

# Mathematical Modeling and Analysis of Hoop Stress in Hydroforming Deepdrawing of n-sided Polygonal Cup



G Kishan, B.V.S. Rao

**Abstract:** The main objective of this paper presents the analytical evaluation and mathematical modelling of hoop stresses of aluminium 7075 alloys in hydro forming deep drawing of n-sided polygonal cup. It is very important to find the magnitude of these stresses generated within the flange region during the deep drawing process for various n-sided polygonal cups. In the flange region two types of stresses will be generated. When is radial tensile stress, it is taking place radially outward direction from the side of the cup to outer side of the blank material. and other is hoop stress it is compressive. It is perpendicular to the radial lines drawn from the job axis to the side of the blank. It is also parallel or tangential to the blank circumference. These two stresses will be generated within the blank material by the application of punch force.. As compared to the conventional deepdrawing process, hydroforming is very convenient. uniform deformation of the blank taking place throughout out the process.  
**Keywords:** Hoopstresses,Hydroformdeepdrawing,n-sided polygonal cup

In this process subjected to equal shear forces by the effect of fluid pressure on both sides of the blank, the blank moves to the center of the gap. The blank is less than the height of the fluid film.

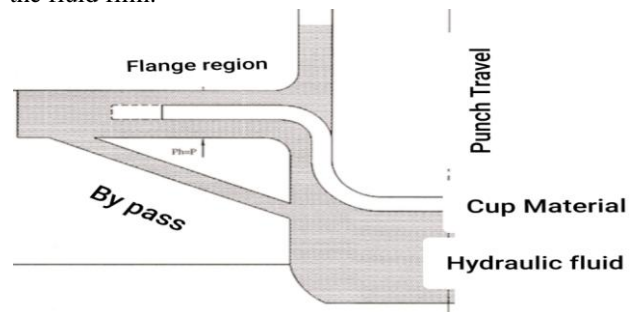


Fig.1: Hydroforming deep drawing process.

## I. INTRODUCTION:

Hydroforming deepdrawing process is one of the most important cold working process to manufacture different shapes with uniform thickness of the deepdrawn cups. In this process pressure of fluid is important parameter. The quality of cup without any failures such as fractures and wrinkles is only depends on this fluid pressure. In this process fluid pressure is applied on either sides of the blank of n-sided polygonal material. Before the deepdrawing operation the hydraulic fluid is placed in between the punching Chambar and die cavity. These two connected by means of bypass Path in the die shown in the fig.(1) some of the empty place is provided in between surface of die and blank holder. The gap is utilised to easy flow of fluid and moment of the blank. While deepdrawing operation taking place punch moves towards die cavity, The hydraulic fluid pressure In the die cavity slowly increases to maximum value. Then fluid passes through the bypass path and enter on the peripheral surface of the blank. On the either side of blank moving fluid film will be generated. When the pressurized fluid film moves either side of the blank surface shear force will be generated. The generated shear force is always proportional to the velocity gradient of the viscous fluid. It is also depends on the viscosity of the fluid.

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\* Correspondence Author

G Kishan\*, Faculty of Mechanical Engineering, University College of Engineering&Technology for Women,Kakatiya University,Warangal(U),Telangana,India-506009.

B.V.S. Rao, Faculty of Mechanical Engineering,Chaitanya Bharathi Institute of Technology,Osmania University,Gandipet,Hyderabad-500075

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## II. LITERATURE SURVEY:

Fakui.S.et al [1], Investigated the most important characteristics of forming process and the use of fluid pressure as a die. They have done mathematical analysis of hydro forming deepdrawing by considering total Strain energy theory. Yossifon et al [2] In this paper by considering classical theory of plasticity, on assuming plane strain tensile failure, rapture by tensile instability in hydroforming deep drawing formulated and tested. A distinct opening zones of fluid pressures upper and lower limits investigated recommended for the practical use.Yossifon et al [3] conducted experiments and developed a mathematical model for hydroforming deepdrawing process. In their experimental work they founded permissible operating fluid pressure path to predict the deep drawn parts from the failures. Zhang et al [4], In this paper basic theory of hydroforming deep drawing process explained. Principles and features are introduced. Theoretical and numerical simulations are explained to get better deep drawn products of highly smooth surface finish and improved mechanical properties.Thiruvvarudehelvan.S et al [5] presented theoretical analysis of hydroforming deep drawing process with constant pressure of hydraulic fluid.In this paper theoretical results are compared with the experimental work and also presented lower and upper bounds for the hydraulic fluid pressure. Thiruvvarudehelvan.S et al [5] investigated merits and demerits of the hydroforming deepdrawing princess by considering different variations of hydraulic pressure. An enhanced processing aspect investigated and discussed many techniques. Lang et al [7] Investigated a new technique in Hydraulic deepdrawing process.



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In this process they applied radial pressure along the rim of the blank material to reduce the drawing force developed in the process. They had done an experimental work with aluminium material and results validated with the finite element analysis. Lang.L et al [8] In this paper extended the same work presented in the [7] paper by considering numerical analysis. Studied the hydraulic deepdrawing mechanism assisted by radial pressure and effect of process parameters. Nader Abedrabbo et al [9] Presented the wrinkling behaviour of 6111-T4 aluminium alloy during sheet hydroforming process. In this paper numerical, experimental analysis is done and finite element method is done for the validation of the experimental work

### III. METHODOLOGY:

In this process of methodology considered to evaluate hoop stress behaviour of aluminium alloy in hydroform deep drawing process. The stresses acting on the flange of blank material during the hydrofarm deepdrawing processes shown in the fig (2), On applying force equilibrium along radial direction that is the summation of forces acting on the element in radial direction is equals to zero.

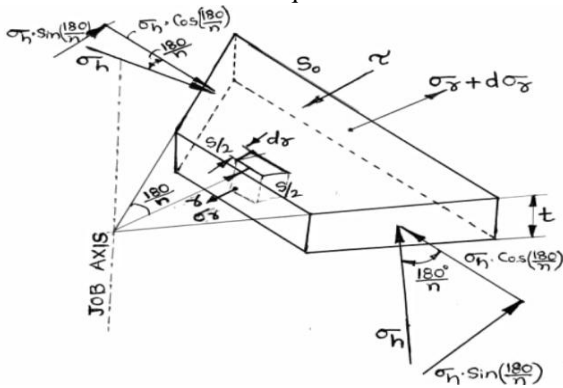


Fig.2: Stresses developed in the flange area.

$$\sigma_r \cdot S \cdot t - (\sigma_r + d\sigma_r) S_0 \cdot t - 2 \cdot \sigma_h \cdot \sin\left(\frac{180}{n}\right) \cdot \left(\frac{dr}{\cos\left(\frac{180}{n}\right)} \cdot t\right) + 2 \cdot \tau \cdot \frac{1}{2} \cdot (S_0 + S) \cdot dr = 0$$

$$S_0 = 2 \cdot (r + dr) \cdot \tan\left(\frac{180}{n}\right) \text{ and}$$

$$S = 2 \cdot r \cdot \tan\left(\frac{180}{n}\right)$$

$$(\sigma_r + \sigma_h) \cdot dr + r \cdot d\sigma_r = \frac{2\tau}{t} \cdot r \cdot dr \dots\dots\dots(1)$$

where  $\sigma_r$ ,  $\sigma_h$  are principle stresses acting on the elemental flange, on applying Tresca's yield criteria.

$$\sigma_r - \sigma_h = \sigma_y$$

$$\sigma_r + \sigma_h = \sigma_y \text{ (where } \sigma_h \text{ is compressive hoop stress)}$$

$$\sigma_r = \sigma_y - \sigma_h \dots\dots\dots(2)$$

$$d\sigma_r = d\sigma_y - d\sigma_h \dots\dots\dots(3)$$

$$\sigma_y \cdot dr + r \cdot d\sigma_y - r \cdot d\sigma_h = \frac{2\tau}{t} \cdot r \cdot dr$$

$$r \cdot d\sigma_h = \sigma_y \cdot dr + r \cdot d\sigma_y - \frac{2\tau}{t} \cdot r \cdot dr$$

$$d\sigma_h = \sigma_y \cdot \frac{dr}{r} + d\sigma_y - \frac{2\tau}{t} \cdot dr \dots\dots\dots(4)$$

On Integrating both sides of the above equation,

$$\int d\sigma_h = \sigma_y \cdot \int \frac{dr}{r} + \int d\sigma_y - \frac{2\tau}{t} \cdot \int dr + C \dots\dots\dots(5)$$

$$\sigma_h = \sigma_y \cdot \ln[r] + \sigma_y - \frac{2\tau}{t} \cdot r + C \dots\dots\dots(6)$$

where C constant of integration, it can be evaluated by applying boundary conditions at  $r = R_0$ ,  $\sigma_r = 0$  then  $\sigma_h = \sigma_y$

$$C = \frac{2\tau}{t} \cdot R_0 - \sigma_y \cdot \ln[R_0]$$

On substituting above C value in the equation (6)

$$\sigma_h = \sigma_y \left[ 1 - \ln\left(\frac{R_0}{r}\right) \right] + \frac{2\tau}{t} \cdot (R_0 - r) \dots\dots\dots(7)$$

In terms of n-sided polygonal side of blank and side of cup

$$\sigma_h = \sigma_y \left[ 1 - \ln\left(\frac{S_0}{S}\right) \right] + \frac{\tau}{t} \cdot \left( \frac{S_0 - S}{\tan\left(\frac{180}{n}\right)} \right) \dots\dots\dots(8)$$

The effect of viscosity in the hoop stresses of the blank material

$$(dy)_1 = (dy)_2 = (dy) = \left\{ \frac{h-t}{2} \right\}$$

$$\text{but } \tau_1 = \tau_2$$

by following Newton's law of viscosity the shear stress developed either sides of the blank are

$$\tau_1 = \mu \left( \frac{du}{dy} \right)_1 = \tau$$

$$\tau_2 = \mu \left( \frac{du}{dy} \right)_2 = \tau$$

total shear stress developed in the blank will be

$$\tau_{\text{blank}} = (\tau_1 + \tau_2) = 2\tau = 2 \cdot \mu \cdot \left( \frac{du}{dy} \right) = 2 \cdot \mu \cdot \left( \frac{u-0}{\frac{h-t}{2}} \right) = \frac{4 \cdot \mu \cdot u}{h-t}$$

...(9)

Now we can evaluate the hoop stresses in terms of viscosity

$$\sigma_h = \sigma_y \left[ 1 - \ln\left(\frac{S_0}{S}\right) \right] + \frac{2 \cdot \mu \cdot u}{(h-t) \cdot t \cdot \tan\left(\frac{180}{n}\right)} \dots\dots\dots(10)$$



The above equation explain the effect of viscosity of fluid on the hoop stress distribution in the flange region of the blank at the time of hydroform deepdrawing process.

**IV. DESCRIPTION OF AA7075 alloy:**

AA7075 alloy is an alloy of aluminium with the primary alloying element is zinc. It has excellent mechanical properties such as and exhibits high strength to weight ratio,good ductility,toughness and good resistance to fatigue. It is more susceptible to fragility than many other aluminium alloys because of microsegregation.It is best material among all the alloys of aluminium.It has good corrosion resistant properties. It is one of the most commonly used aluminium alloy for highly stressed structural applications. It is extensively used in manufacture of aircraft structural parts.So for this analysis AA7075 aluminium material is considered and checked the hoop stress variations for different n-sided polygonal cups.

**V. RESULTS AND DISCUSSIONS:**

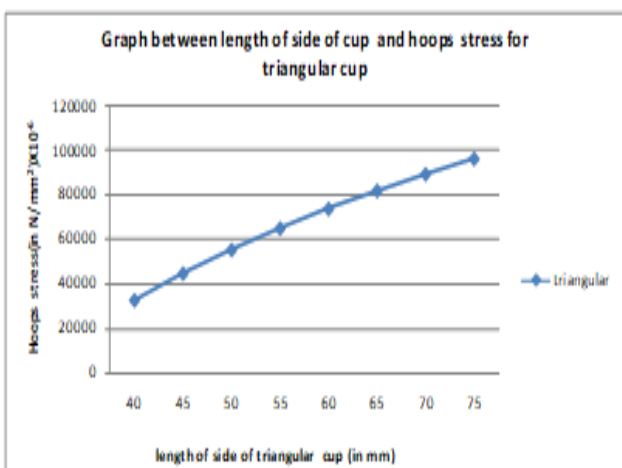
**5.1 Hoop Stress:**

The process parameters of hydroform deepdrawing process considered as follows to evaluate the hoop stresses in the flange of AA7075 aluminium alloy with a hydraulic fluid for the formation n- sided cups without any defects.The hoop stress distribution in the flank region of the blank during the hydraform deep drawing process for n- sided polygonal cup given by the eq(10).. Side of the n-sided polygonal blank,  $S_0 = 80 \text{ mm}$  , Side of the n-sided polygonal blank,  $S = 40\text{mm}$ , Yield stress value of the AA7075 material  $\sigma_y = 103 \times 10^6 \text{ N/mm}^2$ , Blank or Punch velocity of drawing  $u=10\text{mm/s}$ , Hydraulic fluid viscosity (Olive),  $\mu = 0.081\text{N}\cdot\text{s/m}^2$ , Thickness of fluid element,  $h=14\text{mm}$ , Thickness of blank,  $t=3\text{mm}$ .

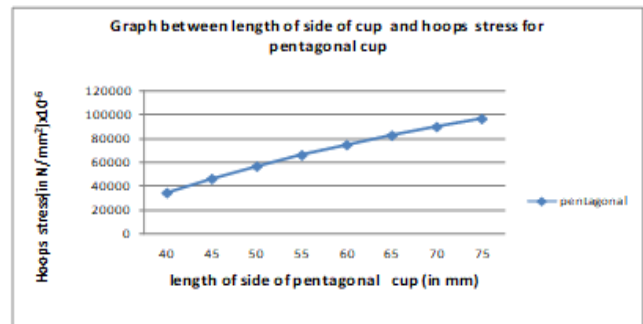
On substituting the above values hoop stress equation as follows

$$\sigma_h = 103 \times 10^6 \cdot \left[ 1 - \ln\left(\frac{80}{S}\right) \right] + 49 \times 10^3 \cdot \frac{(80-S)}{t \cdot \tan\left(\frac{180}{n}\right)} \dots (11)$$

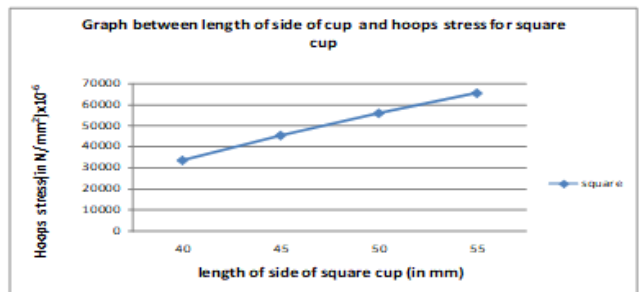
Determination of the values of hoop stresses inAA7075 aluminium alloy for different values is sides of blank from cup size to blank size by following eq.(11)



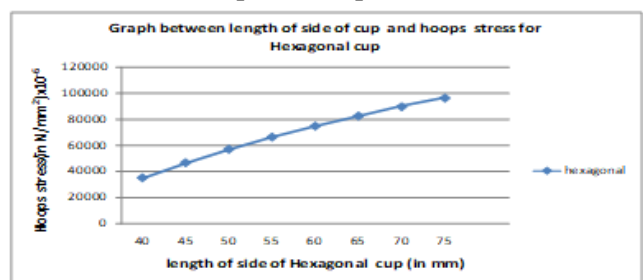
**FIG3: Variation between length of side of triangular cup and Hoop stress**



**FIG4: Variation between length of side of square cup and Hoop stress.**



**FIG5: Variation between length of side of pentagonal cup and Hoop stress**



**FIG6: Variation between length of side of hexagonal cup and Hoop stresses**

The hoop stresses developed in the flange region of aluminium alloy material presented in from fig.3 to fig.6 for triangle, square, Pentagon and hexagon all cups in deepdrawing. Variation of hoop stress values are the function of process parameters. The blank thickness considered as 3 mm. The variation blank side lengths with respect you to job axis taken from s=40 to maximum blank size 80mm. Due to the hydraulic fluid pressure shear stress developed in the flange will be directly proportional to the hoop stress.From the fig.(3) to fig.(6), we can say that, the maximum hoop stress occur for hexagonal cup then pentagonal, square and triangular respectively. As the side length of flange increases from 40mm to 80mm. The variation of hoop stress in the deepdrawing of cup through hexagonal flange will be from  $35 \times 10^{-3} \text{ N/mm}^2$  to  $103 \text{ N/mm}^2$ , for pentagonal flange  $34.303 \times 10^{-3} \text{ N/mm}^2$  to  $103 \text{ N/mm}^2$ , for a square flange  $33.565 \times 10^{-3} \text{ N/mm}^2$  and for triangular flange  $32.737 \times 10^{-3} \text{ N/mm}^2$  to  $103 \text{ N/mm}^2$ . The maximum hoop stress will be developed for each and every blank of different n-sided polygonal cup deepdrawing taking place at the periphery of the blank, it will be equal to yield strength of the material. The minimum hoop stress will be generated at the bent region of neck or at the start of the side of the cup.



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## VI. CONCLUSIONS:

- 1.As the number of sides of blank increases hoop stress developed increases in the flange.
2. As the side of the flange increases to blank size the hoop stress generated increases linearly up to maximum value of yield stress of the material.
- 3.The hoops stresses generated in the hydroform deepdrawing process are responsible to increase the formability of the blank material in the flange region.

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## AUTHORS PROFILE:



**Mr. G KISHAN**, Faculty of mechanical engineering,University College of Engineering& Technology for Women, Kakatiya University Warangal. He got his M.E (CAD/CAM) from Osmania University, Hyderabad, His research area of interest are Metal working Processes,Natural fiber composites, CAD, CAM, FEA, Solid modeling Digital Manufacturing and

multidisciplinary approach. He has 6 years experience in teaching of Engineering Mechanics, strength of materials and Engineering Drawing,He published 7 papers in reputed journals,



**Dr. BVS Rao** is presently working as Assistant Professor in Mechanical Engineering Department of CBIT He has completed his **M Tech in Advanced Manufacturing Systems (AMS)** from JNTU Hyderabad, and did**Ph.D** from the same University in the area of **Deep Drawing**. He has **26 years** of experience in Industry and teaching put together. He

worked as Production In charge in Hyderabad Precision Manufacturing Pvt Ltd for 2 years, then as **Assistant Director in ANDHRA PRADESH PRODUCTIVITY COUNCIL** for 7 years. He then entered into teaching and joined **CBIT** as Assistant professor. He has published 20 Technical Papers in various National and International Journals in the areas of Manufacturing like Metal Forming (Deep Drawing), welding & casting and Industrial engineering etc . He has presented 17 technical papers in National and International conferences, in the same areas as indicated above. His area of interest and **research work** is in **Metal forming, Casting, Welding , CAD/CAM and Industrial Engineering. He got Best papers**, in the National Conference at AVC College of Engineering, Mayiladuthurai (TN), and at the National Conference On Advances In Mechanical Engineering held at CBIT , in 2013.