International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-9 Issue-8, June 2020

Analog Beamforming in Millimeter Wave MIMO Systems

K. Kalyani

***Abstract:** In traditional analog beamforming schemes, like the beam selection method, use the strongest path array steering vector of the channel to generate a beam pointing to the user. In multi-user systems, such schemes will result in the large interference among the users, especially when the users are closely located. In this paper, we designed an analog beamforming scheme for downlink mm-wave multi-user systems to enhance the beamforming gain and suppress the inter-user interference at the same time. A multi-objective problem is developed to beat a balance between the inter-user interference and the beamforming gain. To solve the problem, we firstly use the weighted-sum method and then ε -constraint method to transform the multi-objective problem into a single-objective problem. Then, the analog beamforming is made tractable with the constant-magnitude constraints with the use of semidefinite programing technique. Adding to these, the robust beamforming is designed to mitigate the effects of the channel estimation and to provide the robustness against the imperfect channel information. The simulation results shows that the ε -constraint method outperforms when compared with the weighted-sum method at high SNR's for the robust multi-user analog beamforming.*

***Keywords**: mm-waves, multi-user, analog beam forming, robust , multi-objective.*

I. INTRODUCTION

The millimeter-wave (mm-wave) region is usually considered to be the range of frequencies from 30GHz to 300GHz in the electromagnetic spectrum [4][10]. The high frequency and the propagation characteristics make them, useful for various applications like in the transmission of large amount of computer data, cellular communications and radar. Hence the mm-waves communication is appraised as a key technology for future wireless communication systems. Although, it provides the high data rate and large bandwidth, it suffers from absorption by fog, dust, and smoke because of its shorter wavelength which reduce service coverage and impair communication performance [4][10]. However, the performance can be improved using beam forming with large antenna arrays, usually implemented at both transmitters and receivers to mitigate the severe propagation attenuation. These large antenna arrays leads to high system complexity for implementing conventional full digital beam forming, where each antenna element is connected to a separate radio frequency (RF) chain. Therefore, analog beam forming are used where each RF chain is uses the entire antenna array [1].

Revised Manuscript Received on June 30, 2020.
***** Correspondence Author
K. Kalyani*, Department of ECE, G. Narayanamma Institute of Technology and Sciences, India. Email: kalyanikalakanthi@gmail.com

In this paper, our main aim is to design an analog-only downlink mm-wave MIMO (Multiple Input Multiple Output) system, which enriches the beamforming gain and cancels the inter-user interference at the same time. In the first part of the paper, we mentioned detail information of the analog beamforming method with the assumption of perfect channel information.

They are two objectives, one to enhance the beamforming gain and other to cancel the inter-user interference. So, the multi-objective problem (MOP) is first analyzed and then converted to the single-objective problem (SOP). This conversion is done by two methods in the paper, one by using the weighted-sum method and later one by using ε-constraint method. Then further a semi-definite programming is used to deal with analog beamforming with constant magnitude constraints.

In the second part of the paper, the imperfect channel information is considered for optimizing the above two mentioned problems, i.e.., maximizing the beamforming gain and minimizing the interference. This method also adds the strength against the imperfect channel information. In this paper, we assumed the errors in the AoD/AoA (Angle of Departure/ Angle of Arrival) of the channel. These errored channel information is used to developed the objective problem. This objective problem is further simplified using the weighted-sum and the ε-constraint method similarly as in the non-robust method.

The contributions of our paper can be summarized as follows:

- We developed an analog beamforming scheme based on the path angle information in mm-wave systems. The scheme strikes a balance between beamforming gain and inter-user interference only using partial channel information using -constraint method.
- This method shows the improvement in the sum-rate compared to the weighted-sum method at high snr's.

The further sections are arranged as follows. In section II, we briefly described about the system and the channel models. Section III, deals with the problem formulation for analog beamforming method based on the perfect and imperfect channel information. Section IV, describes the detail information about the weighted-sum method and ε-constraint method for both robust and non-robust analog beamforming for perfect/imperfect channel information. Section V, presents the simulation results for all the mentioned methods. Section VI, provides the concluding remarks.

II. CHANNEL MODEL AND SYSTEM MODEL

A. Channel model

The mm-waves have less scattering characteristics as it suffers from absorption by fog, dust, and smoke because of its shorter wavelength which reduce service coverage and impair communication performance. Hence to characterize the less scattering property, the cluster channel model for mm-waves are adopted with P_k rays for the user k. The channel vector can be as followed by

$$c_k^H = \sqrt{\frac{M_t}{P_k}} \sum_{p=1}^{P_k} \left(\alpha_p^k\right)^* v_t\left(\theta_p^k\right)^H \tag{1}$$

Where $v_t\left(\theta_p^k\right)$ is the antenna array steering vectors of the BS for path p with angle of departure θ_p^k for the user k. The other term $\left(\alpha_p^k\right)^*$ is the complex path gain of path p modelled by a complex Gaussian distribution such as ∊N(0,1). The antenna array response vector of p^{th} path of k user is given by

$$v_t\left(\theta_p^k\right) = \frac{1}{\sqrt{M_t}}[1\ e^{j\frac{2\pi}{\lambda}d\sin(\theta_p^k)}, \dots, e^{j\frac{2\pi}{\lambda}d(M_t-1)\sin(\theta_p^k)}] \tag{2}$$

In the above equation, the terms, d is the spacing between the antenna elements and λ is the wavelength of the signal. θ_p^k , departure angle is assumed to have a uniform distribution over $[0,2\pi]$

The array steering vector of the user k is given as follows

$$V_k = [v_t(\theta_1^k),\ v_t(\theta_2^k), \dots\dots, v_t(\theta_P^k)]^H \tag{3}$$

Matrix, $V_k \in \mathbb{C}^{M_t X P_k}$ is the concatenation of all the array steering vectors of the user k.

B. System model

Consider a downlink multi-user environment with a base station communicating with K users as shown in Fig 1. The number of RF chains is set as M_{RF} and each assigned to each user. The base station generates analog beamforming vector for each user with the estimate multi-path angles of the channels. s_k denotes the transmitted data intended for user k.

The data s_k is assumed to be normalized such as $E[||s_k||^2] = 1$. The beamforming vector for s_k is $w_k \in \mathbb{C}^{M_t X 1}$. The channel matrix for user k is $c_k^H \in \mathbb{C}^{1\, X\, M_t}$. The received signal for user k is as follows,

$$y_k = c_k^H w_k s_k + \sum_{i=1, i\neq k}^{K} c_k^H w_i s_i + n_k \tag{4}$$

Where n_k is the additive white gaussian noise (AWGN) with zero mean and 'σ^2' variance for user k. The first term in the eq (4), signal power and the second term is the co-channel interference caused by other users. The SINR of the user k is given by

$$SINR_k = \frac{|c_k^H w_k|^2}{\sigma^2 + \sum_{i=1, i\neq k}^{K} |c_k^H w_i|^2} \tag{5}$$

The interference for the user k is

$$\text{Interference} = \sum_{i=1, i\neq k}^{K} |c_k^H w_i|^2 \tag{5a}$$

The sum-rate is expressed as

$$\text{Sum-rate} = \sum_{i=1}^{K} \log_2 (1 + SINR_i) \tag{5b}$$

III. PROBLEM FORMULATION

A. For non-robust analog beamforming

In this paper, the main aim is to enhance beamforming gain and cancel the interference simultaneously. The leakage interference can be formulated as

$$I_k = [V_1, \dots, V_{k-1}, V_{k+1}, \dots, V_K]^H \tag{6}$$

Where $I_k \in \mathbb{C}^{\sum_{i=1, i\neq k}^{K} P_k\, X\, M_t}$ is a concatenation of all the array steering vectors of all users except user k . The null space of I_k contains the weight vector w_k according to the traditional zero forcing scheme. The null space of I_k can

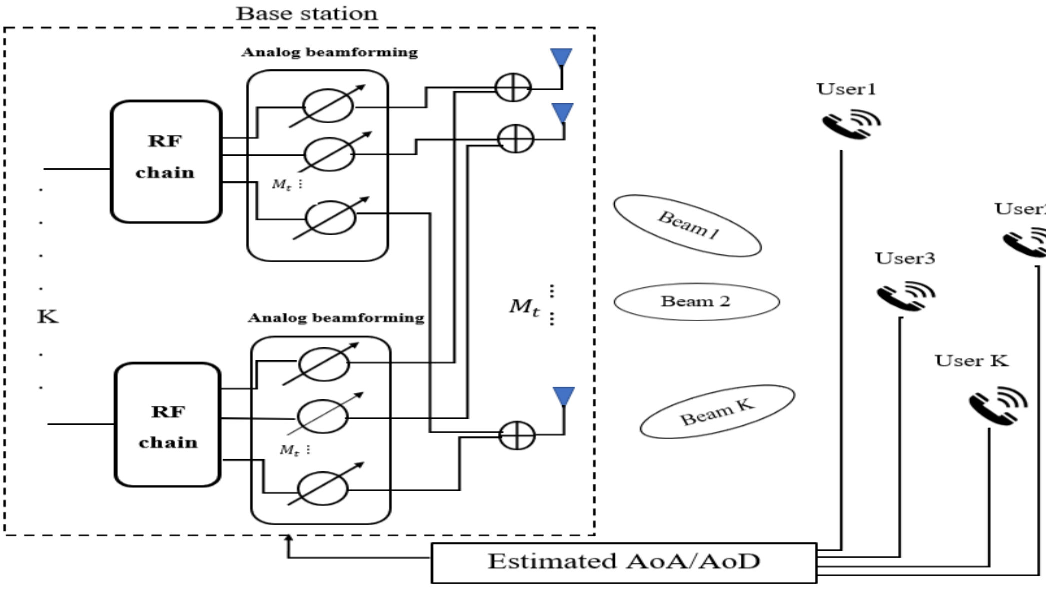

Fig.1 System model

be calculated through singular-value decomposition (SVD). The SVD of I_k can obtained as

$$svd(I_k) = U_k \sum_k [X_k^{(1)} X_k^{(0)}]^H \quad (7)$$

where $X_k^{(1)}$ holds the first $\sum_{i=1, i\neq k}^{K} P_k$ right singular vectors and $X_k^{(0)}$ holds the last $M_t - \sum_{i=1, i\neq k}^{K} P_k$ right singular vectors. M_t is assumed to be very large such that $M_t - \sum_{i=1, i\neq k}^{K} P_k > 0$, to ensure that null space exists. To minimize the interference , it is actually minimizing the projection from w_k to I_k i.e.., w_k must have the larger projection on to the null space of I_k, which means it enough to increase the projection of w_k on to null space of I_k which can be obtained as

$$w_k^{Proj} = X_k^{(0)} (X_k^{(0)})^H w_k \quad (8)$$

The objective problem of minimizing the interference can be simplified to maximizing the projection of w_k on to null space of I_k. For more simplicity, we maximize the square of the norm of w_k^{Proj} and it is taken as

$$||w_k^{Proj}||^2 = w_k^H X_k^{(0)} (X_k^{(0)})^H w_k \quad (9)$$

The problem for minimizing the interference is formulated, but it not enough to optimize the SINR, as it only consider the interference. Hence the beamforming gain is to be maximized as it refer to the receive power. Thus the beamforming gain under partial channel information can be defined as

$$Beamforming\ gain = w_k^H V_k (V_k)^H w_k \quad (10)$$

Thus considering the both the problems, the multi-objective problem can be formulated as

$$w_k^{opt} = argmax\{w_k^H X_k^{(0)} (X_k^{(0)})^H w_k \,, w_k^H V_k (V_k)^H w_k\}$$
$$\text{s.t. } w_k \in W \quad (11)$$

where W is the set of all the constant-magnitude vectors with magnitude of each element as $\frac{1}{\sqrt{M_t}}$.

B. For robust analog beamforming

The previous sub section deals with non-robust analog beamforming where the channel model is developed based on the perfect channel information. This section deals with the imperfect channel information. The error in the channel information can be modelled as

$$v_t(\theta_p^k + \Delta\theta_p^k) = \frac{1}{\sqrt{M_t}} [1\ e^{j\frac{2\pi}{\lambda} d sin(\theta_p^k + \Delta\theta_p^k)}, ..$$
$$..., e^{j\frac{2\pi}{\lambda} d(M_t - 1) sin(\theta_p^k + \Delta\theta_p^k)}] \quad (12)$$

where $\Delta\theta_p^k$ is the angle estimation error for the path p and the user k with gaussian distributed of 0 mean and variance of $(\sigma_p^k)^2$.

The exponential term in above equation (12) can be simplified using the first-order Taylor expansion. To simply the expression, we denote $\frac{2\pi}{\lambda} d = l$, and the exponential term can be written as

$$e^{jmlsin(\theta_p^k + \Delta\theta_p^k)} \approx e^{jmlsin(\theta_p^k)} + jmlcos(\theta_p^k)\Delta\theta_p^k e^{jmlsin(\theta_p^k)} \quad (13)$$

The term $jmlcos(\theta_p^k)\Delta\theta_p^k e^{jmlsin(\theta_p^k)}$ is indicated as $e_p^{k,m}$, which represents the error for the mth element in the steering vector of the pth of user k. Error $e_p^{k,m}$ can be written as:

$$e_p^{k,m} = jmlcos(\theta_p^k)\Delta\theta_p^k e^{jmlsin(\theta_p^k)} \quad (14)$$

$$e_p^{k,m} = jmlcos(\theta_p^k)\Delta\theta_p^k \cos\left(mlsin(\theta_p^k)\right) - mlcos(\theta_p^k)\Delta\theta_p^k \sin\left(mlsin(\theta_p^k)\right)$$
$$\forall m = 0,1, \dots\dots\dots, M_t - 1 \quad (15)$$

The error vector $e_p^k = \frac{1}{\sqrt{M_t}} [e_p^{k,0}, e_p^{k,1}, \dots\dots\dots, e_p^{k,M_t - 1}]^T$ is defined as the error of the pth of user k. Thus the error in AOD is simplified as the additive random error and can be expressed as

$$\widetilde{v_t}(\theta_p^k) = v_t(\theta_p^k + \Delta\theta_p^k) \approx v_t(\theta_p^k) + e_p^k \quad (16)$$

The statistical characteristics of the e_p^k can be calculated as follows based on the mean and variance of $\Delta\theta_p^k$,

$$E[e_p^{k,m}] = E[\Delta\theta_p^k]\left(jmlcos(\theta_p^k)\cos\left(mlsin(\theta_p^k)\right) - mlcos(\theta_p^k)\sin\left(mlsin(\theta_p^k)\right)\right) = 0$$
$$\forall m = 0,1, \dots\dots\dots, M_t - 1 \quad (17)$$

$$Var[e_p^{k,m}] = E\left[\left(e_p^{k,m}\right)^* e_p^{k,m}\right] = E[(mlcos(\theta_p^k)\Delta\theta_p^k)^2 (\cos^2\left(mlsin(\theta_p^k)\right) + \sin^2\left(mlsin(\theta_p^k)\right))]$$
$$(mlcos(\theta_p^k))^2 E[\Delta\theta_p^k]^2 = (mlcos(\theta_p^k)\sigma_p^k)^2$$
$$\forall m = 0,1, \dots\dots\dots, M_t - 1 \quad (18)$$

The covariance matrix of e_p^k can be calculated as

$$D_p^k = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & (lcos(\theta_p^k)\sigma_p^k)^2 & \cdots & (M_t - 1)(lcos(\theta_p^k)\sigma_p^k)^2 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & (M_t - 1)(lcos(\theta_p^k)\sigma_p^k)^2 & \cdots & ((M_t - 1)lcos(\theta_p^k)\sigma_p^k)^2 \end{bmatrix}$$

in the above matrix, the first row and the first column are always 0, which indicates that the first element of array steering vector is always '1' and is independent of error. The modified array steering vector can be expressed as

$$\tilde{V}_k = \tilde{V}_k^{pr} + E_k \quad (19)$$

The error matrix contains all the error vector of user k, can be expressed as

$$E_k = [e_1^k, e_2^k, \dots\dots\dots, e_P^k]^T \quad (20)$$

It is assumed that the paths are independent to each other, hence the errors of different paths of a user k are independent, which means

$$E\left[\left(e_p^k\right)^H e_q^k\right] = 0_{M_t X M_t}, \quad \forall p \neq q \quad (21)$$

The covariance matrix of E_k is

$$D_k = \sum_{p=1}^{P_k} D_p^k \quad (22)$$

The imperfect interference matrix of user k can be modelled as

$$\tilde{I}_k = \tilde{I}_k^{pr} + \tilde{E}_k \quad (23)$$

where the presumed interference matrix of user k as

$$\tilde{I}_k^{pr} = [\tilde{V}_1^{pr}, \dots, \tilde{V}_{k-1}^{pr}, \tilde{V}_{k+1}^{pr}, \dots, \tilde{V}_K^{pr}]^H \quad (24)$$

and $\tilde{E}_k$ is a matrix that contains all the error matrices of all users except the user k and it is calculated as

$$\tilde{E}_k = [E_1, \dots, E_{k-1}, E_{k+!}, \dots, E_k]^H \in \mathbb{C}^{\sum_{i=1, i\neq k}^{K} M_t X P_k} \quad (25)$$

It is assumed that error of different users are independent, i.e.., $E[(E_k)^H E_h] = 0_{M_t X M_t}, \forall k \neq h$. Therefore , the covariance of $\tilde{E}_k$ is

$$\widetilde{D}_k = \sum_{i=1, i\neq k}^{K} D_k \quad (26)$$

The interference matrix is random for uncertainty of the errors, which means it is difficult to find the null space of $\tilde{I}_k$. Hence the probabilistic approach is used to restrict the interference. The outage probability can be written as

$$P_{og} = \Pr\{w_k^H (\tilde{I}_k)^H \tilde{I}_k w_k \le \gamma_k\} \quad (27)$$

where γ_k denotes a pre-specified leakage power level. Similarly the beamforming gain can written as

$$Average\ Beamforming\ gain = E[w_k^H \tilde{V}_k (\tilde{V}_k)^H w_k] \quad (28)$$

The multi-objective problem can be constructed as

$$w_k^{opt} = argmax\{\Pr\{w_k^H (\tilde{I}_k)^H \tilde{I}_k w_k \le \gamma_k\}, E[w_k^H \tilde{V}_k (\tilde{V}_k)^H w_k]\}$$
$$\text{s.t. } w_k \in W \quad (29)$$

where W is set of all constant-magnitude vectors with magnitude of each element as $\frac{1}{\sqrt{M_t}}$. In the MOP, the probabilistic objective is converted into the expectation objective through Markov's inequality. The Markov's inequality is

$$\Pr\{\delta \le \beta\} = 1 - \Pr\{\delta \ge \beta\} \ge 1 - \frac{E[\delta]}{\beta} \quad (30)$$

The probabilistic objective can be simplified as

$$\Pr\{w_k^H (\tilde{I}_k)^H \tilde{I}_k w_k \le \gamma_k\}$$
$$= \Pr\left\{w_k^H \left(\tilde{I}_k^{pr} + \tilde{E}_k\right)^H (\tilde{I}_k^{pr} + \tilde{E}_k) w_k \le \gamma_k\right\} \quad (31a)$$
$$\ge 1 - \frac{E[w_k^H \left(\tilde{I}_k^{pr} + \tilde{E}_k\right)^H (\tilde{I}_k^{pr} + \tilde{E}_k) w_k]}{\gamma_k} \quad (31b)$$
$$= 1 - \frac{E[Tr(\left(\tilde{I}_k^{pr} + \tilde{E}_k\right)^H (\tilde{I}_k^{pr} + \tilde{E}_k) w_k w_k^H)]}{\gamma_k} \quad (31c)$$
$$= 1 - \frac{Tr(E[\left(\tilde{I}_k^{pr} + \tilde{E}_k\right)^H (\tilde{I}_k^{pr} + \tilde{E}_k) w_k w_k^H])}{\gamma_k} \quad (31d)$$
$$= 1 - \frac{Tr((\left(\tilde{I}_k^{pr}\right)^H \tilde{I}_k^{pr} + \tilde{D}_k) W)}{\gamma_k} \quad (31e)$$

The average beamforming gain can simplified as

$$E[w_k^H \tilde{V}_k (\tilde{V}_k)^H w_k]$$
$$= E[w_k^H (\tilde{V}_k^{pr} + E_k)(\tilde{V}_k^{pr} + E_k)^H w_k] \quad (32a)$$
$$= E[Tr(w_k^H (\tilde{V}_k^{pr} + E_k)\left(\tilde{V}_k^{pr} + E_k\right)^H w_k)] \quad (32b)$$
$$= E[Tr((\tilde{V}_k^{pr} + E_k)\left(\tilde{V}_k^{pr} + E_k\right)^H w_k w_k^H)] \quad (32c)$$
$$= Tr(E[(\tilde{V}_k^{pr} + E_k)\left(\tilde{V}_k^{pr} + E_k\right)^H w_k w_k^H]) \quad (32d)$$
$$= Tr((\tilde{V}_k^{pr}\left(\tilde{V}_k^{pr}\right)^H + \tilde{D}_k) W) \quad (32e)$$

The MOP can be re-written for robust beamforming as

$$w_k^{opt} = argmax\{1 - \frac{Tr\left(\left(\left(\tilde{I}_k^{pr}\right)^H \tilde{I}_k^{pr} + \tilde{D}_k\right) W\right)}{\gamma_k}, Tr((\tilde{V}_k^{pr}\left(\tilde{V}_k^{pr}\right)^H + \tilde{D}_k) W)\}$$
$$\text{s.t. } w_k \in W \quad (33)$$

IV. TRANSFORMATION INTO SOP

In the previous section, the MOP is formulated for both non-robust and robust analog beamforming. To convert the MOP into single-objective problem (SOP), the two methods are used, weighted-sum method and the **ε**-constraint methods

A. Weighted-sum method

In the weighted-sum method, all the different objective problems are combined to a single objective problem using the weighted sum i.e.., multiplying the each objective function with a scalar parameter. The solution of the weighted-sum method is strongly depends on the values assigned to the scalar parameters.

a) Non-Robust Analog Beamforming

In the section III, we have formulated the MOP, to convert into a SOP , the scalar parameters are multiplied to each of the objective functions.

$$w_k^{opt} = argmax\{w_k^H X_k^{(0)} (X_k^{(0)})^H w_k, w_k^H V_k (V_k)^H w_k\}$$
$$\text{s.t. } w_k \in W \quad (34)$$

The eq(34) can be transformed as

$$w_k^{opt} = argmax\{\lambda_1 w_k^H X_k^{(0)} (X_k^{(0)})^H w_k + \lambda_2 w_k^H V_k (V_k)^H w_k\}$$
$$\text{s.t. } w_k \in W \quad (35)$$

where $\lambda_{1,2}$ are the weights multiplied to each of the problems. The relation between the weights i.e.., scalar parameters are $\lambda_2 + \lambda_1 = 1$, & $\lambda_{1,2} > 0$. The different values of scalar parameter provides different solutions. For example, to obtain the larger beamforming gain, the λ_2 must be set to larger value in its range.

Even when it changed to SOP, it is highly difficult to tractable as they are non-convex constraints. Hence to convert to convex constraint, the problem is (Semi-Definite Programming) SDP transformed as follows

$$\lambda_1 w_k^H X_k^{(0)} (X_k^{(0)})^H w_k + \lambda_2 w_k^H V_k (V_k)^H w_k$$
$$Tr(w_k^H (\lambda_1 X_k^{(0)} (X_k^{(0)})^H + \lambda_2 w_k^H V_k (V_k)^H) w_k)$$
$$Tr((\lambda_1 X_k^{(0)} (X_k^{(0)})^H + \lambda_2 w_k^H V_k (V_k)^H) w_k w_k^H) \quad (36)$$

$w_k w_k^H$ is denoted as W. Matrix W is a symmetric semi definite matrix with rank one. The constant magnitude constraints can be transformed into

$$W_{kk} = \frac{1}{M_t}, \qquad \forall\, kk = 1,2 \ldots\ldots, M_t$$

where W_{kk} represents the kth diagonal element in W. Then SOP is transformed into SDP as

$$SDP(W_k^{opt}) = argmax\{ Tr((\lambda_1 X_k^{(0)} (X_k^{(0)})^H + \lambda_2 w_k^H V_k (V_k)^H) W)\}$$
$$s.t, \quad W_{kk} = \frac{1}{M_t}, \qquad \forall\, kk = 1,2 \ldots\ldots, M_t$$
$$W \ge 0;$$
$$\text{Rank}(W) = 1 \quad (37)$$

The rank-one constraint is hard to deal with, hence the semi definite programming relaxation (SDR) is used to drop the rank one constraint.

$$SDR(W_k^{opt}) = argmax\{ Tr((\lambda_1 X_k^{(0)} (X_k^{(0)})^H + \lambda_2 w_k^H V_k (V_k)^H) W)\}$$
$$s.t, \quad W_{kk} = \frac{1}{M_t}, \qquad \forall\, kk = 1,2 \ldots\ldots, M_t$$
$$W \ge 0; \quad (38)$$

b) Robust Analog Beamforming

As mention in the previous section, the MOP problem of the robust analog beamforming is rewritten using weighted-sum method as

$$w_k^{opt} = argmax\{\lambda_1 \Pr\{w_k^H (\tilde{I}_k)^H \tilde{I}_k w_k \le \gamma_k\} + \lambda_2 E[w_k^H \tilde{V}_k (\tilde{V}_k)^H w_k]\}$$
$$\text{s.t. } w_k \in W \quad (39)$$

The SOP is then transformed to SDP. Based on the equations (31e, 32e), it can be written as

$$SDP(W_k^{opt}) = argmax\{\lambda_2\, Tr\left(\left(\tilde{V}_k^{pr}\left(\tilde{V}_k^{pr}\right)^H + \tilde{D}_k\right)W\right) + \lambda_1\left(1 - \frac{Tr\left(\left(\left(\tilde{I}_k^{pr}\right)^H \tilde{I}_k^{pr} + \tilde{D}_k\right)W\right)}{\gamma_k}\right)\}$$

$$s.t,\quad W_{kk} = \frac{1}{M_t}, \qquad \forall\, kk = 1,2 \ldots \ldots, M_t$$

$$W \geq 0;$$

$$\mathrm{Rank}(W) = 1 \qquad (40)$$

the SDR is used to drop the rank one constraint can be

$$SDR(W_k^{opt}) = argmax\{\lambda_2\, Tr\left(\left(\tilde{V}_k^{pr}\left(\tilde{V}_k^{pr}\right)^H + \right.\right.$$

Dk W+ λ1(1−Tr Ik prHIk pr+Dk Wγk)}

$$s.t,\quad W_{kk} = \frac{1}{M_t}, \qquad \forall\, kk = 1,2 \ldots \ldots, M_t$$

$$W \geq 0; \qquad (41)$$

B. *ε-constraint method*

In the previous section, the weighted-sum method is used to solve the MOP into a SOP problem, in this section we proposed a method called **ε**-constraint method to transform the MOP into a SOP. The **ε**-constraint is simple and easy to implement. In the method , it is needed to preselect the objective which is to be minimized and its corresponding ε value. In this the interference is minimized and the beamforming gain is maximized at the same time, Hence the interference term is taken as the constraint and the beamforming gain is maximized.

a) Non-robust analog beamforming

The basic MOP problem of the non-robust analog beamforming in the eq(34) is

$$w_k^{opt} = argmax\{w_k^H X_k^{(0)}(X_k^{(0)})^H w_k\ , w_k^H\, V_k(V_k)^H w_k\}$$

$$\text{s.t.}\ \ w_k \in W \qquad (42)$$

The interference term $w_k^H\, V_k(V_k)^H w_k$ can be taken as constraint, and the corresponding ε_k of user k can be calculated as the range between the lower bound and the upper bound values for the given term. In this case of interference, we consider the lower bound values as 0 and the upper bound value as 10.

Hence the MOP problem in eq(42) transformed into SOP as

$$w_k^{opt} = argmax\{w_k^H\, V_k(V_k)^H w_k\}$$

$$\text{s.t.}\ \ w_k \in W$$

$$w_k^H X_k^{(0)}(X_k^{(0)})^H w_k \leq \varepsilon_k \qquad (43)$$

Even when it changed to SOP, it is highly difficult to tractable as they are non-convex constraints. Hence to convert to convex constraint, the problem is SDP transformed as follows

$$SDP(w_k^{opt}) = argmax\{Tr((\,V_k(V_k)^H W)\}$$

$$s.t,\quad W_{kk} = \frac{1}{M_t}, \qquad \forall\, kk = 1,2 \ldots \ldots, M_t$$

$$Tr(X_k^{(0)}(X_k^{(0)})^H W) \leq \varepsilon_k$$

$$W \geq 0;$$

$$\mathrm{Rank}(W) = 1 \qquad (44)$$

Similarly as in weighted-sum method, the rank one is hard to deal. Hence SDR version equation is given as

$$SDR(w_k^{opt}) = argmax\{Tr((\,V_k(V_k)^H W)\}$$

$$s.t,\quad W_{kk} = \frac{1}{M_t}, \qquad \forall\, kk = 1,2 \ldots \ldots, M_t$$

$$Tr(X_k^{(0)}(X_k^{(0)})^H W) \leq \varepsilon_k$$

$$W \geq 0; \qquad (45)$$

b). Robust Analog Beamforming

As mention in the previous section, the MOP problem of the robust analog beamforming is rewritten using **ε**-constraint method as

$$w_k^{opt} = argmax\{E[w_k^H\, \tilde{V}_k\,(\tilde{V}_k)^H w_k]\}$$

$$\text{s.t.}\ \ w_k \in W$$

$$\Pr\{w_k^H\,(\tilde{I}_k)^H \tilde{I}_k\, w_k \leq \gamma_k\} \leq \varepsilon_k \qquad (46)$$

The SOP is then transformed to SDP. Based on the equations (31e, 32e), it can be written as

$$SDP(W_k^{opt}) = argmax\{Tr\left(\left(\tilde{V}_k^{pr}\left(\tilde{V}_k^{pr}\right)^H + \tilde{D}_k\right)W\right)\}$$

$$s.t,\quad W_{kk} = \frac{1}{M_t}, \qquad \forall\, kk = 1,2 \ldots \ldots, M_t$$

$$\left(1 - \frac{Tr\left(\left(\left(\tilde{I}_k^{pr}\right)^H \tilde{I}_k^{pr} + \tilde{D}_k\right)W\right)}{\gamma_k}\right) \leq \varepsilon_k$$

$$W \geq 0;$$

$$\mathrm{Rank}(W) = 1 \qquad (47)$$

And the SDR version can be as follows

$$SDP(W_k^{opt}) = argmax\{Tr\left(\left(\tilde{V}_k^{pr}\left(\tilde{V}_k^{pr}\right)^H + \tilde{D}_k\right)W\right)\}$$

$$s.t,\quad W_{kk} = \frac{1}{M_t}, \qquad \forall\, kk = 1,2 \ldots \ldots, M_t$$

$$\left(1 - \frac{Tr\left(\left(\left(\tilde{I}_k^{pr}\right)^H \tilde{I}_k^{pr} + \tilde{D}_k\right)W\right)}{\gamma_k}\right) \leq \varepsilon_k$$

$$W \geq 0; \qquad (48)$$

The SDR version is the upper bound solution for the SDP problem.

V. SIMULATION RESULTS

In this section, the performance of the non-robust and the robust schemes using the weighted-sum method mentioned in the Section III and the robust scheme using **ε**-constraint method mentioned in Section IV was evaluated. The main aim is to maximize the beamforming gain and minimize the interference at the same time. In the weighted sum method, the $\lambda_{1,2}$ represents the importance given to each term in the MOP for the user k. In **ε**-constraint method, the ε_k represents the constraint applied to the interference for the user k.

A. *Non-Robust Analog Beamforming*

In the simulation, the system model is developed with the large antennas array and K users. The large antenna array at the base station is assumed to have $M_t = 64$ antennas as in [1]. The users are assumed to be $K = 6$, and the number of paths, due to scattering characteristics of mm-waves are assumed to be $P = 6$ for each channel. The distance between the antenna elements are assumed to be half of the signal wavelength d=λ/2. The results are averaged over 1,000 channel realizations.

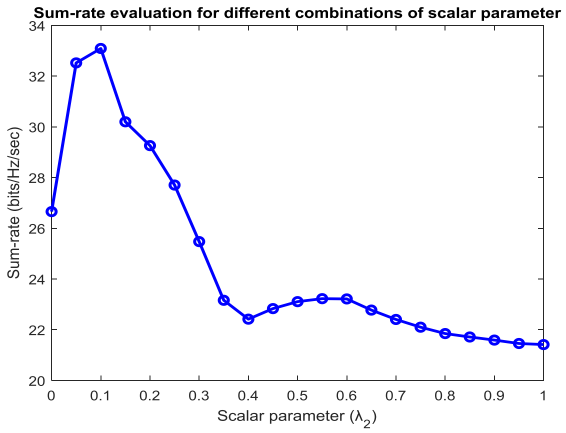

Fig.2 Sum-rate evaluated for different combinations of λ_2

Fig-2, illustrates the sum-rate evaluated for different combinations of λ_2 ranges from 0 to 1 with a step of 0.05. The SNR considered to be 30dB for this plot. The plot depicts that, when $\lambda_2 = 0{,}1$, the sum-rate is maximum. At the beginning of the plot, the sum-rate is increased till λ_2 = 0.1 and reaches the maximum and later on the sum-rate decreases at minimum and maintained which shows the trade off between the beamforming gain and the interference.

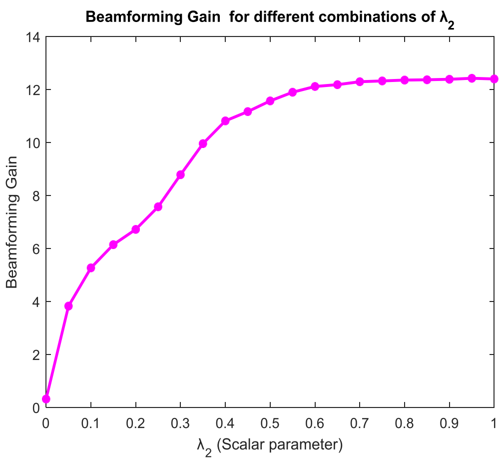

Fig.3 Beamforming gain evaluated for different combinations of λ_2

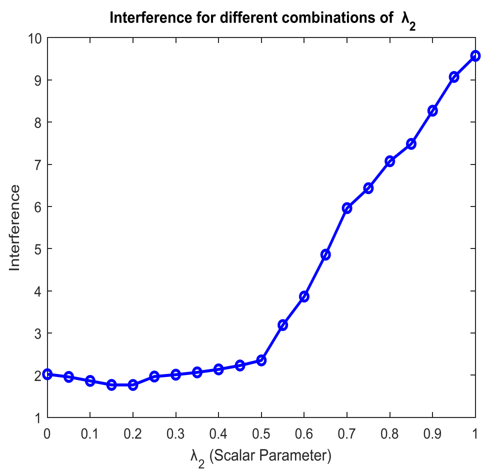

Fig.4 Interference evaluated for different combinations of λ_2

Fig-3 and 4, illustrates the beamforming gain and the interference evaluated for different combinations of λ_2. Similar to the Fig-2, it evaluated for 21 combinations of λ_2. From the Fig-3, it is observed that the beamforming gain is optimized and it increases as the λ_2 increases i.e., at very low λ_2 (=0) value, the beamforming gain is low and when the λ_2 is high , the beamforming gain is also high. Thus the beamforming gain is optimized. Similarly, the leakage interference is optimized and it decreases as the $\lambda_1 (= 1 - \lambda_2)$ increases i.e., at very high λ_1 (=1) value, the leakage interference is low and when the $\lambda_1 (= 0)$ is low, the leakage interference is also high. Thus the beamforming gain and the interference is optimized at the same time.

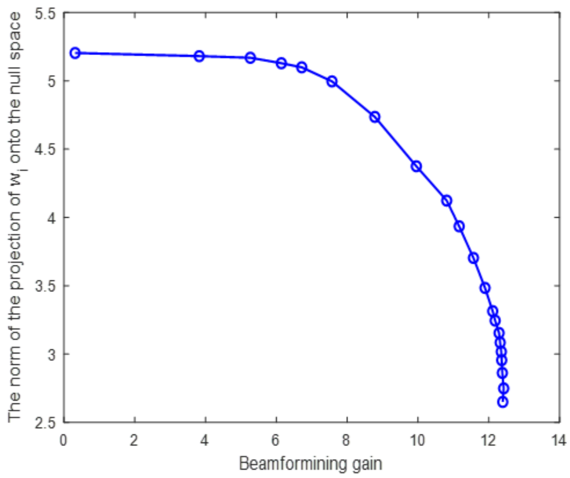

Fig.5 The trade off between the beamforming gain and the norm of the projection of w_k on to the null space.

The above figure depicts the trade off between the beamforming gain and the norm of the projection of w_k on to the null space. It is observed from the plot that, as the beamforming gain increases, the leakage interference decreases which in turn decreases the norm of the projection. Thus justifying the use of the multiple objective.

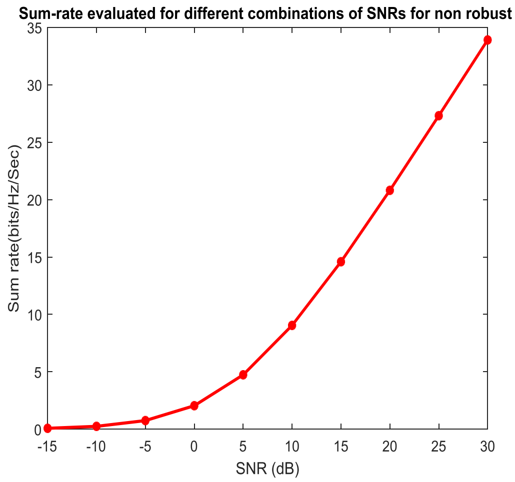

Fig.6	Sum-rate evaluated for different combinations of SNRs for non-robust scheme.

Fig-6 , depicts the sum-rate evaluated for different combinations of SNRs for the weighted-sum method for non-robust scheme. In this plot, λ2 is taken to 0.1 as at this value, it provides maximum sum rate as shown in the fig -2. It is observed that, as the SNR increases the sum-rate also increases. This gives the better results of sum-rate with respect to each SNR.

B. Robust Analog beamforming

For the robust case, the channel is assumed to be imperfect, hence the level of uncertainty i.e., the error variance is assumed as 0.005 as in [1]. The performance of the sum-rate is evaluated for different combinations of the SNR. The SNR are varied from -15 to 30 in this case with the step of 5dB. The leakage power level is set to 0.1 for all the users.

Fig 7, depicts the sum-rate evaluated for different combinations of SNRs for the weighted-sum method. In this plot, λ_2 is taken to 0.1 as at this value, it provides maximum sum-rate. It is observed that, as the SNR increases the sum-rate. But when it compared to the Fig 6, there is a degradation in the performance due the uncertainty in the channel. Thus it shows that with the slight estimation error, there will be severe system performance degradation.

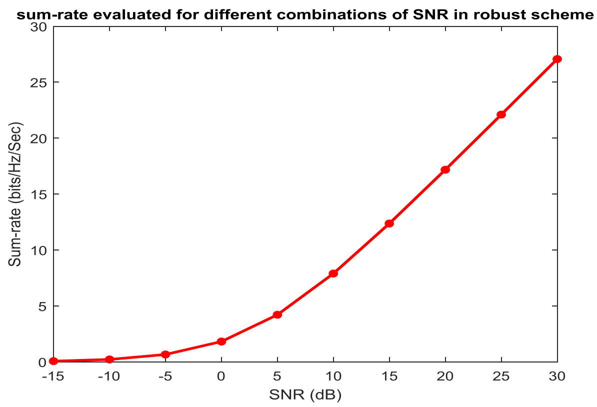

Fig.7 Sum-rate evaluated for different combinations of SNRs for robust scheme for weight-sum method.

For the robust case, the MOP is again simplified by $\boldsymbol{\varepsilon}$-constraint method. The assumptions are same as that of the $\boldsymbol{\varepsilon}$-constraint method i.e.., the level of uncertainty is taken as 0.005. The parameter ε is varied from 1 to 10 with the step of 0.5.

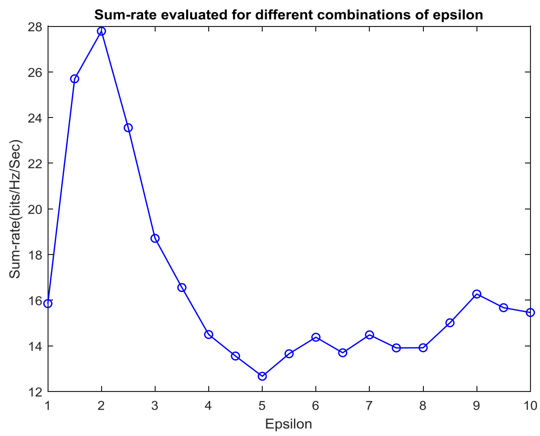

Fig.8 Sum-rate evaluated for different combinations of ε robust scheme.

The above figure depicts the Sum-rate evaluated for different combinations of for ε robust scheme. It is observed that the sum-rate increases at the beginning of the graph, later on it decreases and maintained. when $\varepsilon = 2$, the sum-rate is maximum. In this, ε refers to the interference power level, as the interference increases the sum-rate decreases which is justified in the plot.

Fig 9, depicts the sum-rate evaluated for different combinations of SNRs for the $\boldsymbol{\varepsilon}$-constraint method. In this plot, ε is taken to 2 as at this value, it provides maximum sum-rate. It is observed that, as the SNR increases the sum-rate. But when it compared to the Fig 6, there is a degradation in the performance due the uncertainty in the channel.

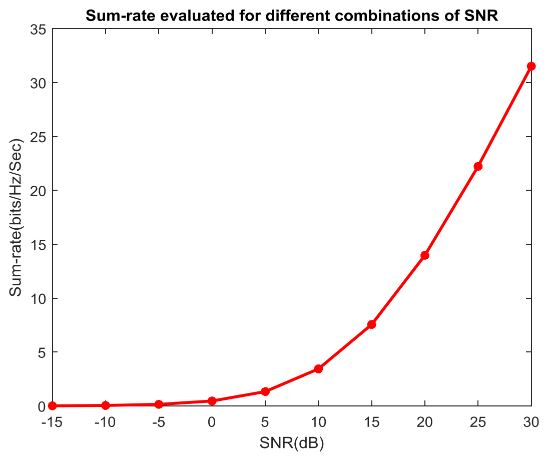

Fig.9 Sum-rate evaluated for different combinations of SNRs robust scheme for $\boldsymbol{\varepsilon}$-constraint method.

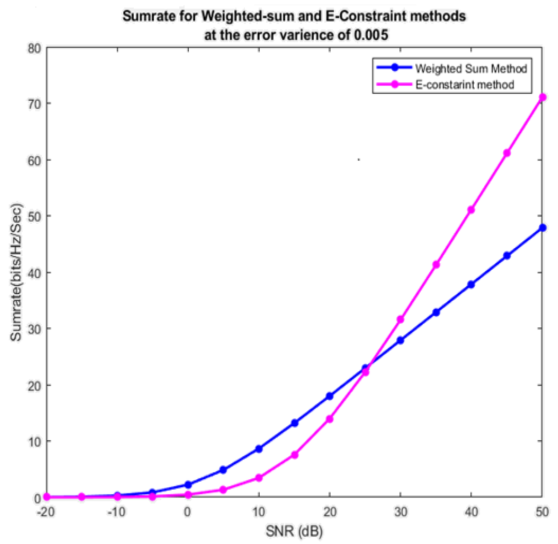

Fig.10 Sum-rate evaluated for different combinations of SNRs robust scheme for weighted-sum and $\boldsymbol{\varepsilon}$-constraint methods.

Fig 10 illustrates the sum-rate evaluated for different combinations of SNRs robust scheme for weighted-sum and $\boldsymbol{\varepsilon}$-constraint methods.

In this plot, it is analyzed that the sum-rate for both the methods are same at very low SNRs and when the SNR increases slightly, the weighted-sum method performs much better compared to ε-constraint method. At high SNRs, the ε-constraint method performance much better than the weighted-sum method.

The comparison table for the weighted-sum method and ε-constraint method

SNR(in dB)	Sum-rate for Weighted-sum method	Sum-rate for ε-constraint method
-20	**0.0226708406**	**0.0048630416**
-15	0.0712800345	0.0153604230
-10	0.2214226247	0.0483968261
-5	0.6640017181	0.1513205531
0	1.8210567058	0.4626217145
5	4.2160835754	1.3353270822
10	7.8974959409	3.4418487491
15	12.369791978	7.5597032935
20	17.177549377	13.980130759
25	22.103519752	22.233190331
30	**27.068257189**	**31.523368448**
35	32.045411014	41.255093578
40	37.026546865	51.144438306
45	42.009073035	61.085792881
50	46.992448737	71.043825560

Table 1: Comparison table for the weighted-sum method and ε-constraint method.

In the table-1, it is noticed that , all the lower SNR i.e.., from -20dB to 20 dB, the weighted-sum method provides the better results compared to ε-constraint method, where as at 25dB of snr both the methods approximately provide the same results and at high SNRs, i.e.., from 30 to 50 dB, the ε-constraint method provides the better results compared to weighted-sum method.

VI. CONCLUSION

In this paper, the analog beamforming scheme is designed to strike a balance between the interference and the beamforming gain. The weighted-sum method and the ε-constraint methods was used to convert the MOP into an SOP. Furthermore, the robust beamforming scheme is designed to overcome the uncertainty of the channel. For the robust case, simulation results showed that the proposed ε-constraint methods outperforms compared to weighted sum method at high SNR thus, demonstrated the highest robustness of the beamforming scheme against channel errors.

REFERENCES

1. Lisi Jiang and Hamid Jafarkhani , "Multi-User Analog Beamforming in Millimeter Wave MIMO Systems Based on Path Angle Information", IEEE Transactions on Wireless Communications, vol. 18, no. 1, January 2019.
2. Monzingo, R. A. and Miller, T. W. (1080): "Introduction to Adaptive Arrays," John Wiley, New York.
3. Capon, J. (1969): "High resolution Frequency Wavenumber Spectral Analysis, "Proc. IEEE, Vol57, no8, pp. 1408 - 1418, Aug., 1969.
4. B. Van Veen and K. Buckley, "Beamforming: A versatile approach to spatial filtering,"IEEE ASSP magazine, vol. 5,no. 2,pp. 4–24,1988.
5. H. Jafarkhani, Space-Time Coding: Theory and Practice. Cambridge, U.K.: Cambridge Univ. Press, Sep. 2005.
6. B. K. Chalise, S. Shahbazpanahi, A. Czylwik, and A. B. Gershman, "Robust down link beamforming based on outage probability specications," IEEE Trans. Wireless Commun.,vol. 6,no. 10,pp. 3498–3503,Oct. 2007.
7. T. S. Rappaport, R. W. Heath, Jr., R. C. Daniels, and J. N. Murdock, Millimeter Wave Wireless Communications. London, U.K.: Pearson, Sep. 2014.
8. L. Liu, Y.-H. Nam, and J. Zhang, "Proportional fair scheduling for multicell multi-user MIMO systems, "inProc. IEEE 44th Annu. Conf. Inf. Sci. Syst. (CISS),Princeton,NJ,USA,Mar. 2010,pp. 1–6.
9. J.Kazemitabar and H.Jafarkhani, "Multiuser interference cancellation and detection for users with more than two transmit antennas," IEEETrans. Commun.,vol. 56, no. 4,pp. 574–583,Apr. 2008.
10. A. Tarighat, M. Sadek, and A. H. Sayed, "A multi user beamforming scheme for downlink MIMO channels based on maximizing signal-to leakage ratios," in Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP), Philadelphia, PA, USA, Mar. 2005,pp. iii/1129–iii/1132.
11. T. S. Rappaport, R. W. Heath, Jr., R. C. Daniels, and J. N. Murdock, Millimeter Wave Wireless Communications. London, U.K.: Pearson, Sep. 2014.
12. YuhanSun, and ChenhaoQi, "Weighted Sum Rate Maximization for Analog Beamforming and Combining in Millimeter Wave Massive MIMO Communications", DOI 10.1109/ LCOMM.2017 .2703113,IEEE.
13. H. Du and P.-J. Chung, "A probabilistic approach for robust leakage based MU MIMO downlink beamforming with imperfect channel state information,"IEEETrans. Wireless Commun.,vol. 11,no. 3,pp. 1239–1247,Mar. 2012.

AUTHOR PROFILE

K. Kalyani received a B.Tech degree in Electronics and Communications from G. Narayanamma Institute of Technology and Sciences, Hyderabad in 2018. Currently, she is pursuing M.Tech degree in Digital Electronics and Communications Engineering at G. Narayanamma Institute of Technology and Sciences, Hyderabad,