3.19%
$f_8 \leq 0$	649 + 894 = 1543	3.78E-07/4.97E-07	1%
$f_9 = 0$	3885 + 5212 = 9097	2.08E-02/4.87E-02	5.85%
$f_{10} \leq 0$	0	0/0	0%

Once the modified model is resolved, with an initial parameter space filling of 3×2^{15} test points within the sequence LPtau, there are 25 valid solutions, they pass through the filter and go directly into the interactive panel. However, they are still useless, because the constraint f_9 was extended moderately. These solutions only imply that it is close to “valid domain”. In order to determine this domain, it needs to observe the solution distribution charts, which are presented in Fig. 3 – 9.

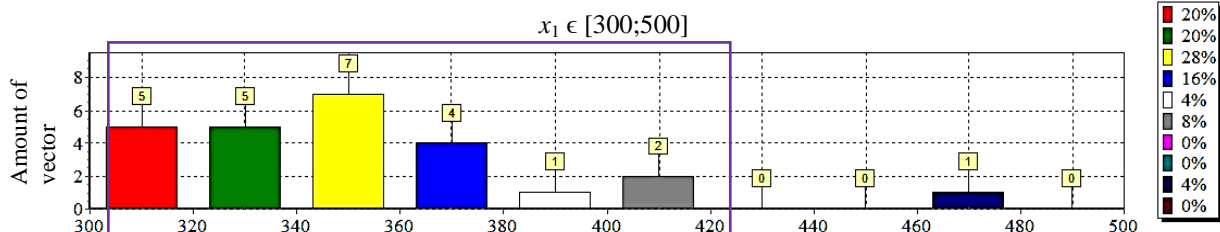


Fig. 3. Distribution of valid solution in accordance with parameter x_1

$$x_2 \in [130;210]$$

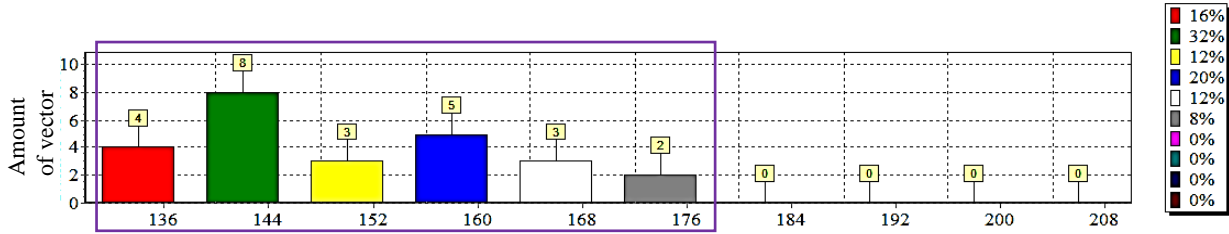


Fig. 4. Distribution of valid solution in accordance with parameter x_2

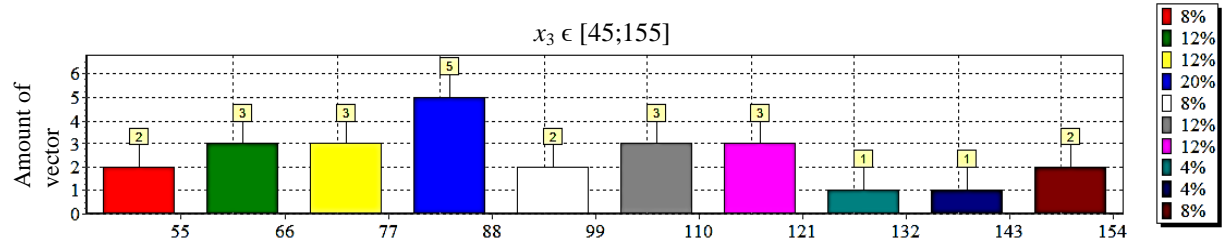


Fig. 5. Distribution of valid solution in accordance with parameter x_3

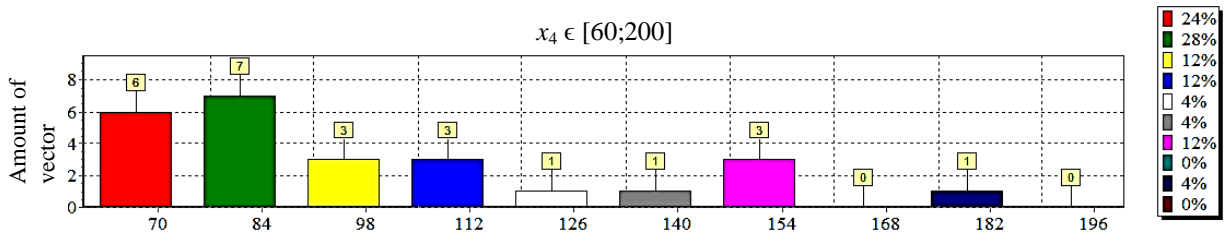


Fig. 6. Distribution of valid solution in accordance with parameter x_4

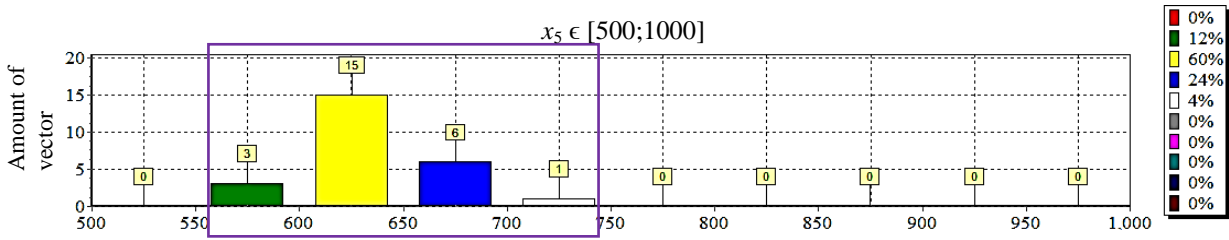


Fig. 7. Distribution of valid solution in accordance with parameter x_5

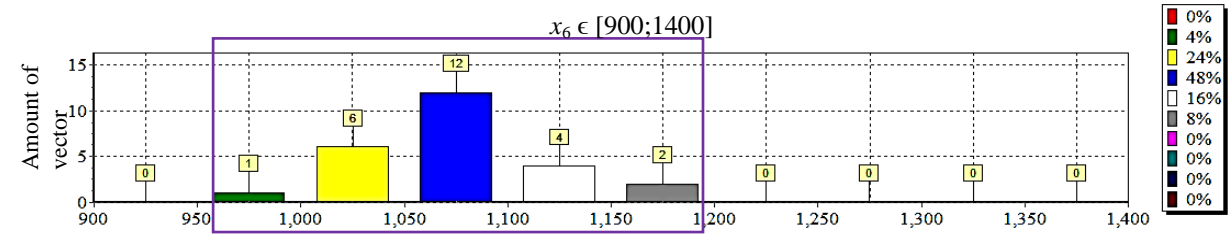


Fig. 8. Distribution of valid solution in accordance with parameter x_6

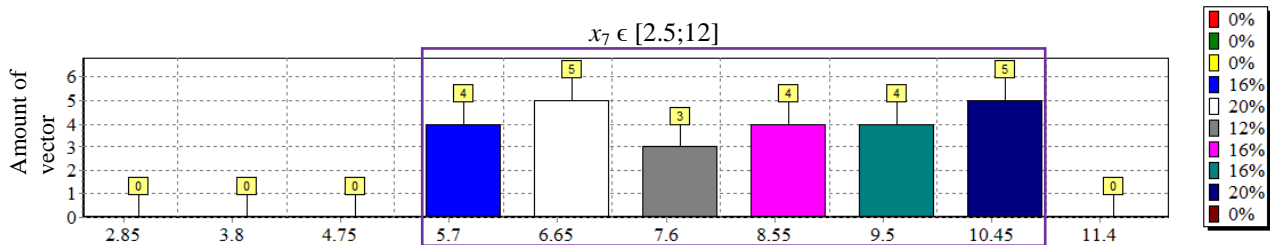


Fig. 9. Distribution of valid solution in accordance with parameter x_7

In these charts, the defined domain of parameters is divided into 10 intervals; each of them presents a rectangular, on top of which there is a number indicating the amount of valid solutions (if available) that parameter value falls into. The violet hollow rectangle surrounding the rectangles represents the domain of valid solution for parameter.

Looking into the parameter x_1 in Fig. 3, the domain of valid solution is distributed in $[300; 480]$, nevertheless in $[420; 460]$ there is no valid solution. Besides, most of valid solutions stay in the range of $[300, 420]$,

thus it is acceptable to eliminate a single solution in [460; 480]. If the space is extended up to 480, solution search at the further step could be complicated and inefficient, because beyond 420 there are a few solutions available. Therefore,

based on these 7 charts (Fig. 3 - 9), a new domain of parameter is obtained and included in Table V.

Table- V: New domain of parameters – 1st modification

$x_1 \hat{I}$	$x_2 \hat{I}$	$x_3 \hat{I}$	$x_4 \hat{I}$	$x_5 \hat{I}$	$x_6 \hat{I}$	$x_7 \hat{I}$
[300;420]	[130;180]	[45;155]	[60;200]	[550;750]	[950;1200]	[4.75;11.4]

With the new domain, the process carries on by filling the space with 3×2^{14} test point vectors of the uniformly distributed sequence LPTau. At this point, there are 192 valid solutions. All of them belong to the 10 intervals of the distribution charts, thus it has not to modify the domain. But, at this time it needs to squeeze again the ninth constraint. It recommends that the admissible value $f_9 = \pm 0.02$ (i.e. in practice there is a deviation of 2% in terms of concrete volume due to air entrainment in mixing process).

On the other hand, using the uniformly distributed sequence Windows with 80000 test points, there are only 4 valid solutions, the distribution charts of these solution are

similar to those presented in Fig. 3 - 9. Based on this, it is possible to modify the domain of parameters for the second time, and a new domain of parameter is included in Table VI. At this time, it has to fill the parameter space in Table VI with 3×2^{15} test points by using the sequence LPTau. Thus, there are 40 valid solutions in compliance with 10 functional constraints. After computing the function values, they are included in the interactive panel, as shown in Table VII and Table VIII.

Table- VI: New domain of parameters – 2nd modification

$x_1 \rightarrow$	$x_2 \rightarrow$	$x_3 \rightarrow$	$x_4 \rightarrow$	$x_5 \rightarrow$	$x_6 \rightarrow$	$x_7 \rightarrow$
[300;360]	[130;165]	[45;110]	[70;170]	[620;700]	[975;1200]	[4.75;9.975]

Step 2: Setting the interactive panel and determination of threshold values for the function

In the interactive tables (Table VII – Table VIII), the function of every test point parameter vector is analyzed and set in ascending order from top to bottom. Based on this, it is able to know the domain value of function, from which it is possible to establish the admissible “threshold”. In the table, these thresholds are vectors and the corresponding values are highlighted: yellow, red and green colors refer to the threshold values for the criteria in the scenarios 1, 2 and 3, respectively. In addition, vectors #42617 and #52220 (highlighted in 2 colors) and vector #95759 (highlighted in 3 colors) are set as the threshold values in two and three scenarios respectively.

1) Scenario 1: Assumed that the first priority is cost of 1m³ high performance concrete (HPC), and according to market survey that the cost function Φ_2 should not be greater than 1.3 mil. VND per cubic meter at local place (Vietnam). In order to achieve that, from technical point of view it has to

minimize cement content in 1m³ HPC, because it effects highly on the cost. Thus, the function Φ_3 of cement content is set as the second place based on importance grade, and it should not excel the threshold value of 400 kg/m³. Next, HPC durability is represented by the function Φ_1 - chlorine ion diffusion coefficient on the 28th day – is the third priority, and its value would not be greater than 4.5×10^{-13} m²/s. While, the function Φ_5 (amount of fine slag per 1m³ HPC) and Φ_4 (amount of fly ash per 1m³ HPC) follow after the function Φ_1 in order to improve concrete strength as they make concrete matrix denser [9]. In theory, the amount of mineral admixtures (fine slag and fly ash) also affects HPC cost and it also needs to be minimized as same as cement content, because their percentage is correlated closely to the cement content owing on chemical reaction during hydration process. Thus, it recommends that the threshold values for these functions that: $\Phi_5 \rightarrow \text{MIN} \leq 100 \text{ kg/m}^3$ and $\Phi_4 \rightarrow \text{MIN} \leq 100 \text{ kg/m}^3$.

Table- VII: Short version of interactive panel of 40 valid vector with function $\Phi_1 \div \Phi_3$

Function Φ_1		Function Φ_2		Function Φ_3	
Vector	Value Φ_1	Vector	Value Φ_2	Vector	Value Φ_3
#64666	1.49825E-14	#38772	1262428.5	#90240	300.2403259
#97433	4.17022E-14	#25193	1278677.625	#95760	301.9761658
#95759	4.23573E-14	#42617	1315916.25	#10512	302.010498
...
#21589	1.77858E-13	#33084	1376642.75	#77772	312.1769714
#17891	2.0504E-13	#21589	1426249.875	#24604	313.1304932
#51774	2.12792E-13	#71836	313.3818054
...
#25193	4.25888E-13	#95759	1766834	#30223	356.350708
#42617	4.58956E-13	#67875	1793932.25	#95759	356.3511658

Table- VIII: Short version of interactive panel of 40 valid vector with function $\Phi_4 \div \Phi_6$

Function Φ_4		Function Φ_5		Function Φ_6	
Vector	Value Φ_4	Vector	Value Φ_5	Vector	Value Φ_6
#42617	47.264328	#30223	70.65002441	#42617	5.159239769
...
#24949	57.77664185	#94858	99.6043396	#63593	7.286520004
#46670	58.20610046	#77733	7.413884163
#94858	62.26016998	#10512	139.6228027	#52220	7.955909252
...	...	#63593	141.3851929	#11755	8.176035881
#50022	101.4554596	#27462	141.6583252	#29771	8.254647255
...
#66234	109.9370193	#71622	166.3554382	#30223	9.91297245

Table- IX: Scenario 1

$\Phi_2 \rightarrow$	$\Phi_3 \rightarrow$	$\Phi_1 \rightarrow$	$\Phi_5 \rightarrow$	$\Phi_4 \rightarrow$	$\Phi_6 \blacksquare$
1.3×10^6	400	4.5×10^{-13}	100	100	8

Eventually, the amount of costly chemical admixture per 1m^3 HPC or function Φ_6 also needs to be optimized, and technically it should be smaller than 8 kg/m^3 , unless it would work inefficiently. Therefore, importance order and initial thresholds of function need to be achieved are presented in Table IX.

With this particular requirement, the corresponding threshold values of the function (Table VII - Table VIII) are determined. The vector, at which there is the closest value to the threshold value, is highlighted with yellow color. For instance, a vector #42617 is opted for the condition $\Phi_2 \rightarrow \text{MIN} \leq 1.3 \times 10^6$ and $[\Phi_2] = 1315916.25$; for the condition $\Phi_3 \rightarrow \text{MIN} \leq 400$, vector #95759 - $[\Phi_3] = 356.3511658$; $\Phi_1 \rightarrow \text{MIN} \leq 4.5 \times 10^{-13}$, vector #42617 - $[\Phi_1] = 4.58956\text{E-}13$; $\Phi_5 \rightarrow \text{MIN} \leq 100$, vector #94858 - $[\Phi_5] = 99.6043396$; $\Phi_4 \rightarrow \text{MIN} \leq 100$, vector #50022 - $[\Phi_4] = 101.4554596$; and $\Phi_6 \rightarrow \text{MIN} \leq 8$, vector #52220 - $[\Phi_6] = 7.955909252$. The function value of these vectors might be slightly greater than the recommended threshold value, though they are still acceptable, while other functions have values moderately smaller than the corresponding threshold values.

After setting the threshold values, from initial 40 test point vectors there are only 2 valid vectors, #42617 and #25193, they are also Pareto optimal solutions. However, in order to analyze more thoroughly and determine more valid solutions, it continues to set parameter space for searching.

Step 3.1: Determine more Pareto optimal solutions for the scenario 1.

Using valid solution distribution chart, it is possible to obtain new domain of parameters for two solutions #42617 and #25193, which are given in Table X.

Filling the space with 2^{16} test point with the uniformly distributed sequence LPtau, though it needs to make the requirements slightly stricter in comparison with those mentioned above such as: $\Phi_1 \rightarrow \text{MIN} \leq 4\text{E-}13$; $\Phi_2 \rightarrow \text{MIN} \leq$

$1.3\text{E}6$; $\Phi_3 \rightarrow \text{MIN} \leq 342$; $\Phi_4 \rightarrow \text{MIN} \leq 72$; $\Phi_5 \rightarrow \text{MIN} \leq 90$; $\Phi_6 \rightarrow \text{MIN} \leq 6.3$. Doing so, there are 4 Pareto optimal solutions more for this scenario. The function values are provided in Table XI, and parameter values are in Table XII.

2) Scenario 2: Assumed that, the engineer focuses on three most important functions, which are chlorine ion diffusion coefficient on the 28th day (Φ_1), cost of 1m^3 HPC (Φ_2), amount of cement per 1m^3 HPC (Φ_3), and they have similar importance grade. Besides, it definitely does not desire to diminish the threshold values of these three functions, which are given in Table XIII. While, the other functions at this circumstance might have arbitrary values.

Under this scenario, the corresponding threshold values of the function are determined. These values are highlighted with red color at the vectors with the closest values. A vector #21589 is opted for the condition $\Phi_1 \rightarrow \text{MIN} \leq 1.8 \times 10^{-13}$ and $[\Phi_1] = 1.77858\text{E-}13$; $\Phi_2 \rightarrow \text{MIN} \leq 1.3 \times 10^6$, vector #42617 - $[\Phi_2] = 1315916.25$; $\Phi_3 \rightarrow \text{MIN} \leq 400$, vector #95759 - $[\Phi_3] = 356.3511658$. The other functions are neglected; i.e. they can be least values in Table VII – Table VIII. However, after setting the threshold values for three abovementioned functions in those tables there is no valid vector. This does not mean that in the whole parameter space there is no any valid solutions, the problem here is how to find them. Even though, the number of test points increases from 3×2^{15} up to $2^{17} = 131072$, there is still no satisfied solutions. Table VII shows that the least value of function 3 (356.35) is even better than the limit value (390), this means that solution nonexistence is due to the inconsistency between the first two functions Φ_1 and Φ_2 . Here, it needs to extend the threshold of one of those functions, even both of them, in order to determine at least three valid solutions which are the closest one to the limits set in Table XIII- Table XIV.

Table- X: New domain of parameters – modification at step 3.1

$x_1 \rightarrow$	$x_2 \rightarrow$	$x_3 \rightarrow$	$x_4 \rightarrow$	$x_5 \rightarrow$	$x_6 \rightarrow$	$x_7 \blacksquare$
[330; 342]	[150.5; 161]	[45; 71.5]	[70; 90]	[640; 696]	[1080; 1147.5]	[4.75; 6.27]

Table- XI: Function values of 4 Pareto optimal solutions at step 3.1

Vector	$\Phi_1 \rightarrow \text{MIN}$	$\Phi_2 \rightarrow \text{MIN}$	$\Phi_3 \rightarrow \text{MIN}$	$\Phi_4 \rightarrow \text{MIN}$	$\Phi_5 \rightarrow \text{MIN}$	$\Phi_6 \rightarrow \text{MIN}$
Min:	3.57E-13	1.28E+06	3.34E+02	6.41E+01	7.60E+01	5.02E+00
Max:	3.99E-13	1.30E+06	3.39E+02	6.78E+01	8.36E+01	6.00E+00
#13101	3.81E-13	1.29E+06	3.38E+02	6.41E+01	7.87E+01	5.46E+00
#31630	3.57E-13	1.30E+06	3.35E+02	6.78E+01	8.36E+01	5.02E+00
#56451	3.99E-13	1.28E+06	3.39E+02	6.55E+01	7.60E+01	5.68E+00
#62298	3.95E-13	1.30E+06	3.34E+02	6.67E+01	7.95E+01	6.00E+00

Table- XII: Parameter values of 4 Pareto optimal solutions at step 3.1

Vector	x_1	x_2	x_3	x_4	x_5	x_6	x_7
Min:	3.34E+02	1.51E+02	6.41E+01	7.60E+01	6.41E+02	1.08E+03	5.02E+00
Max:	3.39E+02	1.53E+02	6.78E+01	8.36E+01	6.93E+02	1.14E+03	6.00E+00
#13101	338.474	151.485	64.109	78.653	646.135	1140.088	5.464
#31630	335.338	152.525	67.803	83.631	664.191	1111.713	5.015
#56451	339.058	152.573	65.509	76.041	641.204	1142.312	5.677
#62298	334.257	153.261	66.740	79.516	692.846	1083.905	6.001

Table- XIII: Scenario 2

$[\Phi_1]$	$[\Phi_2]$	$[\Phi_3]$
1.8×10^{-13}	1.3×10^6	390

- If setting $[\Phi_1] = 1.77858\text{E-}13$ and $[\Phi_2] = 1480537.125$ (yellow-highlighted in Table XIV), there are 3 valid solutions such as #21589, #97627, #26225 in accordance with the function 1. It is named as direction 2.a.

- If setting $[\Phi_1] = 4.58956\text{E-}13$ and $[\Phi_2] = 1315916.25$ (green-highlighted in Table XIV), there are 3 valid solutions such as #42617, #38772, #25193 in accordance with function 2. It is named as direction 2.b.

- If setting $[\Phi_1] = 2.57721\text{E-}13$ and $[\Phi_2] = 1376642.75$ (blue-highlighted in Table XIV), there are 3 valid solutions such as #17891, #33084, #95760 in accordance with functions 1 and 2. It is named as direction 2.c.

However, after examining the distribution of the solutions according to those directions, the image shows that their distribution spread out, i.e. surveying each direction separately will not be different from the combination of all 3 directions. Thus, it is possible to take all 3 directions into consideration to set threshold value (violet-highlighted in Table XIV). Doing so, there are 10 valid solutions, and based on distribution charts a new domain of parameters is obtained as shown in Table XV. The only is that at the right limit of the

parameter x_3 there are 2 solutions, it means that there might be more valid solutions. Thus, it is decided to extend the parameter domain as same as initial one [45; 155].

Step 3.2: Determine more Pareto solutions in Scenario 2.

In the new space, it needs to fill up with 100,000 points in evenly distributed sequence of Windows. Only 77 vectors pass through functional constraints and go into the test table. After resetting the threshold value for the first 3 function in Table XIII, there are 2 satisfied solutions #43219 and #28150, as shown in Table XVI, which are both Pareto solutions. It is observed that these solutions are better than 4 solutions in scenario 1 at the first 5 functions. This is reasonable because at scenario 2 the effort is to optimize the first 3 functions, hence one of the rest functions might obtain the least value.

3) Scenario 3: Here, the threshold value is set for all of functions, as shown in Table XVII. It has to determine a valid vector for all of functions simultaneously.

Table- XIV: Short interactive panel of 40 valid vector with values of functions $\Phi_1 \div \Phi_2$

Function Φ_1		Function Φ_2	
Vector	Value Φ_1	Vector	Value Φ_2
...	...	#38772	1262428.5
#21589	1.77858E-13	#25193	1278677.625
#17891	2.0504E-13	#42617	1315916.25
...
#33084	2.45384E-13	#33084	1376642.75
#24604	2.57511E-13	#21589	1426249.875
#95760	2.57721E-13	#94858	1435086.5
...
#80138	3.09363E-13	#97627	1480537.125
...	...	#87753	1482874.375
#42617	4.58956E-13

Table- XV: New domain of parameters – modification at step 3.2

$x_1 \rightarrow$	$x_2 \rightarrow$	$x_3 \rightarrow$	$x_4 \rightarrow$	$x_5 \rightarrow$	$x_6 \rightarrow$	x_7
[300;348]	[140;164.5]	[45;155]	[70;120]	[620;690]	[1012.5;1147.5]	[5;7.837]

It is seen the condition of functions Φ_2, Φ_4, Φ_5 is not as strict as that in scenario 1, except for the functions Φ_1 . To deal with this, it has to look into the interactive panel Table VII-VIII, here it decides that the chlorine ion diffusion coefficient on the 28th day (Φ_1) needs to be minimized, and for the worst case, its magnitude should not be greater than 2.0504E-13 (corresponding to test vector #17891); cost of 1m3 concrete (Φ_2) should not be more than 1426249.875 cu (corresponding to test vector #21589). Next, the functions Φ_3 (cement content) and Φ_4 (fly ash content) are intentionally satisfactory, thus their magnitudes are set as minimum in the threshold. Finally, the functions Φ_5 (slag content) and Φ_6

(admixture content) need to be minimized and their magnitudes should not be greater than 139.6228027 (#10512) and 7.955909252 (#52220) respectively. Doing so, from 40 vectors in Table VII - Table VIII only #21589 and #17891 are left, they are Pareto optimal solutions. However, in order to examine more closely and find more solutions to help engineer evaluate the domain of function values, it need to shrink the parameter space to search.

Step 3.3: Determine more Pareto solutions in Scenario 3.

By using solution distribution chart, it is possible to obtain new domain of parameters around solutions #21589 and #17891, as shown in Table XVIII.

Table- XVI: Pareto optimal solutions in scenario 2

Vector	i	1	2	3	4	5	6	7
#43219	x_i	319.492	142.720	138.644	74.692	660.436	1070.693	6.451
	Φ_i	1.38E-13	1.29E+06	3.19E+02	1.39E+02	7.47E+01	6.45E+00	—
#28150	x_i	308.878	145.840	138.427	76.143	643.135	1086.826	7.039
	Φ_i	1.71E-13	1.30E+06	3.09E+02	1.38E+02	7.61E+01	7.04E+00	—

Table- XVII: Scenario 2

$[\Phi_1]$	$[\Phi_2]$	$[\Phi_3]$	$[\Phi_4]$	$[\Phi_5]$	$[\Phi_6]$
2×10^{-13}	1.4×10^6	400	110	140	8

Table- XVIII: New domain of parameters – modification at step 3.3

[336;348]	[140;151]	[71.5;91]	[80;110]	[656;696]	[1035;1147.5]	[4.75;7.315]
-----------	-----------	-----------	----------	-----------	---------------	--------------

By filling the space with 90 000 test points by Windows sequence and applying conditions as follows: $\Phi_1 \rightarrow \text{MIN} \leq 2\text{E-}13$; $\Phi_2 \rightarrow \text{MIN} \leq 1.4\text{E}6$; $\Phi_3 \rightarrow \text{MIN} \leq 400$; $\Phi_4 \rightarrow \text{MIN} \leq 110$; $\Phi_5 \rightarrow \text{MIN} \leq 140$; $\Phi_6 \rightarrow \text{MIN} \leq 8$, at this step there are 10 Pareto optimal solutions together with #21589 and #17891

at the step 3.2, in total there are 12 valid solutions. Their function values and parameter magnitudes are tabulated in Table XIX and Table XX respectively. On the basis of these solutions, engineer can select one of them to make HPC mix with criteria set in Table XVII above.

Table- XIX: Function values of 12 Pareto optimal solution in scenario 3

Vector	$\Phi_1 \rightarrow \text{MIN}$	$\Phi_2 \rightarrow \text{MIN}$	$\Phi_3 \rightarrow \text{MIN}$	$\Phi_4 \rightarrow \text{MIN}$	$\Phi_5 \rightarrow \text{MIN}$	$\Phi_6 \rightarrow \text{MIN}$
Min:	1.56E-13	1.35E+06	3.37E+02	7.35E+01	8.19E+01	5.24E+00
Max:	2.05E-13	1.43E+06	3.48E+02	9.04E+01	1.06E+02	7.11E+00
#17891 (Step 2)	2.05E-13	1.36E+06	3.47E+02	8.23E+01	8.19E+01	7.11E+00
#21589 (Step 2)	1.78E-13	1.43E+06	3.40E+02	7.96E+01	1.06E+02	5.45E+00
#3351	1.56E-13	1.39E+06	3.40E+02	9.04E+01	9.60E+01	5.54E+00
#21629	1.95E-13	1.39E+06	3.45E+02	7.50E+01	9.71E+01	5.30E+00
#26522	1.87E-13	1.39E+06	3.42E+02	7.89E+01	9.51E+01	5.83E+00
#28684	1.94E-13	1.38E+06	3.45E+02	8.20E+01	9.01E+01	6.61E+00
#56021	1.94E-13	1.35E+06	3.37E+02	8.45E+01	8.64E+01	6.30E+00
#56394	1.99E-13	1.39E+06	3.42E+02	7.69E+01	9.42E+01	6.29E+00
#60029	1.87E-13	1.39E+06	3.46E+02	8.12E+01	9.18E+01	6.57E+00
#63389	1.61E-13	1.39E+06	3.40E+02	8.91E+01	9.28E+01	6.23E+00
#67933	1.77E-13	1.38E+06	3.38E+02	7.81E+01	9.68E+01	5.51E+00
#75706	1.97E-13	1.37E+06	3.48E+02	7.35E+01	9.24E+01	5.24E+00

Regarding multi-objective optimization model used for proportioning HPC mix, it is observed that the solution is always not unique, because it is a series of objective functions, and each of them has a different meaning from one

to another engineer.

However, VIAM with an application of parameter space investigation allow for altering flexibly the constraints and requirements in the model, besides there is a possibility of analyzing on the outcomes among engineers of different

background. Thus, the resultant solutions are not only a series of Pareto ones but also they meet the requirements of actual production.

Table- XX: Parameter values of 12 Pareto optimal solution in scenario 3

Vector	x_1	x_2	x_3	x_4	x_5	x_6	x_7
Min:	3.37E+02	1.40E+02	7.35E+01	8.19E+01	6.57E+02	1.05E+03	5.24E+00
Max:	3.48E+02	1.48E+02	9.04E+01	1.06E+02	6.95E+02	1.12E+03	7.11E+00
#17891 (Step 2)	346.789	141.421	82.330	81.887	661.106	1113.050	7.110
#21589 (Step 2)	339.882	147.569	79.640	105.507	688.284	1054.850	5.449
#3351	339.760	142.796	90.403	95.971	663.614	1090.502	5.542
#21629	344.963	144.970	75.018	97.055	660.785	1102.372	5.299
#26522	342.030	143.397	78.920	95.057	695.309	1068.744	5.834
#28684	345.161	143.784	81.987	90.133	680.972	1079.532	6.611
#56021	337.245	140.463	84.516	86.358	656.924	1122.777	6.297
#56394	341.983	143.829	76.854	94.206	672.212	1094.371	6.290
#60029	346.380	143.756	81.237	91.787	686.636	1071.759	6.573
#63389	339.789	141.462	89.097	92.801	694.859	1064.713	6.233
#67933	338.134	141.188	78.059	96.784	668.436	1106.554	5.510
#75706	347.811	142.682	73.495	92.443	672.705	1100.846	5.244

IV. CONCLUSION

By using VIAM with an application of PSI, after the solution search for three different production scenarios in automation and process control of proportioning HPC mix, there are 18 solutions available (four for scenario 1, two in scenario 2, and twelve in scenario 3). They are all Pareto solutions, allowing engineers to select the optimal mix in a certain circumstance. The problem can also be broadened with different parameters, constraints, and criteria when varying quantity and characteristics of the HPC components.

Regarding an application of PSI technique, several aspects can be withdrawn:

- A quantity of trial points: it is necessary to fill the parameter space with a relatively large number of points ($\approx 90000 \div 120000$).

- Uniformly distributed sequences: it is necessary to combine various uniformly distributed sequences simultaneously to maximize the advantages of each sequence in order to fill the parameter space compactly and to define valid solutions.

- The statistical analysis of invalid constraints and objective functions, their close and distant values in comparison with the requirements, allowing engineers to determine quantitatively a satisfactory alteration level for a mathematical model including constraints, condition types, thresholds, and domain parameters.

- A gradual alteration of the mathematical model and continuous re-solving allow engineers to achieve the valid domain effectively.

Last but not least, VIAM can also be applied widely to automation and control in many other engineering areas.

ACKNOWLEDGMENT

The authors would like to express gratitude to late Professor Statnikov R., who inspired the idea of a toolkit for visual interaction and more effectively later being named as

visual interactive analysis method (VIAM), which was introduced in this paper.

REFERENCES

1. M.F. Alves, R.A. Cremonini and D.C.C. Dal Molin, "A comparison of mix proportioning methods for high-strength concrete", *Cement and Concrete Composites*, vol. 26, no. 6, pp. 613-621, 2004.
2. Z. Yunsheng, S. Wei, L. Sifeng, J. Chujie and L. Jianzhong, "Preparation of c200 green reactive powder concrete and its static-dynamic behaviors", *Cement and Concrete Composites*, vol. 30, no. 9, pp. 831-838, 2008.
3. K. Celik, C. Meral, M. Mancio, P.M. Mehta and P.J.M. Monteiro, "A comparative study of self-consolidating concretes incorporating high-volume natural pozzolan or high-volume fly ash", *Construction and Building Materials*, vol. 67, no. 4, pp. 14-19, 2014.
4. P.C. Aitcin, "Cements of yesterday and today: Concrete of tomorrow", *Cement and Concrete Research*, vol. 30, no. 9, pp. 1349-1359, 2000.
5. A.M. Neville and P.C. Aitcin, "High-performance concrete - An overview", *Materials and Structures*, vol. 31, no. 3, pp. 111 - 117, 1998.
6. P.C. Aitcin, "The art and science of high-performance concrete", in: O.K. Metha (Ed.), *Advances in Concrete Science and Technology*, Proceedings of the M. Collepardi Symposium, Rome vol. 10, pp. 107- 124, 1997.
7. Federal Highway Administration, *High-Performance Concrete Bridges*, Building Bridges for the 21st Century, U.S. Department of Transportation, FHWA-SA-98-084.
8. C.H. Goodspeed, S. Vanikar and R. Cook, "High-performance concrete defined for highway structures", *Concrete International*, vol. 18, no. 2, pp. 62-67, 1996.
9. K.E. Hassan, J.G. Cabrera and R.S. Maliche, "The effect of mineral admixtures on the properties of high-performance concrete", *Cement and Concrete Composites*, vol. 22, no. 4, pp. 267-271, 2000.
10. C.H. Lim, Y.S. Yoon and J.H. Kim, "Genetic algorithm in mix proportioning of high-performance concrete", *Cement and Concrete Research*, vol. 34, pp. 409-420, 2004.
11. R. Parichatprecha and P. Nimityongsukul, "Analysis of durability of high performance concrete using artificial neural networks", *Construction and Building Materials*, vol. 23, pp. 910-917, 2009.
12. A. Yadollahi, E. Nazemi, A. Zolfaghari and A.M. Ajorloo, "Application of artificial neural network for predicting the optimal mixture of radiation shielding concrete", *Progress in Nuclear Energy*, vol. 89, pp. 69-77, 2016.
13. X. Xie, Y. Zheng and F. Tian, "Multi-objective Optimized Design of High-Performance concrete Based on Matlab", *Advanced Materials Research*, vol. 261, pp. 202-207, 2011.

14. B.H. Bharatkumar, R. Narayanan, B.K. Raghuprasad and D.S. Ramachandramurthy, "Mix proportioning of high-performance concrete", *Cement and Concrete Composites*, vol. 23, no. 1, pp. 71–80, 2001.
15. P.A. Gutierrez and M.F. Canovas, "High-performance concrete: requirements for constituent materials and mix proportioning", *ACI Material Journal*, vol. 5, pp. 233–241, 1996.
16. V.D. Nguyen and H.M. Dang, "High performance concrete mixture proportioning: Multi-objective optimization approach", *Journal of Science of Ho Chi Minh City Open University*, vol. 20, no. 4, pp. 65-76, 2016.
17. S.S. Gavriushin and H.M. Dang, "Multi-criteria design pressure vessel, manufactured from composite materials by the method of winding", *Advanced Composite Materials and Technologies for Aerospace Applications*, Wrexham, North Wales, UK, pp. 60-66, 2013.
18. S.S. Gavriushin and H.M. Dang, "Multi-criteria management of the metal cutting process. *Journal of higher educational institutions: Machine building*, vol. 10, pp. 82-95, 2016.
19. R.B. Statnikov, S.S. Gavriushin, H.M. Dang and A.R. Statnikov, "Multicriteria design of composite pressure vessels", *International Journal of Multicriteria Decision Making*, vol. 4, no. 3, pp. 252-278, 2014.
20. R. Statnikov and J. Matusov, "Multicriteria Analysis in Engineering", Kluwer Academic Publishers, Dordrecht, Boston, London, 2002.
21. R. Statnikov, V. Astashev, J. Matusov, M. Toporkov, K. Piankov, A. Statnikov, E. Yakovlev and I. Yanushkevich, "Software Package MOVI 1.4 for Windows: User's Manual, Certificate of Registration, Register of Copyrights, USA, Registration Number TXu 1-698-418, 2010.
22. R. Statnikov, J. Matusov and A. Statnikov, "Multicriteria engineering optimization problems: statement, solution and applications", *Journal of Optimization Theory and Applications*, vol. 155, no. 2, pp. 355–375, 2012.
23. R.B. Statnikov and A.R. Statnikov, "The Parameter Space Investigation Method Toolkit", Artech House, Boston/London, 2011.
24. K. Doi, Y. Chujo, M. Yoshimura, S. Nishiwaki and K. Izui, "Construction of an optimum system design method considering product lifecycle", *International Journal of Sustainable Engineering*, vol. 2, no. 3, pp. 171–183, 2009.
25. E. Papageorgiou, M.H. Eres and J. Scanlan, "Value modelling for multi-stakeholder and multi-objective optimization in engineering design", *Journal of Engineering Design*, DOI: 10.1080/09544828.2016.1214693, 2016.
26. Y. Chen, Z.L. Liu and Y.B. Xie, "A multi-agent-based approach for conceptual design synthesis of multi-disciplinary systems", *International Journal of Production Research*, vol. 52, no. 6, pp. 1681–1694, 2014.
27. A. Lamghabbar, S. Yacout and M.S. Ouali, "Concurrent optimization of the design and manufacturing stages of product development", *International Journal of Production Research*, vol. 42, no. 21, pp. 4495-4512, 2004.
28. H. Shidpour, M. Shahrokhi and A. Bernard, "A multi-objective programming approach, integrated into the TOPSIS method, in order to optimize product design; in three-dimensional concurrent engineering", *Computers & Industrial Engineering*, vol. 64, pp. 875–885, 2013.
29. C. Renzi and F. Leali, "A Multicriteria Decision-Making Application to the Conceptual Design of Mechanical Components", *Journal of Multi-Criteria Decision Analysis*, vol. 23, no. 3-4, pp. 87-111, 2016.

AUTHORS PROFILE



Dr. Minh-Hong Nguyen is currently working at Faculty of Control Engineering, Le Quy Don Technical University, 236 Hoang Quoc Viet, Hanoi, Vietnam. Throughout the years, he has published more than 30 scientific articles.



Dr. Hoang-Minh Dang is currently working at Faculty of Mechanical Engineering, Industrial University of Ho Chi Minh City, Ho Chi Minh City, Vietnam. He has published more than 20 research and conference papers in the last decade.