International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-8, June 2020

Algorithm for Eliminating of the Limiting Disambiguation of Measurement Made by Phase Radio Direction Finders by Sorting Out the Abnormally Large Errors

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Abstract: ***The aim of the article is to find the upper probability limit of the measurement results to correct disambiguation in case of multi-base phase direction finders, where all bases are ambiguous. Direction finding is done using the maximum likelihood method based on a set of measured phase differences and an algorithm of rejecting (erasing) abnormally large measurement errors. The theoretical background of the article is the maximum likelihood method applied to disambiguate results of the phase measurements in multi-base measuring systems. The physical meaning of the method is that if the disambiguation process is correct, the results of angular measurements for each base are grouped around the true value of bearing. The mathematical background of the article are methods of linear algebra based on the geometric interpretation of disambiguation measurement results. We obtained formulas for calculating upper bounds for the probability correct disambiguation of measurement results, which are applicable to direction finders with linear, planar and conformal antenna arrays.***

The obtained theoretical relations are exemplified by a numerical calculation of error probability including the upper bounds for a specific three-base direction finder 'bad' measurement results. The calculations proved effectiveness of the proposed algorithm, which depends on the accuracy of phase measurements.

The proposed algorithm is applicable not only in case of the direction finders, but also for other multi-base phase measurements.

The work may be interesting for designers of direction finders in terms of achievable accuracy of measurement results even if some of the results are rejected.

Keywords : ambiguity vector, antenna array, bearing estimation, likelihood function, multibase phase direction finder

Revised Manuscript Received on June 30, 2020.
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I. INTRODUCTION

A phase direction finder is a device to find the direction to a radio source. [1] - [4]. A bearing of a radio emission source can be found by processing the phases of the signals received by low-directivity antennas forming the linear, planar or three-dimensional (conformal) arrays. To achieve the high accuracy of the direction finding, it is necessary to use a spacing of antenna arrays by the distance significantly exceeding the signal wavelength [3], [5], [6]. In this case, the phase difference of the signals will exceed the range of the unambiguous direction finding of 360 degrees. Therefore, to determine a radio source bearing, it is first necessary to disambiguate the results of the phase measurements, which are random due to internal noise of the direction finder receivers, parasitic phase shifts in receivers and amplifiers of the direction finder and phase distortions of the radio waves on the propagation path.

There are various algorithms to disambiguate the results of the phase measurements [7] - [12]. Their quality is characterized by the probability of the correct disambiguation. The highest probability of the correct disambiguation provides the algorithm of maximal plausibility [3]. For this algorithm, the limiting relations to set the upper bound for the probability of the correct disambiguation were written as a function of the level of the phase errors, the number of the phase-measuring bases, the type of the antenna array, and the accuracy of the direction finding [13] - [15]. Rough (anomalous) errors [16] are often observed while using disambiguating results of the phase measurements in multi-base phase direction finders. There are descriptions [17] - [20] of the method allowing to disambiguate the results of the phase measurements with errors in case of direction finders having an arbitrary number of phase-measuring bases. The use of the said method made it possible to reduce significantly (several times) the probability of anomalous errors and exceed the bounds obtained in the relevant works [13] - [15]. In this paper, we specify relations to determine maximal probability values (bounds) for the correct disambiguation of phase measurement results while using this rejection method. The procedure to obtain the probability bounds was exemplified by a four antenna phase direction finder.

But due to universality of the algorithm for maximal plausible disambiguation, this technique be applied to phase direction finders with low-directivity antenna systems forming linear, planar or three-dimensional (conformal) arrays. In this case, all the phase-measuring bases can be ambiguous, and the ratio of their sizes form a set of co-prime numbers. In other words, the biggest common divisor of numbers in the given set is 1.

II. METHODOLOGICAL FRAMEWORK

The proposed rejection method is demonstrated for the simplest case – when a plane wave comes to a linear antenna array of a direction finder (Fig. 1). To determine the source bearing α_x there is used the direction cosine $v = \cos\alpha_x$, described by the set of phase differences $\varphi_1, \varphi_2, ..., \varphi_n$ measured between the elements of the antenna array.

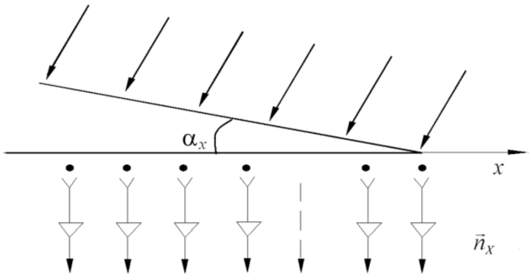

Fig. 1. Diagram of the antenna array

The total phase difference $\vec{\Phi}$ for each base is the sum

$$\vec{\Phi} = \vec{\varphi} + \vec{k} \ ,$$

where $\vec{k}$ is the ambiguity vector, and its *i*-th coordinate is equal to the whole number of phase periods (2*π*) fit in the *i*-th base and lost in the process of measurements; $\vec{\varphi}$ is the vector of the measured phase differences in the range from -*π* to + *π* in fractions of period 2*π* (modulus of the vector elements are less than 0.5).

The total phase difference is random because of the noise influence in the phase meter, and due to the uncertainty of the number of the lost phase periods during the measurement.

According to the maximum likelihood principle, the direction cosine estimate v^*, requires (as shown in [3]) calculating the weighted values of the total phase difference, that can be written as:

$$v^* = \vec{\Phi}^T \vec{q}_v, \tag{1}$$

where $\vec{\Phi}^T$ is the vector of the total phase difference at the bases, T is the index denoting the sign of the transposition,

$\vec{q}_v$ is the weighting vector: $\vec{q}_v = \dfrac{\mathbf{B}_\varphi^{-1}\vec{n}_X}{\vec{n}_X^T \mathbf{B}_\varphi^{-1}\vec{n}_X}$;

$\mathbf{B}_\varphi$ is the correlation matrix of the phase errors;

$\mathbf{B}_\varphi^{-1}$ is the inverse of $\mathbf{B}_\varphi$ matrix;

$\vec{n}_X$ is the vector of scale the factors, the *i*-th element of which is the ratio of the *i*-th phase-measuring base and the length of the incident wave.

The accuracy of the direction cosine estimate v^* is characterized by the dispersion $\sigma^2_{v^*}$, for which, in view of Gaussian error probability distribution in phase measurements, the following expression was obtained in [3], [17] - [19]:

$$\sigma^2_{v^*} = \frac{\sigma^2_\varphi}{\vec{n}_X^T \mathbf{B}_\varphi^{-1} \vec{n}_X},$$

where σ^2_φ is the dispersion of the measured phase difference.

To determine the bearing of a radio emission source by formula (1), it is necessary to disambiguate the results of the phase measurements. The problem of disambiguating results of phase measurements is reduced to finding the ambiguity vector $\vec{k}$.

The quality of disambiguation can be characterized by the probability of a correct solution P_0, if disambiguation has been performed. The highest value of this probability is achieved by applying the algorithm of maximal plausibility (optimal algorithm). As shown in [3], [10], this is equivalent to the process of choice from the set of possible ambiguity vectors $\{\vec{k}\}$ such a $\vec{k}^*$ value that minimizes the quadratic form $\Pi_{\vec{\varphi}}(\vec{k})$:

$$\Pi_{\vec{\varphi}}(\vec{k}) = (\vec{\varphi} + \vec{k})^T \cdot \mathbf{G} \cdot (\vec{\varphi} + \vec{k}) \tag{2}$$

where $\mathbf{G}$ is the square orthogonal matrix $\vec{n}_X$,

$$\mathbf{G} = \mathbf{B}_\varphi^{-1} - \frac{\mathbf{B}_\varphi^{-1}\vec{n}_X \vec{n}_X^T \mathbf{B}_\varphi^{-1}}{\vec{n}_X^T \mathbf{B}_\varphi^{-1} \vec{n}_X}.$$

The vector estimate $\vec{k}^*$ is calculated as follows:

$$\vec{k}^* = \arg \min_{\vec{k}_i \in \{\vec{k}\}} (\vec{\varphi} + \vec{k}_i)^T \, \mathbf{G} \, (\vec{\varphi} + \vec{k}_i).$$

In [3], [5], [6] there was described the way to find the upper bound of the correct disambiguation P_0 probability of.

An algorithm to reject abnormally large errors was proposed in [17] - [20]. It is based on the use of the quadratic form (2). If its value does not exceed a certain threshold Δ, i.e.

$$\Pi_{\vec{\varphi}}(\vec{k}^*) < \Delta, \tag{3}$$

then a 'positive decision' is taken – to continue calculating the direction cosine estimate v^* based on phase measurement data $\vec{\varphi}$. If condition (3) is not satisfied, then a "negative decision" is taken.

That is, the corresponding set of the measured phase differences $\vec{\varphi}$ is rejected or "erased".

If we denote P_M and P_R as the corresponding probabilities of the 'measurement' or the 'erasure', then $P_M + P_R = 1$.

However, the rejection performed in this way does not exclude an incorrect decision of calculation using allowed measurement results. Relation (3) is unconditionally satisfied only for vector $\vec{k}^*$, which provides the least bearing error, but can (with a smaller probability) be used for the other $\vec{k}$. Therefore, the following additional criterion for rejecting measurements from the group of allowed ones was proposed in [3]: an ambiguity is correctly resolved if during calculation of the direction cosine

$$\left|v_{cor} - v^*\right| < \frac{0,5}{\left|e_X\right|_{\max}},$$

where $\left|e_X\right|_{\max}$ is the maximum modulus of the integer vectors $\vec{e}_X$ collinear to $\vec{n}_X$; v_{cor} is the correct value of the direction cosine.

Otherwise, the decision should be considered false. That means the one corresponding to anomalous bearing estimation error is too big.

Thus, four events form a complete group. One - the measurements are correct: the bearing is estimated without the abnormal error. The probability of this event is P_{CM} (probability of the correct bearing measurement). Second, the decision of measurements is wrong: the bearing estimated with an abnormal error. The probability of this event is P_{WM} (probability of wrong bearing measurement). Third, the correct decision is to reject the measurement results: the bearing estimate would be with the abnormal error. The probability of this event is P_{CR} (probability of the correct rejection). Fourth, the wrong decision is to reject the measurement results: the bearing estimate would be without an abnormal error. The probability of this event is P_{WR} (probability of wrong rejection).

Then,

$$P_M = P_{CM} + P_{WM}; \quad P_R = P_{CR} + P_{WR};$$
$$P_{CM} + P_{WM} + P_{CR} + P_{WR} = 1.$$

The probability of the correct disambiguation of phase measurements P_0 when the algorithm for rejecting abnormally large errors is used is now described as follows: $P_0 = P_{CM} / P_M$ and the probability of an abnormally large error P_A: $P_A = P_{WM} / P_M$.

In the absence of the restriction (3) imposed on the quadratic form (2), the probability of correct disambiguation is determined by (n-1)-dimensional integral [3].

$$P_0 = \int_D \cdots \int W(\vec{y})\, d\vec{y}, \qquad (4)$$

where $W(\vec{y})$ is (n-1)-dimensional distribution density of the random variables

$$\vec{\eta}_i = \vec{k}_i^T \mathbf{G} \vec{\delta}, \quad i = 1, 2, ..., (n-1), \qquad (5)$$

where $\vec{\delta}$ is the vector of errors the phase measurements; $\vec{k}_i$ are the vectors of ambiguity from the reference set, that is those for which the values $d_i^2 = \vec{k}_i^T \mathbf{G} \vec{k}_i$, $i = 1, 2, ..., (n-1)$ are the smallest among the other integer vectors and which in combination with $\vec{n}_X$ form a complete system of linearly independent n-dimensional vectors.

The random variables $\vec{\eta}$ were received from the linear transformations $\vec{\delta}$. Their distribution also follows the normal law. Elements of the correlation matrix $\mathbf{B}_\eta$ of the random values $\vec{\eta}$ (5) are determined as follows: $b_{ij} = \vec{k}_i^T \cdot \mathbf{G} \cdot \vec{k}_j$, $i = 1, 2, ..., (n-1)$, $j = 1, 2, ..., (n-1)$.

After transformations (4) takes the following form:

$$P_0 = \frac{1}{\sqrt{(2\pi)^{n-1} \det \mathbf{B}_\eta}} \int_D \cdots \int \exp\left\{-\frac{1}{2} \vec{y}^T \mathbf{B}_\eta^{-1} \vec{y}\right\} d\vec{y}. \qquad (6)$$

Let's give the geometric interpretation of a radio emission source angular position finding (Fig. 2) with a three-base ($n = 3$) phase direction finder, antenna system of which forms a linear array. In this case, the integral (4) becomes a double one. The bearing found by the method of maximum likelihood is correct for any measurement vector realization $\vec{\varphi}$, since its projection on a subspace orthogonal to $\vec{n}_X$ falls in one of the proper domains of the ambiguity vectors. Algorithm (3) allows to carry out the similar actions if the projection of the measurement vector $\vec{\varphi}$ falls in the ellipsoids $\vec{y}^T \mathbf{B}_\eta^{-1} \vec{y} = \Delta$, lying within one of their own domains.

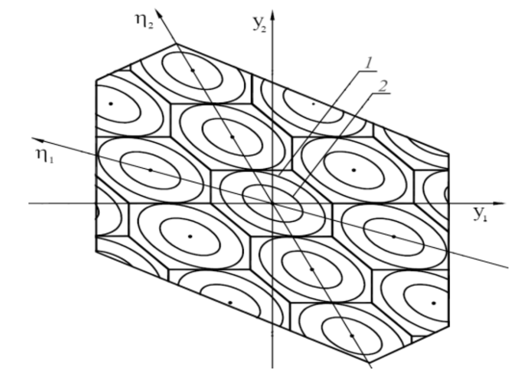

Fig. 2. The proper domains of the ambiguity vectors and ellipses of the equal probability density: 1 - $\vec{y}^T \mathbf{B}_\eta^{-1} \vec{y} = Q_н$; 2 - $\vec{y}^T \mathbf{B}_\eta^{-1} \vec{y} = Q$, $Q < Q_н$

Integrating (6) is carried out for (n-1)-dimensional domain D, which we call the proper domain of the ambiguity vector. The probability P_0 does not depend on the position of the radio emission source within the sector of the unambiguous bearing if disambiguation is done using the algorithm of maximal plausibility.

Therefore, hereafter it is assumed that the emission source is normal to the antenna array. In this case, the value of the direction cosine $v = 0$, all coordinates of the ambiguity vector $\vec{k}$ are zero: $\vec{k} = \vec{k}_0$, where $\vec{k}_0^T = (0;0;...;0)$.

Fig. 3 shows a proper domain of a zero ambiguity vector $\vec{k}_0^T = (0;0;...;0)$, concentration ellipses of the phase errors projection and the threshold limitation of a direction finder with a linear antenna array having four antenna elements.

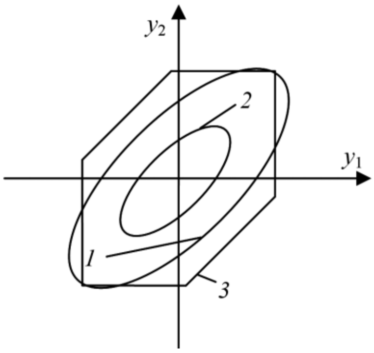

Fig. 3. Concentration ellipse of the phase errors projection (1), threshold limitation ellipse E_0 (2), and the proper domain of zero ambiguity vector D_0 (3)

Introduction of the threshold limitation (3) means that only measured phase differences, projections of which are inscribed in the ellipse shown in Fig. 3 E_0, which follows the equation $\vec{y}^T \mathbf{B}_\eta^{-1} \vec{y} = \Delta$ take place in finding the bearing. To determine the probability of the measurement, it is necessary to find the (n-1)-dimensional integral of the domains E_i of all ambiguity vectors $\vec{k}_i$ from the total set $\{\vec{k}\}$. P_M can be calculated as follows:

$$P_M = \sum_{i=0}^{M} \left[\int_{E_i}\!\!\cdots\!\int W(\vec{y})d\vec{y} \right]. \qquad (7)$$

Fig. 4 shows some proper domains: the proper domain of the vector $\vec{k}_0^T = (0;0;0)$ and the domains adjacent to it contribute most to the sum (7).

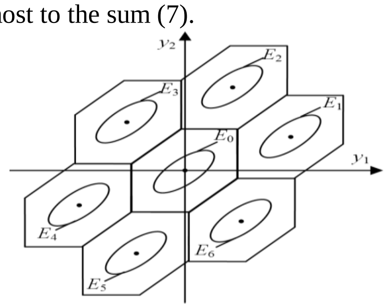

Fig. 4. The threshold limitation ellipses E_i for the proper domains of ambiguity vectors.

III. RESULT AND DISCUSSION

The threshold limitation (3) in Fig. 4 corresponds to the ellipse E_0. Its introduction provides the probability of the correct measurement. P_{CM} is calculated using a $(n-1)$-dimensional integral for the domain E_0.

$$P_{CM} = \int_{E_0}\!\!\cdots\!\int W(\vec{y})d\vec{y}. \qquad (8)$$

Having in mind that the equation for the integration domain $\vec{y}^T \mathbf{B}_\eta^{-1} \vec{y} = \Delta$ coincides with an ellipsoid of the equal probability density, the probability is equal to P_{CM}

$$P_{CM} = P\left\{\chi_{n-1}^2 \le \Delta\right\}.$$

The random value of χ_{n-1}^2 is distributed according to the chi-square law with (n-1) degrees of freedom.

The probability of the correct disambiguation P_0 is calculated as:

$$P_0 = \int_{E_0}\!\!\cdots\!\int W(\vec{y})d\vec{y} \Bigg/ \sum_{i=0}^{M} \left[\int_{E_i}\!\!\cdots\!\int W(\vec{y})d\vec{y} \right], \qquad (9)$$

where E_0 and E_i are the threshold limitation ellipsoids located within their proper domains of zero ambiguity vector and ambiguity vector $\vec{k}_i \in \{\vec{k}\}$, respectively; $i = \overline{0, M}$, $M+1$ is the number of ambiguity vectors of the total set $\{\vec{k}\}$.

A decrease in the threshold Δ leads to a decrease in the probability of measurements P_M and the correct measurement P_{CM}, as well as to the increase in the probability of the correct disambiguation P_0.

We shall find the upper bound for the probability P_0 using the algorithm of rejecting abnormally large errors in the process of the direction finding. To do this, we consider the geometrical meaning of the integral (4) for a three-base direction finder, antenna system of which forms a linear array. To obtain a value of P_{CM}, it is necessary to calculate the two-dimensional integral for the domain E_0

$$P_{CM} = \int_{E_0}\!\!\int W(y_1, y_2)dy_1 dy_2. \qquad (10)$$

The geometric meaning of the integral (10) is the volume of the cylinder (located in the centre of Fig. 5) with the ellipse E_0 at the base of it bounded at the top by the probability density function $W(y_1, y_2)$. A decrease in the threshold Δ results in a decrease in the area of the ellipse E_0. Then the volume of the cylinder tends to the product of the area of the ellipse S by the distribution density $W(y_1, y_2)$ in the centre of the ellipse $y_1 = 0$, $y_2 = 0$. Approximate value of P_{CM} is determined as:

$$P_{CM} \approx SW(0;0) = \frac{S}{2\pi\sqrt{\det \mathbf{B}_\eta}}. \qquad (11)$$

The distribution density $W(y_1, y_2)$ in the centre of other ellipses $E_1, E_2, \ldots, E_M$ (around the central cylinder in Fig. 5) is respectively equal to

$K\exp\{-0,5d_1^2\}$, $K\exp\{-0,5d_2^2\}, \ldots$, $K\exp\{-0,5d_M^2\}$,

where $K = \dfrac{1}{2\pi\sqrt{\det \mathbf{B}_\eta}}$.

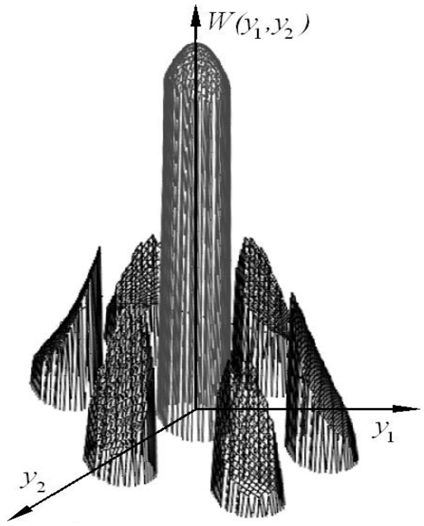

Fig. 5. Geometry in the space of phase differences projections on the orthogonal subspace $\vec{n}_X$

Approximate probability of the wrong measurement P_{WM}, as the threshold Δ decreases, can be determined as follows:

$$P_{WM} \approx \frac{S\exp\{-0,5d_1^2\}}{2\pi\cdot\sqrt{\det \mathbf{B}_\eta}} + \frac{S\exp\{-0,5d_2^2\}}{2\pi\sqrt{\det \mathbf{B}_\eta}} + \ldots \frac{S\exp\{-0,5d_M^2\}}{2\pi\sqrt{\det \mathbf{B}_\eta}}.$$

Taking into account that the areas of all ellipses are the same, and by substituting the obtained relations into formula (9), after all transformations, we obtain the expression determining the probability of the correct disambiguation in the process of rejection abnormally large errors

$$P_0 = 1\Bigg/\left(1+\sum_{i=1}^{M}\exp\{-0,5d_i^2\}\right). \qquad (12)$$

This is the universal formula. It is valid for phase direction finders, antenna systems of which consist of N antenna elements forming linear, planar or three-dimensional arrays, if phase-measuring bases are linearly independent. The difference is only in the matrix $\mathbf{G}$, which is used in calculating d_i^2. The formulas for calculating matrix $\mathbf{G}$ for phase direction finders with planar arrays are given in [3], [6], and with conformal arrays – in [3].

The number of summands of the sum (12) can be limited, since the probability density function decreases exponentially. The summands corresponding to ambiguity vectors of the reference set and their simplest linear combinations have the most significant influence on the sum (12). Fig. 4 shows the proper domains for this kind of vectors, the total number of which is J. In general, for the phase direction finders with antenna systems in the form of linear arrays $J = 2(2^{N-2}-1)$, in case of the planar arrays $J = 2(2^{N-3}-1)$ and of the three-dimensional arrays $J = 2(2^{N-4}-1)$, where N is the number of antenna elements [3]. Thus, we obtain the relation to determine the upper bound of the correct disambiguation probability in case of phase measurements $P_0 < P_0^{\max}$ when there is used algorithm for rejecting abnormally large errors

$$P_0^{\max} = 1\Bigg/\left(1+\sum_{i=1}^{J}\exp\{-0,5d_i^2\}\right). \qquad (13)$$

Taking into account the fact that the proper ambiguity vectors form a regular array, the number of summands in (13) can be reduced by half.

As far as $\vec{k}_4 = -\vec{k}_1$,

$$d_4^2 = \vec{k}_4^T\mathbf{G}\vec{k}_4 = (-\vec{k}_1)^T\mathbf{G}(-\vec{k}_1) = \vec{k}_1^T\mathbf{G}\vec{k}_1 = d_1^2.$$

Similar actions can be performed for vectors $\vec{k}_2$ and $\vec{k}_5$, $\vec{k}_3$ and $\vec{k}_6$, etc.

Expression (13) takes the following form:

$$P_0^{\max} = 1\Bigg/\left(1+2\sum_{i=1}^{J/2}\exp\{-0,5d_i^2\}\right). \qquad (14)$$

The research results in [17] - [20] confirm the effectiveness of the threshold limitation (3) in case of the phase direction finder with the antenna spacing diagram shown in Fig. 6 (phase-measuring bases are in millimeters).

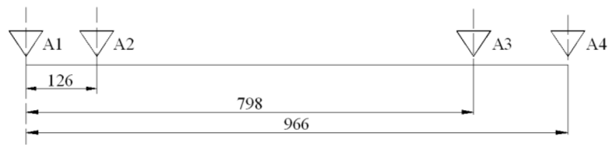

Fig. 6. Antenna system of the phase direction finder

We shall calculate the bound limit probability of this direction finder correct disambiguation $P_0^{\max}$. The number of ambiguity vectors, the proper domains of which are adjacent to the proper domain of zero vector $\vec{k}_0^T = (0;0;0)$, is 6. These vectors have the following values:

$\vec{k}_1^T = (1;5;6)$, $\vec{k}_2^T = (1;6;7)$, $\vec{k}_3^T = (0;1;1)$,
$\vec{k}_4^T = (-1;-5;-6)$, $\vec{k}_5^T = (-1;-6;-7)$, $\vec{k}_6^T = (0;-1;-1)$.

Parameters d_i^2 of these ambiguity vectors are:

$d_1^2 = d_4^2 = 0,053\sigma_\varphi^{-2}$, $d_2^2 = d_5^2 = 0,059\sigma_\varphi^{-2}$,
$d_3^2 = d_6^2 = 0,064\sigma_\varphi^{-2}$.

Table I shows the values $\exp\{-0,5d_i^2\}$ for phase difference measurement errors of 20, 25, 30, and 35 degrees.

Fig. 7 shows the dependence of the conditional probability of the phase measurements P_0 correct disambiguation and the upper bound $P_0^{\max}$ on the root-mean-square deviation (RMSD) of the phase difference σ_φ, provided that the measurement has taken place. The threshold Δ is defined as a fraction of the maximum possible value, which is equal to $\Delta_{\max} = \sigma_\varphi^2 \left(d_{\min}/2\right)^2$.

Table- I: The values $\exp\{-0,5d_i^2\}$ for the phase difference measurement errors

Ambiguity vectors $\vec{k}_i$	Values of probability density at the centre of proper domains $W(y_1, y_2)/K = \exp\{-0,5d_i^2\}$			
	RMSD of phase difference, degree			
	20	25	30	35
$\vec{k}_1$	0,0002	0,0039	0,0213	0,0591
$\vec{k}_2$	0,0001	0,0023	0,0148	0,0452
$\vec{k}_3$	0,0001	0,0013	0,0102	0,0345
$P_0^{\max}$	0,9994	0,9851	0,9153	0,7828

Fig. 7 shows that implementation of the threshold quadratic limitation (2) resulted in the reduction of the measurement P_M probability due to the rejection of 'bad' measured phase differences and to a significant reduction in the probability of abnormal errors. For all values of RMSD of phase difference, the curve of $P_0^{\max}$ passes above the curves P_0 for any values of the threshold in case of the quadratic form (2). This means that the boundary to improve the radio direction finding accuracy has been found, and confirms the applicability of expressions (13) or (14) for the practical calculations.

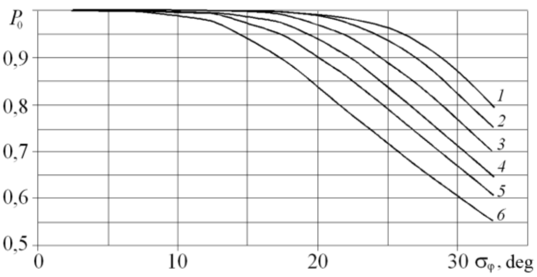

Fig. 7. Dependence of the upper bond of the correct disambiguation $P_0^{\max}$ probability (curve 1) and the conditional probability of the correct disambiguation P_0 on RMSD of the phase difference σ_φ at $\Delta = 0.2\Delta_{\max}$; $\Delta = 0.5\Delta_{\max}$; $\Delta = 0.7\Delta_{\max}$; $\Delta = \Delta_{\max}$ (curves 2–5, respectively) without the threshold limitation (curve 6)

IV. CONCLUSION

There are suggested the method to calculate the upper bound of the probability of the correct disambiguation in multi-base phase direction finders using the algorithm for rejecting the measurement results with abnormally big errors in the disambiguation process. The physical nature of this method is demonstrated. In order to test the theoretical positions of the method we made a computer-based simulation. The method is applicable to the phase direction finders with linear, planar or conformal antenna arrays and can also be applied to other types of multi-scale phase-measuring systems.

ACKNOWLEDGMENT

The work was performed as part of the state task of the Ministry of Education and Science of the Russian Federation number FEWM-2020-0039.

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