Mathematical Modeling of the Processes Formations of Stocks in Low Water Period (on the example of the Kitab-Shahrisabz aquifer)

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Abstract*: *In this study, we consider mathematical modeling of the dynamic state of groundwater aquifers, i.e., the process of groundwater formation in dry years under intensive (forced) groundwater intake withdrawal, i.e. operational selection exceeds the value of groundwater resources and depletion of capacitive reserves occurs) on the example of the Kitabo-Shakhrisabz groundwater deposits, of the Kashkadarya area of the Republic of Uzbekistan, which has a long period of regime observations and comparatively correct information on the groundwater level regime, groundwater intake withdrawal and interconnections within surface runoff. The data of hydrogeological area obtained as a result of analysis and schematization of hydrogeological conditions are generalized, and the hydrogeological parameters of the aquifer are calculated. The hydrogeological factors of groundwater formation are given and evaluated taking into account changes in water intake conditions, their current state is highlighted, and recommendations are given for substantiating the tasks of groundwater automations of monitoring in these territories..

Keywords*: *Mathematical modeling, boundary conditions, groundwater formation, geofiltration process, groundwater intake, automations of monitoring water resource.

I. INTRODUCTION

By administrative division the territory is part of the Kashkadarya area of the Republic of Uzbekistan. In limits of Kitab-Shahrisabz aquifers inter mountainous depression the great practical value have the underground waters located in thickness of quaternary sediments. The last are deposited on a dim surface of the neogen, and in the east, northeast parts of a depression contact to limestone's, to dolomite and sandstones of the paleogenous, that creates a condition for hydraulic connection between underground waters of quaternary sediments and underground waters of the paleogenous [8, 9]. The detailed studies on the channel balance [9], the selection and regime of groundwater, and experimental filtration work made it possible (using literature data on irrigation systems and the irrigated field) to estimate the groundwater balance of the Kitab-Shakhrisabz for 1976. (54% of the supply, i.e., the average water availability per year). Therefore, a typical base model was created precisely for 1976. and on this model, computational experiments were carried out for different years of water availability (high-water - 1969 and low-water - 1986, 2000 and 2012) [9].

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II. FORMULATION OF PROBLEM

Mathematical models describing geofiltration processes represent a system of partial differential algebraic equations with corresponding initial and boundary conditions. The non-stationary filtering model can be represented in the following form [1, 3, 11, 15].

$$\mu\frac{\partial h}{\partial t}=\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(Kh\frac{\partial h}{\partial y}\right)+w+\delta_1\cdot Q_1+\delta_2\cdot Q_k-\delta_3\cdot Q_g \qquad (1)$$

with initial and boundary conditions:

$$h(x,y,t)=F_1(x,y), \qquad (x,y)\in G, \qquad t=t_0, \qquad (2)$$

$$h(x,y,t)=F_2(x,y,t), \qquad (x,y)\in\Gamma_i, \qquad t\geq t_0, \qquad (3)$$

$$kh\frac{\partial h}{\partial n}=F_3(x,y,t); \qquad (x,y)\in\Gamma_i, \qquad t\geq t_0 \qquad (4)$$

$$-kh\frac{\partial h}{\partial n}=\gamma(h_в-h); \qquad (x,y)\in\Gamma_i, \qquad t\geq t_0 \qquad (5)$$

where, $h(x,y,t)$-groundwater level, m;

$K(x,y)$ - coefficient of filtration of the integument, m/day;

$f(x,y,t)$ - infiltration, m/day;

$w(x,y,t)$ - evaporation from a mirror of groundwater, m/day;

μ - free water loss;

Q_k - filtration losses from the channels, m/day;

Q_g - pinch out of groundwater into rivers, m/day; Q_1 -well intake, m/day;

G -area, with boundary Γ_i,

$$G = \sum_{i=1}^{3} \Gamma_i;$$

$F_1 - F_3,$ γ - predefined functions;

$\delta_1 - \delta_3$ - Dirac function.

III. SUGGESTED APPROACH

To solve equation (1)-(5) the boundary value problem by the method of F.B.Abutaliev, W.U. Umarov, I.Khabibullaev et al. uses the numerical method [1, 3, 4, 6, 15]. To convert the original partial differential equation to a system of algebraic equations, we ware chooses the finite difference method [11].

The filtration region *G* with boundary is replaced by a uniform grid region, i.e. is covered by an orthogonal uniform grid with steps *x* and *t*. Then the values of the function *h* and the arguments *x, y* time *t* are replaced by grid analogues [16]:

$$h_{i,j,t} = h(x_i, y_j, t_\tau);\ x_i = \Delta x \times i; \quad y_j = \Delta y \times j;$$

$$t_\tau = \Delta x \times \tau; i = \overline{1, N};\ j = \overline{1, M} \quad \tau = \overline{1, T}$$

where *N, M* is the number of rows and columns of the grid domain, T is the final calculation time, step sizes, Δx, Δy, Δt. For each moment of time t, we determine the state *h(i, j)* and then, using the formula, we calculate the filtration rates [3].

$$V_{i,j} = k_{i,j} \frac{h_{i+1,j} - h_{i,j,}}{\Delta x}$$

and the paths of motion are $S = V_{i,j}/T$. If $S/\Delta x \geq \Delta k$, then the boundary will move by “*k*” steps along the *x* coordinate, as well as along the *y* coordinate. And on these bases a package of application programs was compiled, the functional part of the package is organized in the form of components consisting of modules that provide the definition of a function specific to a given subject area, for example, reading the initial data and outputting the results in the form of a matrix; , the values of the filtration coefficient, the marks of the stopper, the flow rate of wells, etc., information for the grid model, the parameters of the grid region and the filtration region, according to their numbering. In the same way, arrays of inflow-outflow at the border, channel flow, etc. are calculated.

Based on the hydrogeological data and the schematization of the hydro-geological conditions for modeling, a piecewise-homogeneous single-layer scheme of the structure of the aquifer with horizontal aquifer was adopted.

The underground waters contained in quaternary sediments are among themselves in the hydraulic connection as in a vertical section, and in the plan and form the uniform hydro dynamical system. Within the limits of this system are separated [9, 12, 13, 14]: The aquiferous horizon of upper quaternary alluvial - proluvial sediment; (Q_{III}). The aquiferous horizon of middle quaternary alluvial - proluvial sediment; (Q_{II}). The waters circulating in upper quaternary aquifer horizon concern to the type of subsoil waters with the free surface having slope of 0,004-0,005. The regional impervious bed of described complex aquifer is neogenous sediment.

The maximal capacity of water containing gravels is observed in the central part of a depression. Here it makes more than 300 m. Depth of bedding of subsoil water level in the top parts of cones influence reach 20 m and more. In the central part of a depression in connection with immer-sing aquifer of gravels under the cover of loams, and also due to occurrence of loamy layers thickness gravels, waters get weak pressure.

Depending on a relief of a contour pressure of the surface levels are established on depth from 17,45 m up to +12,3 m from a surface of the ground. A mineralization of water up to 0,85 g/l, type of water, hydrocarbon – calcium and hydrocarbon – sulphatic [9, 14].

A feed of aquifer horizons occurs basically, due to filtration losses of a superficial drain of the rivers, channels from irrigated fields and an atmospheric precipitation. Besides, underground inflow takes place on the part of mountains surrounding a depression.

The discharge of underground waters occurs by vertical unloading to the top horizons and bottoming in valleys of the rivers in a replacement zone of gravel - grit sediment by gritty-loamy breeds.

On the conditions of feed and draining underground waters three zones are divided.

A. A zone of formation of underground waters (absorption of a superficial drain). The area 326 кm^2, amplitude of fluctuation of a level of subsoil waters Δh= 5-7m depending on water abounding year. Factor of the variability of a selection K_1=30-50.

B. A zone of intensive bottoming, the area 332 км2. Δh= 2-3 m. Factor of dynamism K_2=2-2,5.

C. A zone of weak bottoming, the area 218 кm^2, Δh= 2-3 m [9, 13].

The underground drain is formed in foothill parts and directed downwards on valleys from the west to the east according to hypsometric decrease of a relief from 700 m up to 500 m though the impervious bed of deposited is practically horizontal, and especially in the central part of a depression of 340-380 m, with slope of 0,04-0,005 absolute marks of water table change from 670 up to 500 m, [9] water-conductivity of sediment changes from 1000 (northern part of a depression) up to 6000 m_2/d the central and east part of a depression.

The productive aquiferous complex has universal distribution. Ground waters of the given complex are dated to gravel-grit alluvial sediments of quarternary age. Average thickness of the horizon is equal to 200-300 m. General thickness of the quarternary aquiferous complex increased in the western direction (downwards on a valley). The debit of the wells change from 20 up to 100,0 l/s., and specific debit from 2,2 up to 81,4 l/s., water abounding of these sediment changes on the area and in a section.

The level of underground waters on a considered site is located at depth of 3-20 m. On the base of known data of balance of 1976 the model was made and the stationary task was solved. At the specification of values of factors of filtration water labels (WL) are accepted for a basis (initial WL) as of September, 1976.

During its solution some parameters of aquifer horizons and boundary conditions were specified. For example, in model

μ=0,15 (in a place of μ=0,1-0,19), water-conductivity, infiltration, bottoming in a hydro network is updated. Thus, after calibration of model sizes of settlement values of all parameters and clauses of balance on the basis of which the further numerical experiments with various years of water- maintenance were obtained [8, 9, 13]. Clauses of balance of 1969 (rich water) and 1986, 2000, 2001 (low water) were calculated. Elements of balance were defined as follows:

A. Infiltration from the rivers - on the factor of the infiltration feed from the charge expense determined by experimental data in 1974-1976 years which has made:

B. Factor of the infiltration feed of underground waters (K) r. Kashкadarya K= 0,127; C) r.Aкdarya K = 0,086; D) r. Tanhezdarya K = 0,194; E) r. Yakkabagdarya K = 0,104;

C. Infiltration from irrigated fields and sprinklers was accepted equal in 1976 y. with some reductions of its value depending on water abounding year.

D. Bottoming was defined as a member of the balance equation of underground waters under the formula [5]:

$Q_{bottoming}=P\pm Q_{capacity}-(Q_{selection}+Q_{outflow})$

where, P-inflowing clauses of balance (including inflow in a balance contour); $\pm Q_{capacity}$- downturn or completions of capacitor stocks for the settlement period; $Q_{selection}$.- selection of underground waters; $Q_{outflow}$ - outflow of underground waters [7, 10]. The maps of hydroisogips for these years are obtained and adjusting stocks for years of various securities, since 1966 on 2002 year are determined. On the basis of the numerical information obtained at modeling it is possible to draw the following conclusions [6]:

Kitab-Shahrisabz underground water deposit has significant natural accumulation (about 7,5 billion m3) and operational regulating (about 1 billion m3) capacity. Adjusting stocks are defined under the formula:

$$V_p = \mu \cdot F \cdot \Delta h,$$

where, μ- water-feedback, the F-area, m^2, Δh- amplitude of fluctuation, m.

In the natural conditions in a long-term section the amplitude of fluctuation of mid-annual WL did not exceed 5 m (in zone A), 2,5 m (in zone B) and 1 m (in zone C). And therefore adjusting stocks were defined by the prisoned capacitor stocks in the most low water year (2001 y.) and by the position of WL in the various researched years. By intensity of mode of operation of underground waters is subdivided: on a) normal (at $Q_{exploit}\approx Q_{resource}$) b) forced ($Q_{exploit}>Q_{resource}$) c) slowed down ($Q_{exploit}<Q_{resource}$).

The important factor influencing formation of underground waters (and underground water basins) is feeding sources and their mode (adjusting stocks), and also exploitations of underground waters.

During intensive pump out of underground waters in 1976-86 (for irrigation and water supply) the ground (subpressure head) waters has essentially changed. If in natural conditions (up to 1970) the balance of underground waters changed depending on water abounding of the rivers during 1971-86 (the forced use except 1979-82) there was constant decrease of WL (and subpressure head waters) and owing to development of the regional depression cone the operational regulating capacity in volume about 1 billion m^3 was generated.

In a zone A average size of WL downturn has reached about 10 m (23,46 m - 13,19 m), and the amplitudes of seasonal fluctuations have increased [9]. In 1987-1992 use of underground waters did not exceed their feed (about) 6,5 m^3/s), from 1994 till 1999 slowed down mode of exploitation was made ($Q_{exploit}$=3,2-4,4 m^3/s<$Q_{resource}$).

In low water years (2010-2012) the forced pump out of underground waters $Q_{exploit}$=(6,95 -10,74 m^3/s) > $Q_{resource}$ was made. In the years of 1994 and 1999 WL was restored up to conditionally natural size (1969 -11.28 m, 1999 -11.53 m, 1994 -11.11 m) (fig. 1.)

As a result of the analysis of the regime information, 1966-2002 and modeling it is established that for the low water period 545 million m^3 of capacitor stocks (exploitation regulating capacities) is worked, maximal downturn capacitor stocks was observed in 1986 after several low water years and pump out of underground waters in the forced mode (7 m^3/s).

Since 1986 after several water abounding years and exploitation selection in the slowed down mode ($\approx$3,5 m^3/s) the level was restored up to the natural size. In 2002 adjusting stocks made 270 million m^3 (about 9 m^3/s).

In a long-term section operational regulating capacity makes $\approx$1 billion m^3, as it was confirmed with results of modeling. Besides by results of numerical modeling the forecast of UWL change was given at selection of underground water in the amount of 10,7 m^3/s within 3 low water years (see fig.2.)

For the initial hydrodynamic conditions of the filtration flow, the distribution of UWL 1976 is taken. A discrete grid model that carries information about the initial boundary conditions and design parameters consists of 43x24 = 1032 modal points (scale 1:1,000,000) with a grid spacing x = y = 1000 m. (Fig. 2-3).

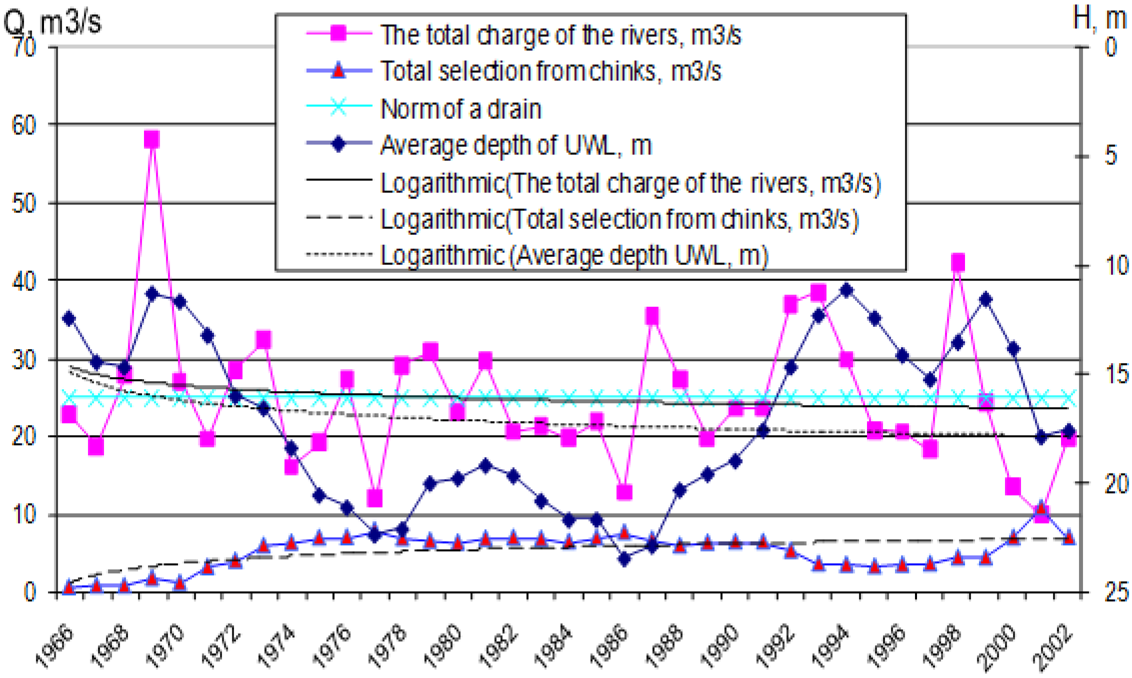

Fig 1. The combined diagram of annual of the rivers of Kashkadarya basin, average depth of water and selection of underground water

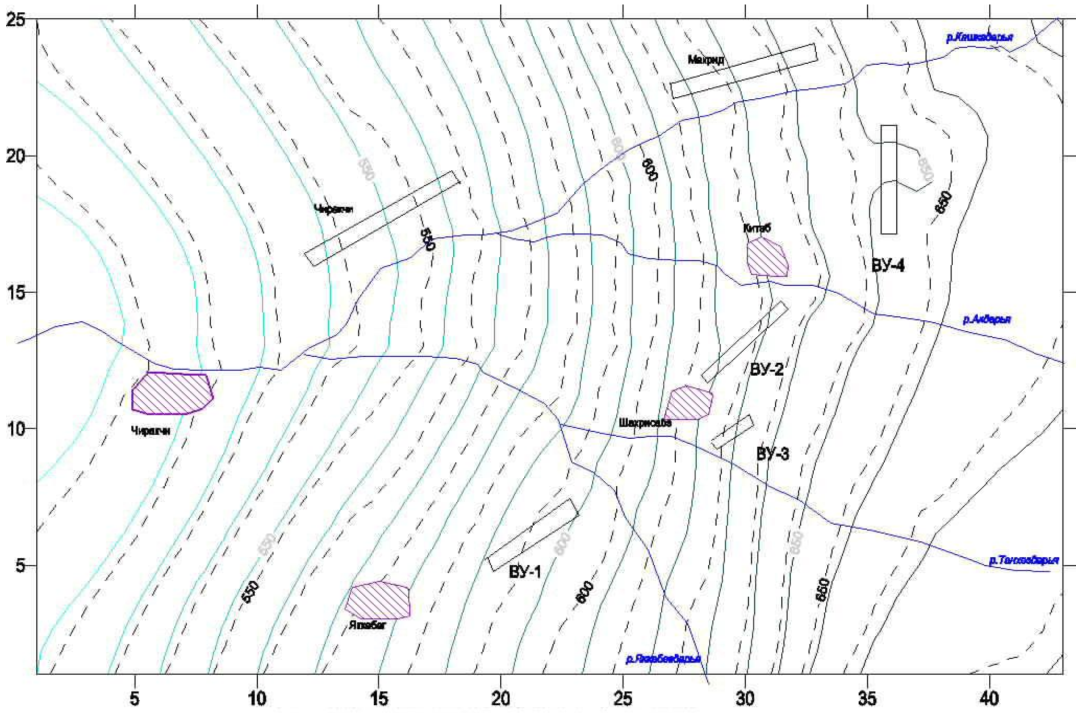

Fig. 2. Schematic map of the boundary conditions and hydroisogypsum of the Kitab-Shakhrisabz aquifers.
1976 (- - -); 2012 (——) absolute mark, m.; the numbers on the frame 5-40 are the conditional coordinates of the grid model.

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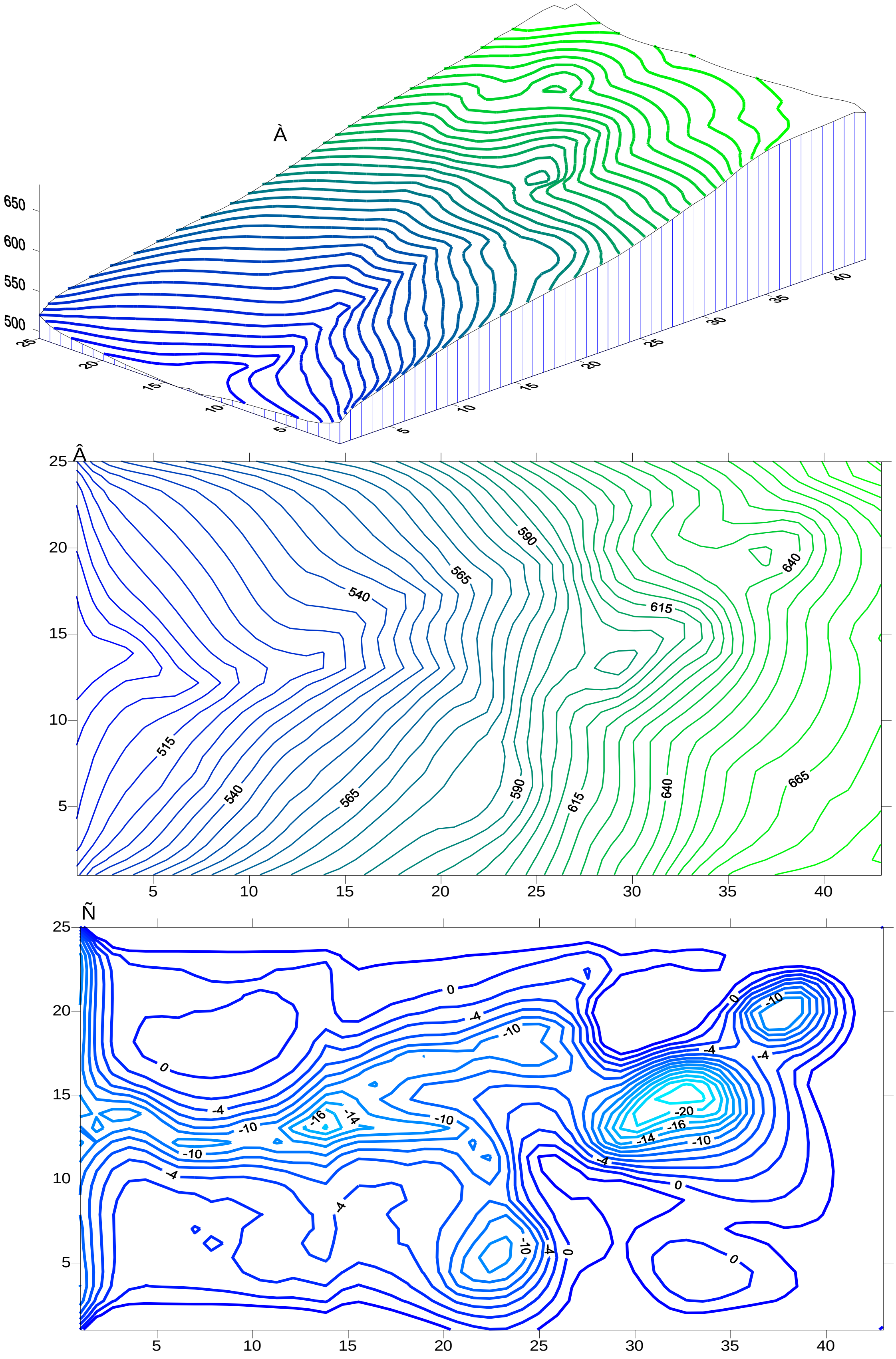

Fig.3. The block-diagram of underground water of Kitab -Shahrisabz aquifers at the pump out of underground waters of 10.7 m^3/s, and the balance (table I.) of subsoil waters on 2012 -A; Contour of underground water –B; Downturn of underground water label, as a result of the selection of underground waters in amount of 10.7 m^3/s within three low water years –C

Table- I: Groundwater balance of the Kitab-Shakhrisabz aquifers

Balance elements		1976	1986	2000	2012	
		Estimated data	Factual data	Estimated data	Estimated data	Model data.
Inflow balance sheet item						
Groundwater inflow ($Q_{in.}$)		1,9	1,9	1,9	1,9	1,9
Infiltration losses from rivers: 1) Kashkadarya River		0,15	0,96	0,8	0,54	0.6
2) Aksu River		0,63	0,97	0,49	0,3	0,5
3) Tankhazdarya River		0,25	1,04	1,45	0,32	1,45
4) Yakkabagdarya		0,5	0,97	0,46	0,41	0,46
Total: Infiltration losses from rivers		1,53	3,94	2,2	1,57	2,2
5) Infiltration from irrigated fields and canals	Zone A	3,3	3,87	3,2	3,2	3.3
	Zone B+C	4,57	5,36	4,4	4,4	4.5
Generally:		11,3	13,17	11,7	11,57	11,7
Outflow balance sheet item						
7) Groundwater abstraction $Q_{abst.}$		7,6	6,92	6,95	10,74	7,5
8) Groundwater outflow		1,25	1,25	1,25	1,25	1,25
9) Pinch out by Zone B+C		5,86	6,93	3,44	2,59	4.4
Total outflow:		10,14		11,64	14,58	11,64
Change in capacitive stocks $Q_{емк.}$		-3,4	-1,93	-3,81	-3,0	-3,0
Generally:		1,13	13,17	11,7	11,57	11,7

IV. EXPERIMENTS AND RESULTS

On a total elapsed time of exploitation (3 years) downturn UWL has made from 10 up to 20 m, in the centre of depression cone that is allowable (S_{max}<0,5 H). Thus, it is possible to recommend use of underground waters in the shallow period at a rate of 10 m^3/s, as it was confirmed in practice in 2002 with the subsequent completion of capacitor stocks in the next years at the slowed down mode of exploitation (3,5-4 m^3/s), within 3-5 years.

Modeling was carried out in stages, stage I - calibration of the model. According to the well-known balance sheet for 1976 a model was compiled and a stationary problem was solved. Stage II, in the course of its solution, some parameters of aquifers and boundary conditions were clarified. For example, on the model $\mu = 0.15$ (in place of $\mu = 0.1$-0.19), the conductivity, infiltration, and wedging in the hydraulic network were adjusted. Control was carried out on a well-known map of hydroisogypsum. Stage III, thus, after calibrating the model, the values of the calculated values of all parameters and balance items were obtained, on the basis of which further numerical experiments were carried out for different years of water availability.

Stage IV, the balance sheet items for 1969 were calculated (high water) and 1986, 2000, 2001 and 2012 (low-water). Estimated, actual and model balance elements for years with different water availability: 1969 (catastrophic high water), 1976 (average water content), 1986, 2000, 2001, 2012 (dry years, that is to say low water period) that is, we can predict the year 2022 as low-water.

V. CONCLUSION

According to the results of numerical modeling of the Kitab-Shakhrisabz aquifers, the selection of airflow in the amount of 10.7 m^3 / s for 3 dry years. For the final term of operation (3 years), the decrease in UWL in the center of the depression funnels was from 10 to 20 m, which is permissible (S_{max} <0.5 H).

It is necessary to indicate that the “model” predicted decreases do not take into account the additional decrease in the well due to their imperfection, which in experience is 15-30% of the estimated decrease. Thus, it is possible to recommend the extraction of groundwater in a dry period of размере10 m^3 / s., which will be restored in high-water years.

The Kitab-Shakhrisabz aquifers has sufficient capacitive reserves, but due to the difficult hydrochemical conditions (the presence of variegated mineral waters, both in plan and in section) and the regulated surface runoff (and groundwater supply conditions), it is not recommended to take underground water higher than replenished groundwater resources (approximately equal to the size of regulatory reserves).

Research methods for modeling geofiltration processes for determining the balance of groundwater have been developed and the hydrogeological parameters of the aquifer have been calculated from it.

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